Dynamical Effects of Weak Magnetic Fields

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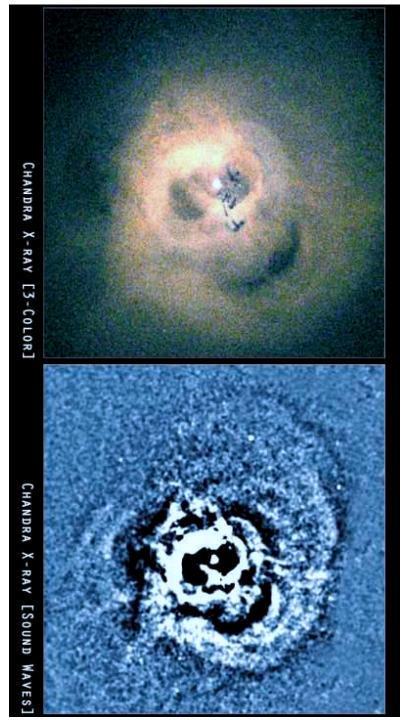
Motivation

- How is matter added to & lost from clusters?
- How is material cycled into & out of galaxies?
- How is ICM gas mixed, heated, & cooled?
- How viscous is the ICM?

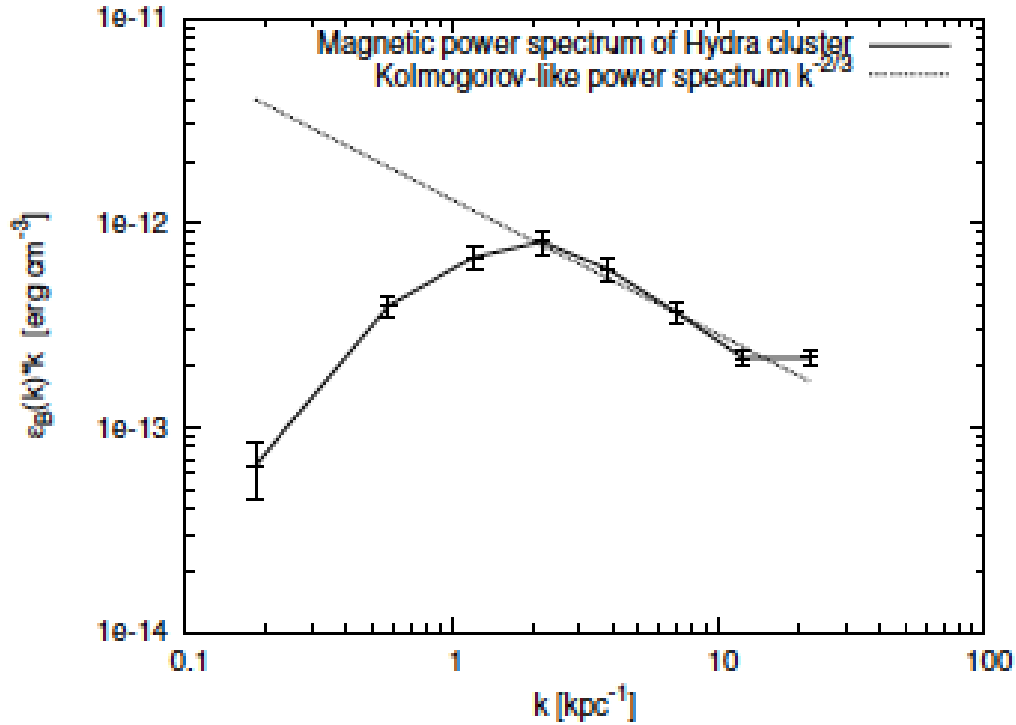
What is the role of the ICM magnetic field in addressing these questions?

Example: Bubbles



X-ray cavities in the Perseus cluster. How do such structures transfer heat to the gas, and what stabilizes them?

Magnetic Field Properties



Magnetic power spectrum in Hydra (Vogt & Ensslin 2005). Cluster fields are weak (high β) & tangled. This talk focusses on *mesoscale* aspects in contrast to micro & macroscales.

Effects of Tangled B

- Thermal conduction (widely studied)
- Stresses (weak due to high β, difficult to evaluate rigorously)
- Couple cosmic rays to thermal gas
- Contribute to dissipation
  - Anisotropic plasma viscosity
  - Cosmic ray viscosity
  - *Turbulent magnetic viscosity*
  - Collisionless damping (collaboration with Vladimir Mirnov)

Turbulent Magnetic Viscosity

- Turbulence model of Ryutov & Remington
- Dynamical equations with this turbulence model
  - Dissipation leads to an effective viscosity
- Evaluate for ICM plasma
- Examples
  - Waves
  - Rayleigh-Taylor instability

Turbulence Model

- Irregular magnetic field B on scale $\leq l$.
- B relaxes to its minimum energy state, consistent with constraints such as global helicity, on timescale τ.
- Response to a flow $\delta\mathbf{V}$:

$$\frac{\partial \delta \mathbf{B}}{\partial t} = \nabla \times (\delta \mathbf{V} \times \mathbf{B}) - \frac{\delta \mathbf{B}}{\tau}.$$

- Relaxation *ansatz* replaces scale dependent resistive term $\nabla^2 \delta \mathrm{B}$ with scale independent term $-\delta \mathrm{B}/\tau$.
- We'll assume $\delta\mathbf{V}$ varies on a scale $L \gg l$

Turbulent Stress

- The perturbation $\delta\mathbf{B}$ induces

$$\delta\mathbf{F} = -\nabla \cdot \delta\mathbf{T}$$

- Perturbed stress tensor

$$4\pi\delta T_{ij} = \delta_{ij}\mathbf{B}\cdot\delta\mathbf{B} - B_i\delta B_j - B_j\delta B_i$$

- Find turbulent stress on large scale L by averaging over l.

Homogeneous, Isotropic Turbulence

$$\langle B_i B_j \rangle = \frac{1}{3} \langle B^2 \rangle \delta_{ij}$$

$$\left(\frac{\partial}{\partial t} + \frac{1}{\tau} \right) \langle \delta T_{ij} \rangle = -\frac{1}{3} \frac{\langle B^2 \rangle}{4\pi} \left(\frac{\partial \delta V_i}{\partial x_j} + \frac{\partial \delta V_j}{\partial x_i} \right)$$

Similar but messier for anisotropic turbulence. Note that the RHS has the form of a strain tensor. The ∂_t part is dynamical and the τ^{-1} part is viscous.

Waves with HIT

There are 2 wave modes (3 if we add pressure):

- Shear waves: $\omega = kv_A/\sqrt{3} + \frac{i}{2\tau}$
- Compressive waves: $\omega = \sqrt{\frac{2}{3}}kv_A + \frac{i}{2\tau}$

Compare to a uniform field

- Shear waves: $\omega = \mathbf{k} \cdot \mathbf{v}_A$
- Compressive waves: $\omega = kv_A$

Relaxation dissipates wave energy (transferred where?). These expressions assume $kv_A\tau \gg 1$; if $kv_A\tau \ll 1$ the waves can't propagate.

Viscosity from HIT

When $\omega\tau \sim \tau/\tau_{dyn} \to 0$,

$$\langle \delta T_{ij} \rangle = -\frac{\tau v_A^2}{3} \left(\frac{\partial \delta V_i}{\partial x_j} + \frac{\partial \delta V_j}{\partial x_i} \right)$$

Coefficient of viscosity ν_T

$$\nu_T \equiv \frac{1}{3} v_A^2 \tau$$

Magnitude of Viscosity

Compare magnetic & plasma viscosities:

$$\frac{\nu_T}{\nu_p} \sim \frac{v_A^2 \tau}{v_i^2 \tau_i}.$$

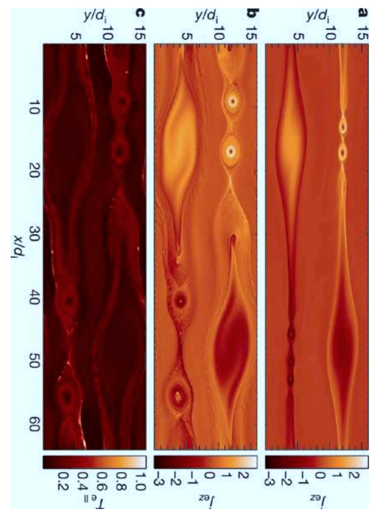
Define $\epsilon \equiv l/(v_A \tau)$; then

$$\frac{\nu_T}{\nu_p} \sim \frac{1}{\epsilon} \frac{v_A}{v_i} \frac{l}{\lambda_i}.$$

If l = 3 kpc, $\nu_T/\nu_p \sim 10^{18} n_i T^{-2} \epsilon^{-1} v_A / v_i$. Elephant in the room: what to use for plasma viscosity? Perpendicular to local B, it's reduced by 1-2 powers of $\omega_{ci}\tau \gg 1$. But it seems ν_T competes well with the largest possible ν_p.

Relaxation Process

What is relaxation process? Most likely, collisionless magnetic reconnection at some fraction of v_A ($\epsilon \sim$ 0.01 - 0.1).



Note that reconnection heats both electrons & ions; viscosity heats mainly ions.

Rayleigh-Taylor Instability

Review of classic setup: sharp interface separating ρ_2 from ρ_1 below. Ripple interface as e^{nt+ikx}. With no B or ν,

$$n^2 = \gamma_{RT}^2 \equiv kg\frac{\rho_2 - \rho_1}{\rho_2 + \rho_1}$$

- Laminar magnetic field $\hat{x}B$ stabilizes perturbations with $k > g\Delta\rho/B^2$, B_y has no effect.
- With viscosity, fastest growth rate is at $k \sim (g/\nu^2)^{1/3}$

Add Turbulence

Assume 2D turbulence is present, no B threading the interface. Derive the dispersion relation

$$n^2(n\tau + 1) + n\tau(\gamma_A^2 - \gamma_{RT}^2) - \gamma_{RT}^2 = 0$$

where $\gamma_A^2 \equiv k^2 B^2/(4\pi(\rho_2 + \rho_1))$.

- Without relaxation, modes with $\gamma_A^2 - \gamma_{RT}^2$ are stable (no directional effect)
- With relaxation, $n_1 n_2 n_3 = \gamma_{RT}^2/\tau$: instability for $\rho_2 > \rho_1$.
- With short relaxation time, fastest growing mode has $k \sim (g/\nu_T^2)^{1/3}$.

Summary

- The RR model of turbulence invokes scale independent relaxation on a timescale τ
- Included this in a standard derivation of averaged magnetic stress.
- Without relaxation, isotropic turbulence supports shear & compressive waves; 2D turbulence stabilizes RT instability at small enough scales.
- Relaxation introduces a viscous response with viscosity $\nu_T = v_A^2 \tau / 3$ & destabilizes RT modes, similar to classical viscosity.
- Can be added to large scale calculations without explicit small scale modeling; just characterization of turbulent spectrum.