The role of stellar dynamics in bringing massive black holes close together

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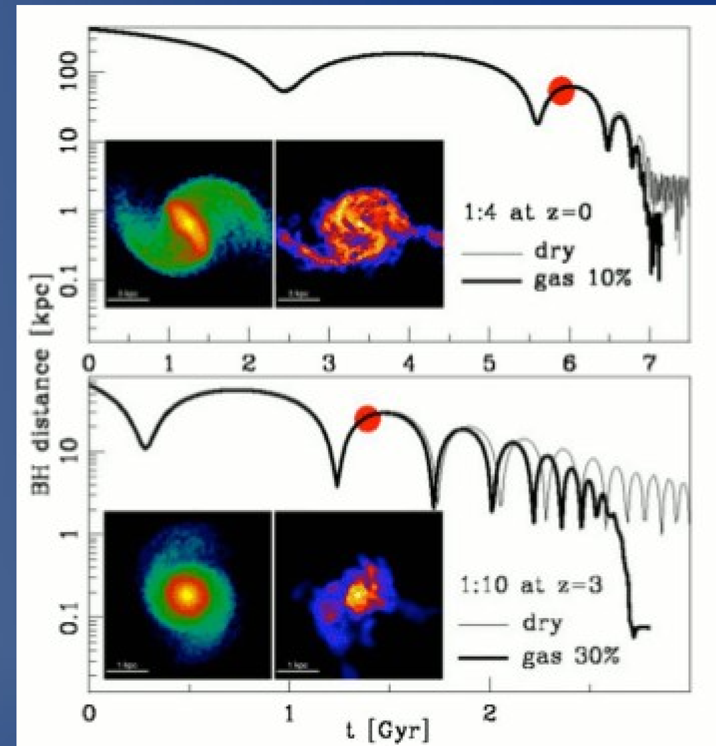
Do MBHs form gravitationally bound binaries?

q	SF	f_g	z	BH final distance[a]
0.25	No	0	0	2–4 kpc
0.25	Yes	0.1	0	200 pc
0.1	No	0	3	1–6 kpc
0.1 (Hi-Res)	No	0	3	1–5 kpc
0.1	Yes	0.1	3	400 pc
0.1	Yes	0.3	3	70 pc



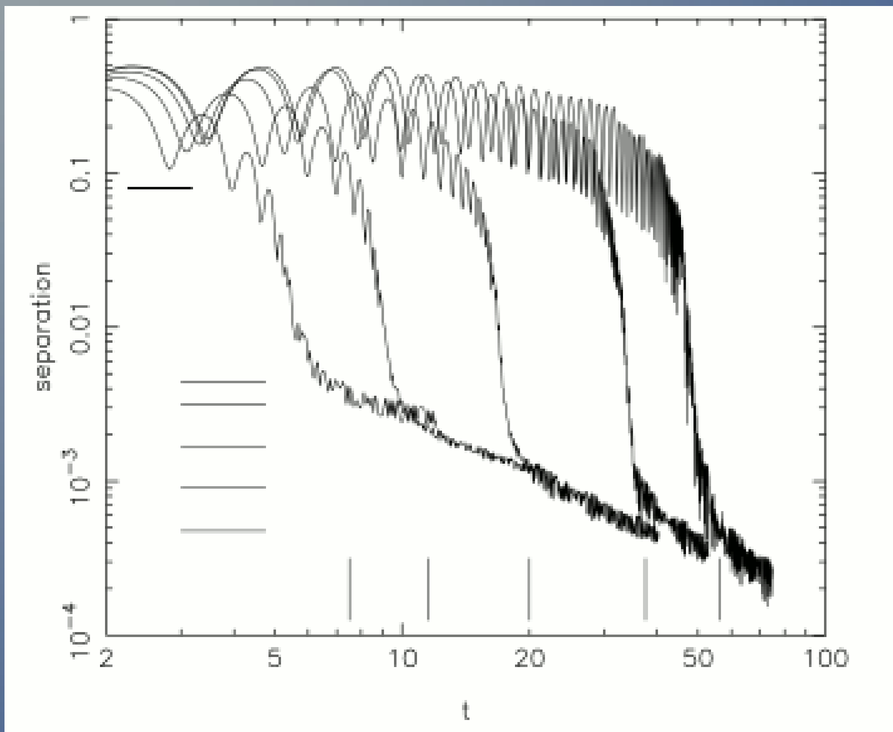
-- Yes, if it results from a nearly equal mass merger
-- Yes, if the merger is sufficiently gas-rich

If a binary with $M_{BH} = 3 \times 10^6\ M_S$ reaches a separation of ~ 100 -200 pc, then it can become bound in ~ few X 10^7 yr by dyn. friction

Callegari et al 2010, 2011

$$T_{DF} \sim 3.5\, x\, 10^8\ \mathrm{yr} \left(\frac{5}{\ln\left(M_{TOT}(<R_c)/M_{BH} \right)} \right) \left(\frac{R_c}{300\ \mathrm{pc}} \right)^2 \left(\frac{V_c}{100\ \mathrm{Km/s}} \right) \left(\frac{10^6 M_{Sun}}{M_{BH}} \right) \epsilon^{0.4}$$

Dynamical Friction efficient down to a_h ~ 0.1 - 1 pc scales



Merritt 2006

Hard Binary Radius

$$a_h := \frac{G\mu_r}{4\sigma^2} \sim \frac{q}{4(1+q)^2} r_h$$

$$0.1\,\mathrm{pc}\,\frac{1}{q+1}\left(\frac{M_2}{10^6 M_S}\right)\left(\frac{100\,\mathrm{Km/s}}{\sigma}\right)^2$$

GW emission shrinks the orbit on timescale:

$$T_{GW} \sim 5\times10^{10}\,\mathrm{yr}\,\frac{q^3}{(1+q)^6}\left(\frac{10^6 M_S}{M_{12}}\right)^{0.65}\left(\frac{a}{0.01\,a_h}\right)^4$$

So for an equal mass binary, stars must shrink it by ~ 50 times below $a_{h,}$ but eccentricity can help!

Binary evolution in a *fixed* stellar background

From 3-body scattering experiments (Quinlan 96), the binary hardening rate can be derived

$$\frac{1}{a(t)} - \frac{1}{a_h} = H \frac{G\rho}{\sigma}(t - t_h), \text{ for } \; t \geq t_h, \; \text{ and } \; a \leq a_h$$

The resulting coalescence time would be

$$T_{coal} \sim 5 \times 10^5 \,\text{yr}\, q\,(1+q)^2 \left(\frac{\sigma}{200\,\text{Km/s}}\right)^3 \left(\frac{10^3\, M_s\, pc^{-3}}{\rho}\right)\left(\frac{10^8\, M_S}{M_{12}}\right)$$

If background properties remained unchanged during inspiral, coalescence of MBHBs would be prompt; but stars are ejected on the (local) dynamical time and resupply of stars is not as fast

Only stars that cross the binary can drive its inspiral, these are the stars that belong to the loss cone orbits. The mass in these stars is

$$M_{lc}(a) = m_* \int dE \int_0^{J_{lc}} dJ\, N(E, J^2), \text{ so } M_{lc}(a_h) \sim 3 f \mu_r$$

If all the stars in the loss cone interacted with the binary, and equating the average energy they carry away with the change in the binary's E_{bind}

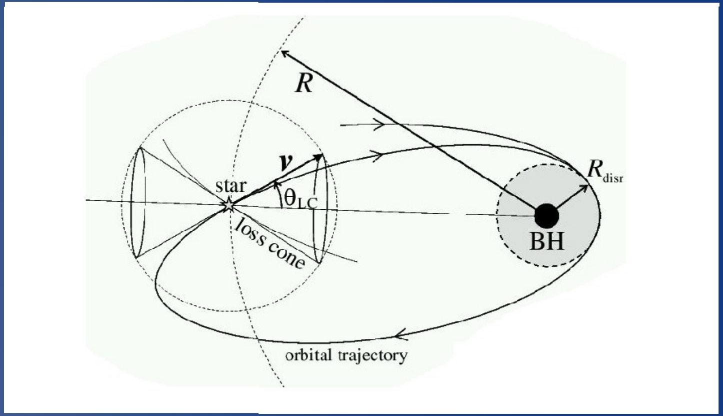
$$\frac{3}{2} \frac{G \mu_r}{a} dM \sim \frac{G M_1 M_2}{2} \frac{d}{dt}\left(\frac{1}{a}\right)$$

This means an equal-mass binary can shrink **at most by a factor of ~ 10** as it clears the loss cone orbits

$$\ln\left(\frac{a_h}{a}\right) \sim 9 f \frac{q}{(1+q)^2}$$

But this is an **overestimate** as its loss cone shrinks as well.

Refilling on loss cone by 2-body relaxation is too slow for $M_{12} > 10^6 M_S$



$$J_{lc} = \sqrt{G M_{BH} f a}$$

$f = O(1)$, a: binary's semimajor axis

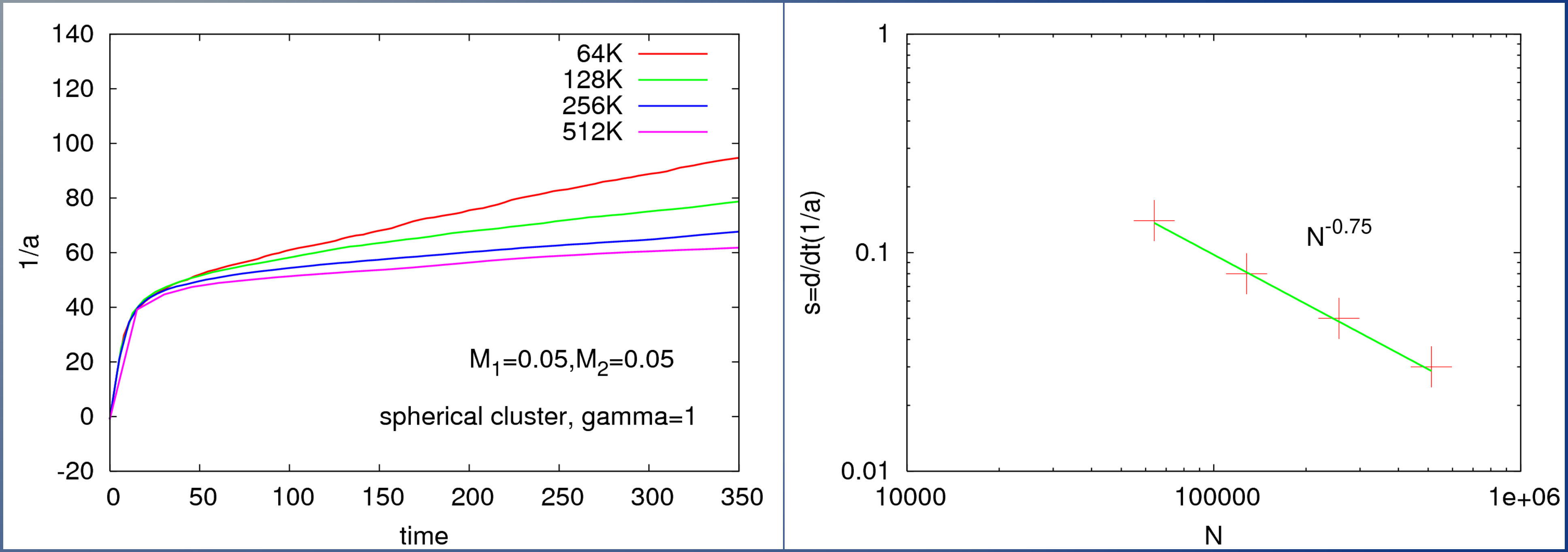
$$4\pi^2 p(E) \frac{\partial f}{\partial t} = -\frac{\partial F_E}{\partial E} - R_{\mathrm{lc}}(E,t)$$

$\boldsymbol{f(E,t)}$: phase-space density

$\boldsymbol{F_E(E,t)} = -D_{EE}\frac{\partial f}{\partial E} - D_E f$: flux of stars in energy space driven by two-body relaxation

$\boldsymbol{R_{lc}(E,t)} \approx \dfrac{N(E,t)}{\log(J_c/J_{lc})\, T_{rlx}(E)}$: flux of stars into the loss cone, and ejected with high-velocity through 3-body interaction

MBHB Stalling in a spherical nucleus

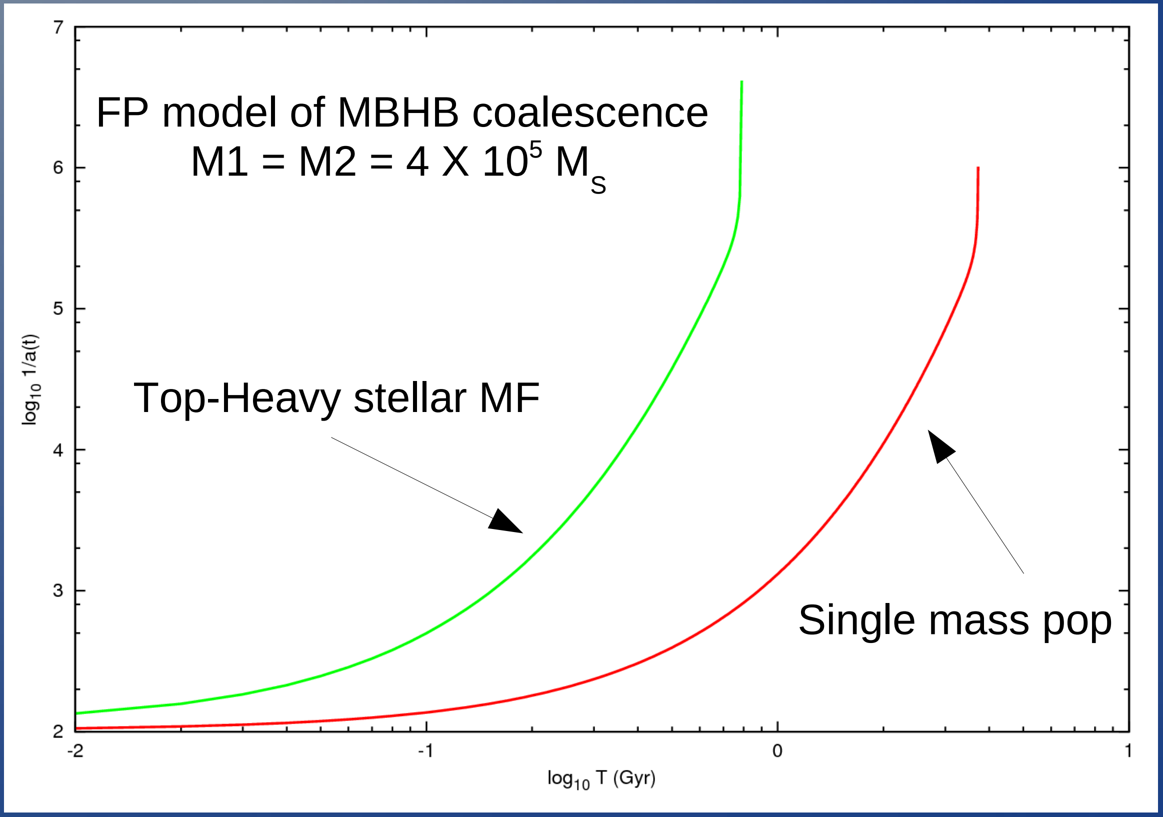


$$s(t)=\frac{d}{dt}\left(\frac{1}{a}\right) = \frac{2\langle C\rangle}{aM_{12}}\int dE\,R_{\rm lc}(E,t) \propto \frac{M_{\rm lc}(<r)}{T_{\rm rlx}(r)} \propto \frac{1}{N}$$

(Quinlan 96, Yu 02, Milosavljevic & Merritt 03, Makino & Funato 04, Berczik et al 05, Merritt et al 07)

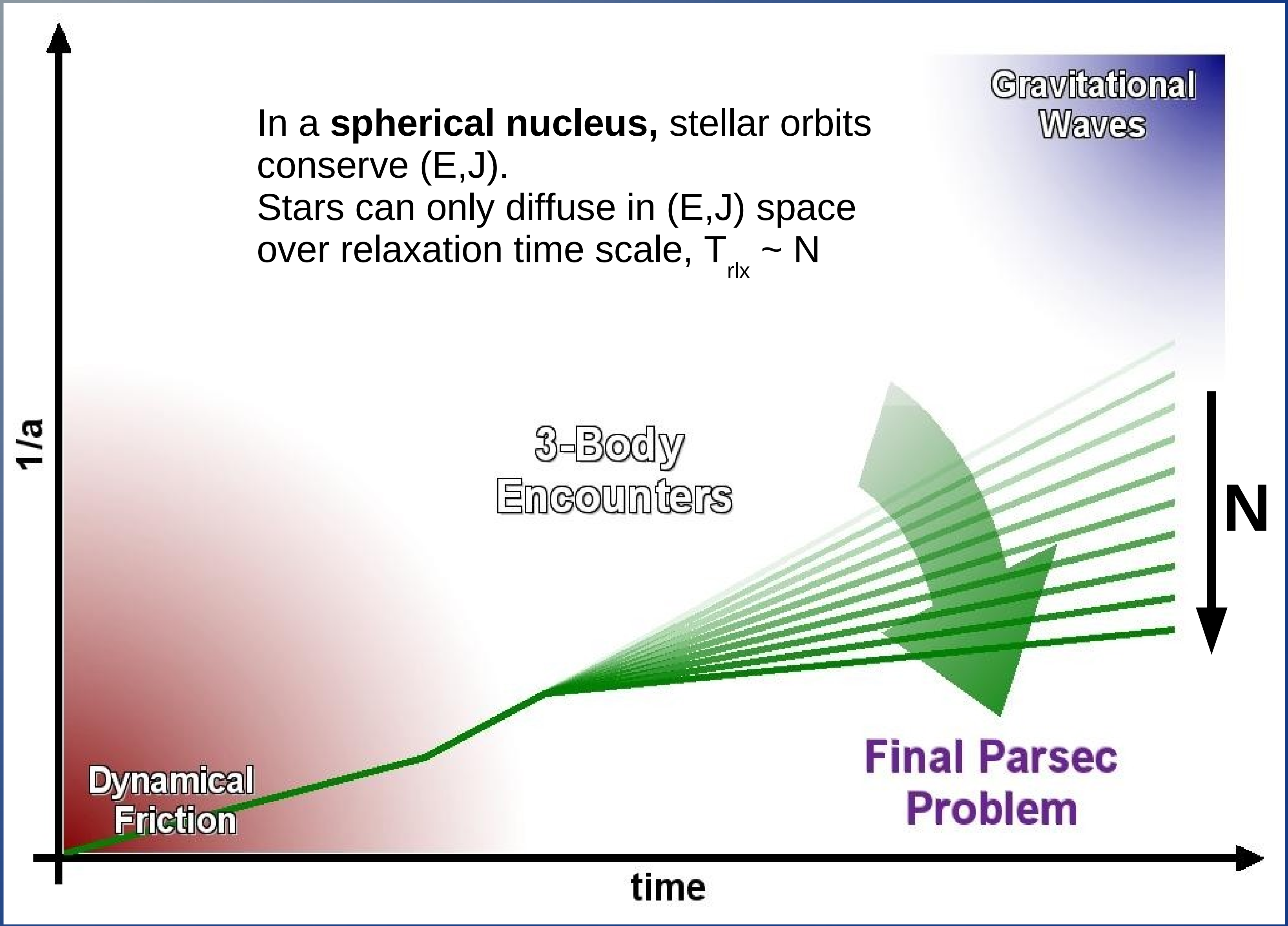
Binary hardens as stars diffuse into its loss cone:

$$\frac{d}{dt}\left(\frac{1}{a}\right) = -\frac{2mC}{M_{12}a}\int_0^{E_c} dE\, R_{lc}(E,t) + \frac{d}{dt}\left(\frac{1}{a}\right)_{GW}$$



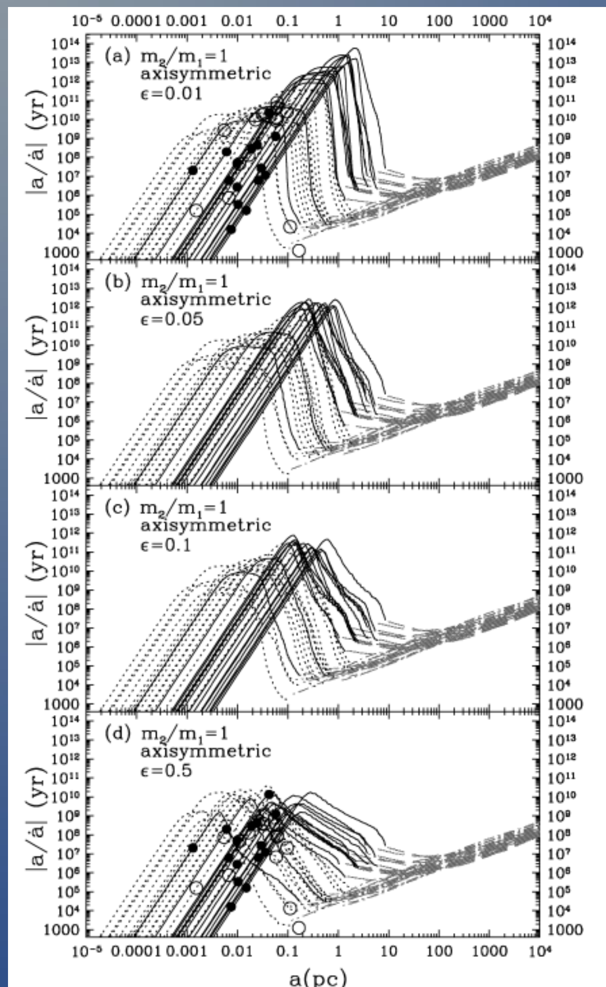
Binary Black Hole Evolution

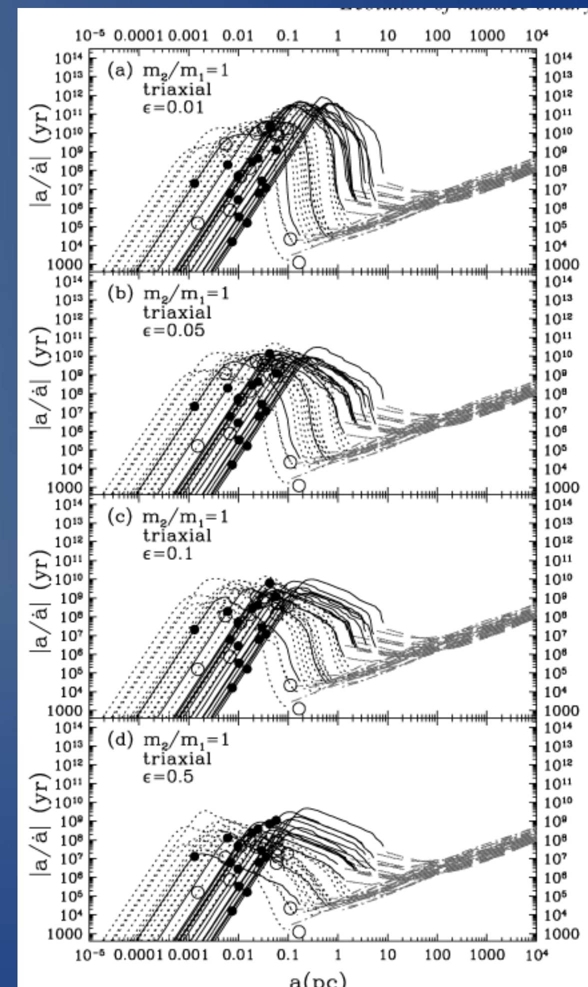
In a **spherical nucleus,** stellar orbits conserve (E,J).
Stars can only diffuse in (E,J) space over relaxation time scale, $T_{rlx} \sim N$



a_0~200 pc (Milky way)

From the HST sample of galaxies from Faber et al 1997, earlier Yu 2002 pointed out that **flattening and non-axisymmetries** would help stellar dynamics in bringing MBHBs to coalescence





Evolution bottleneck is less severe and $T_{coal} < T_H$ for:

-- galaxies with low velocity dispersion, $\sigma_e \leq 90$ Km/s

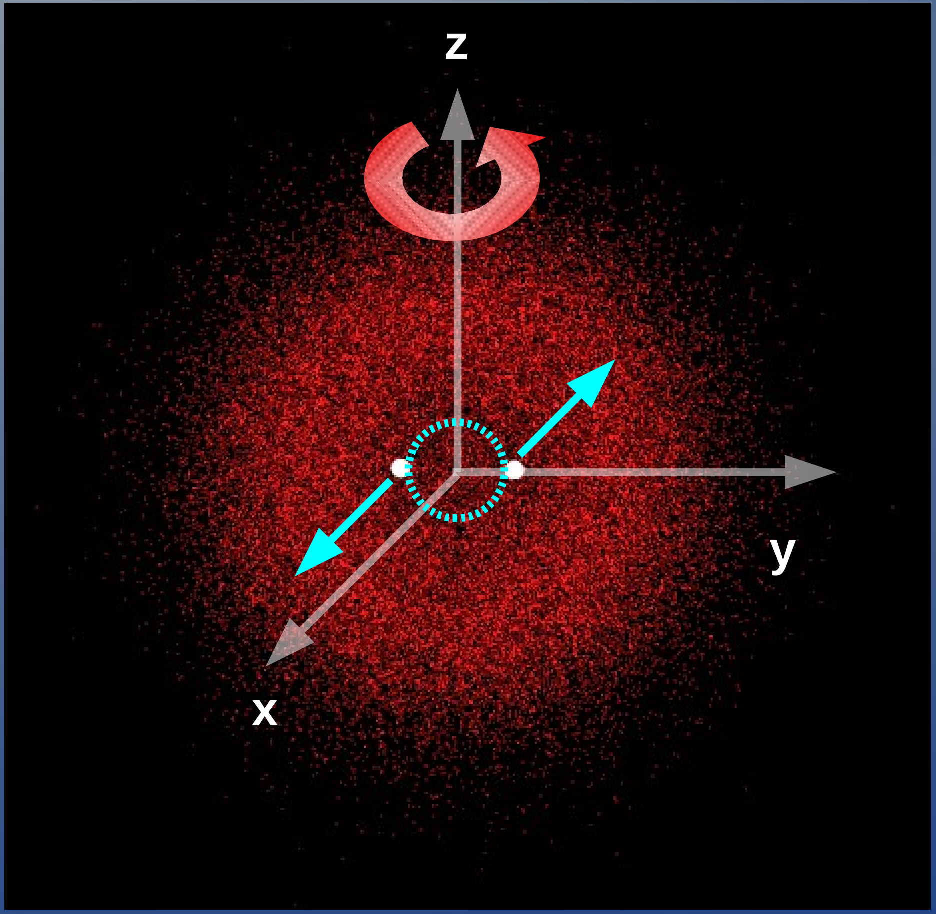
-- highly flattened galaxies, $\epsilon \geq 0.5$

-- or mildly triaxial $T \geq 0.05$ galaxies

Yu 2002

Initial Conditions

Schematic view



Rotating King Model

$$f(E, L_z) = \text{const} \times \left(e^{-\beta E} - 1\right) e^{-\beta \Omega_0 L_z}$$

$$E = \frac{1}{2} v^2 + \Phi(R, z),\ J_z = R v_\phi$$

Rotation parameter:

$$\omega_0 = \sqrt{9/(4\pi G \rho_0)}\, \Omega_0 = 0.3, 0.6, 0.9, 1.2, 1.8$$

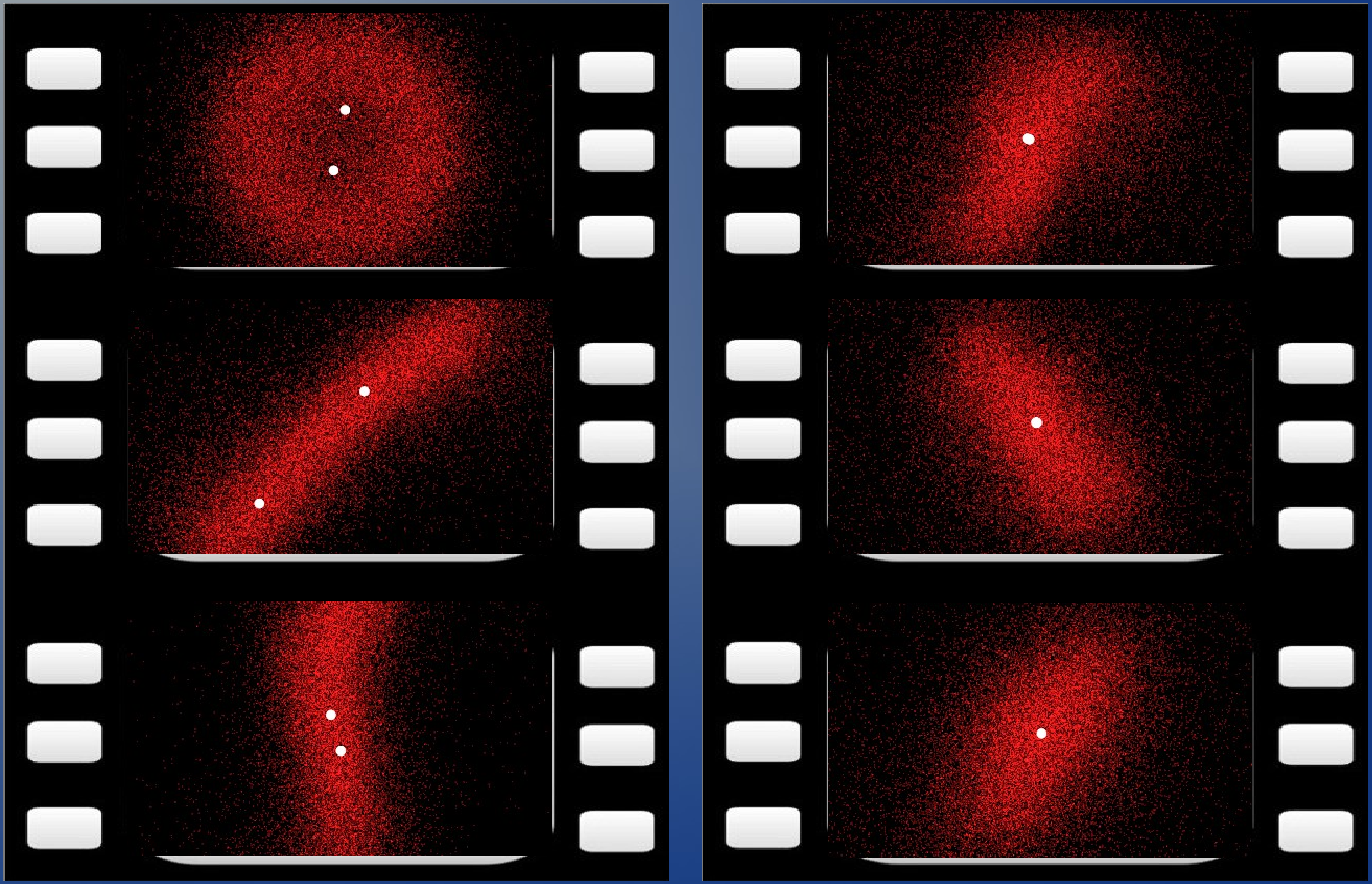
$$M_1 = M_2 = 0.01\, M_{cl}$$

$$Y_1 = -Y_2 = 0.3,\ \textit{with circular velocity}$$

Particle number: ¿ $N = (0.025 - 1.0) \times 10^6$

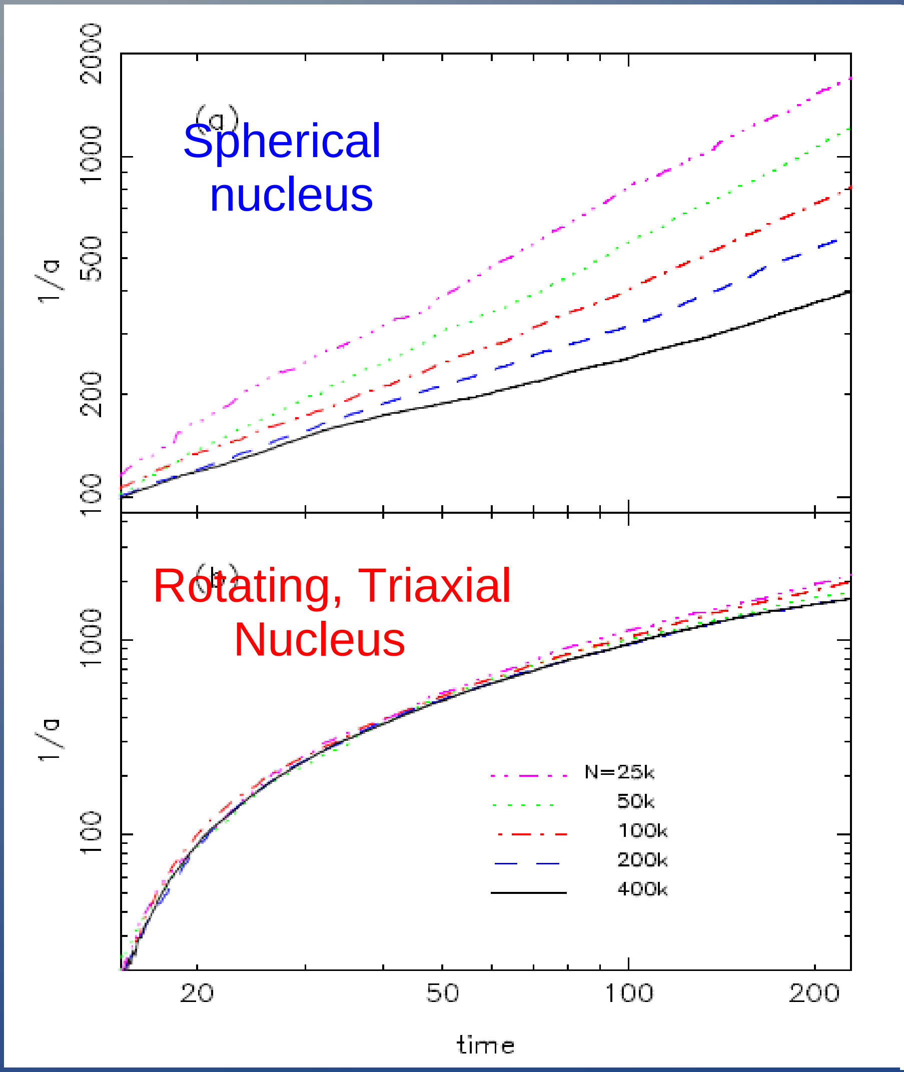
Berczik et al 2006

Rotating triaxial stellar cluster

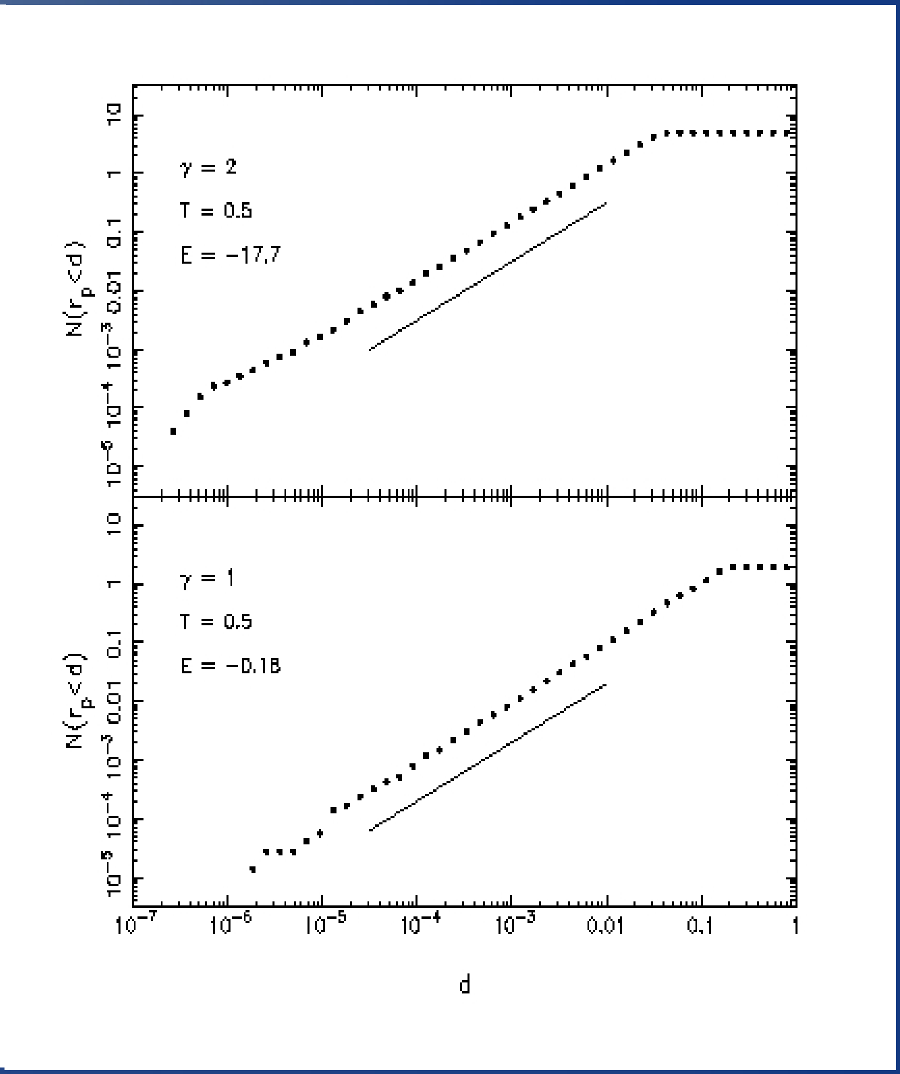


Berczik et al 2006   Berentzen, MP et al 2009

TRIAXIAL GALACTIC NUCLEI: NO STALLING

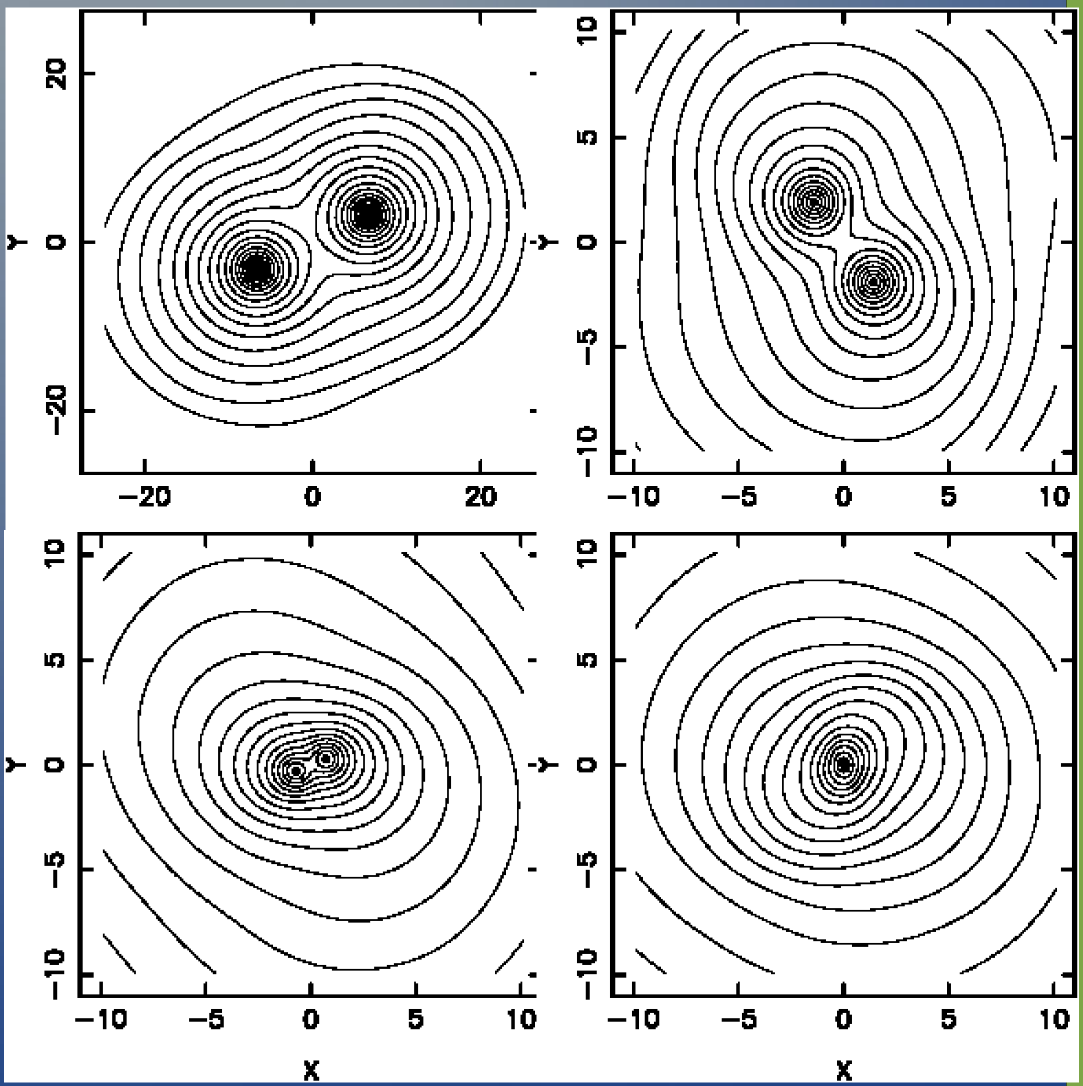


Berczik et al. 2006



Merritt & Poon 2004

Models and initial conditions for mergers



Our Models:

-- Spherical Models
-- Rotating, Triaxial Models
-- Merging Spherical Models

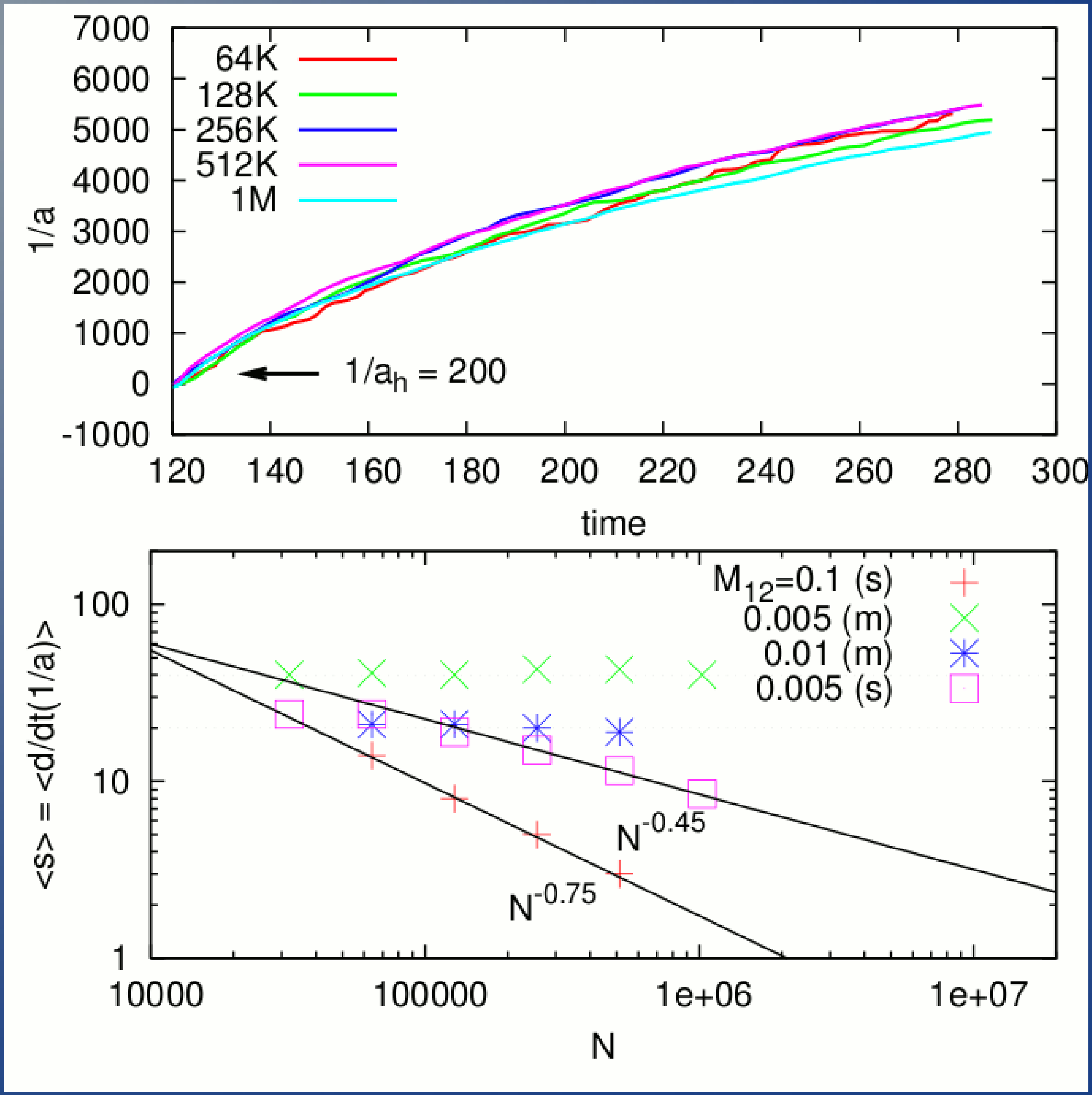
$M_{BH}/M_{nuc} = 0.001, \ldots, 0.1$

$q = M_{BH,2}/M_{BH,1} = 1/20, 1/10, 1/8, 1/4, 1/2, 1$

$N = 32\mathrm{K}, \ldots, 1\mathrm{M}$

$L/L_c = 0.15, 0.6, 1.0$

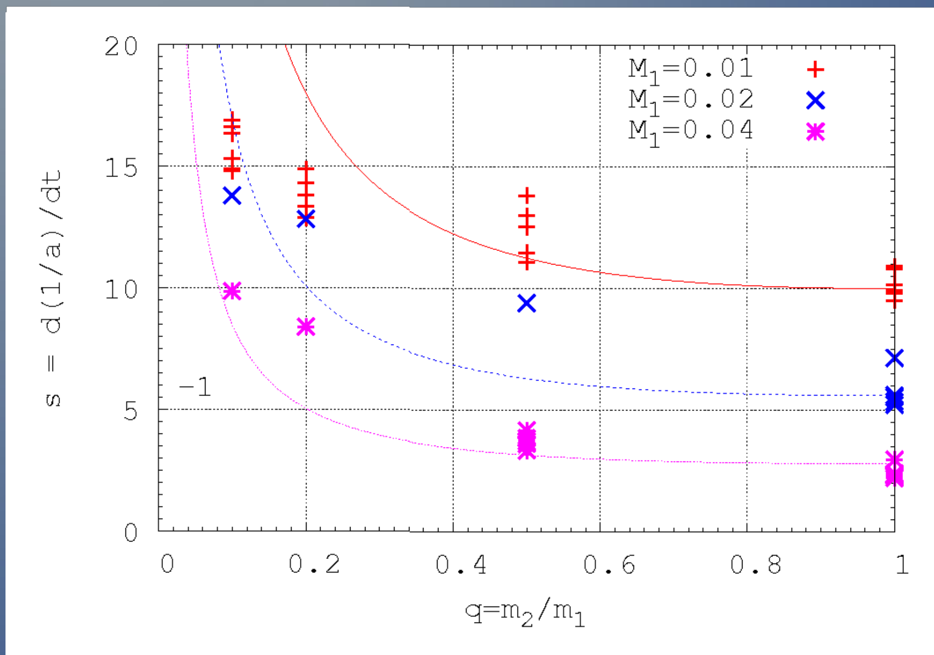
Equal mass mergers



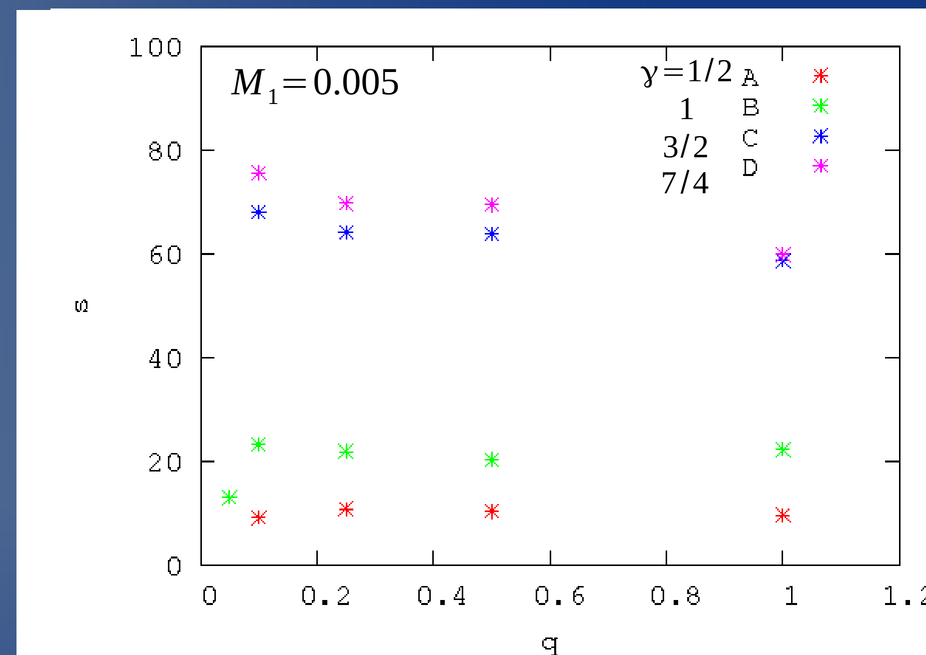
MP et al 2011 (also Khan et al 2011)

Unequal-mass mergers

Rotating Triaxial Nucleus



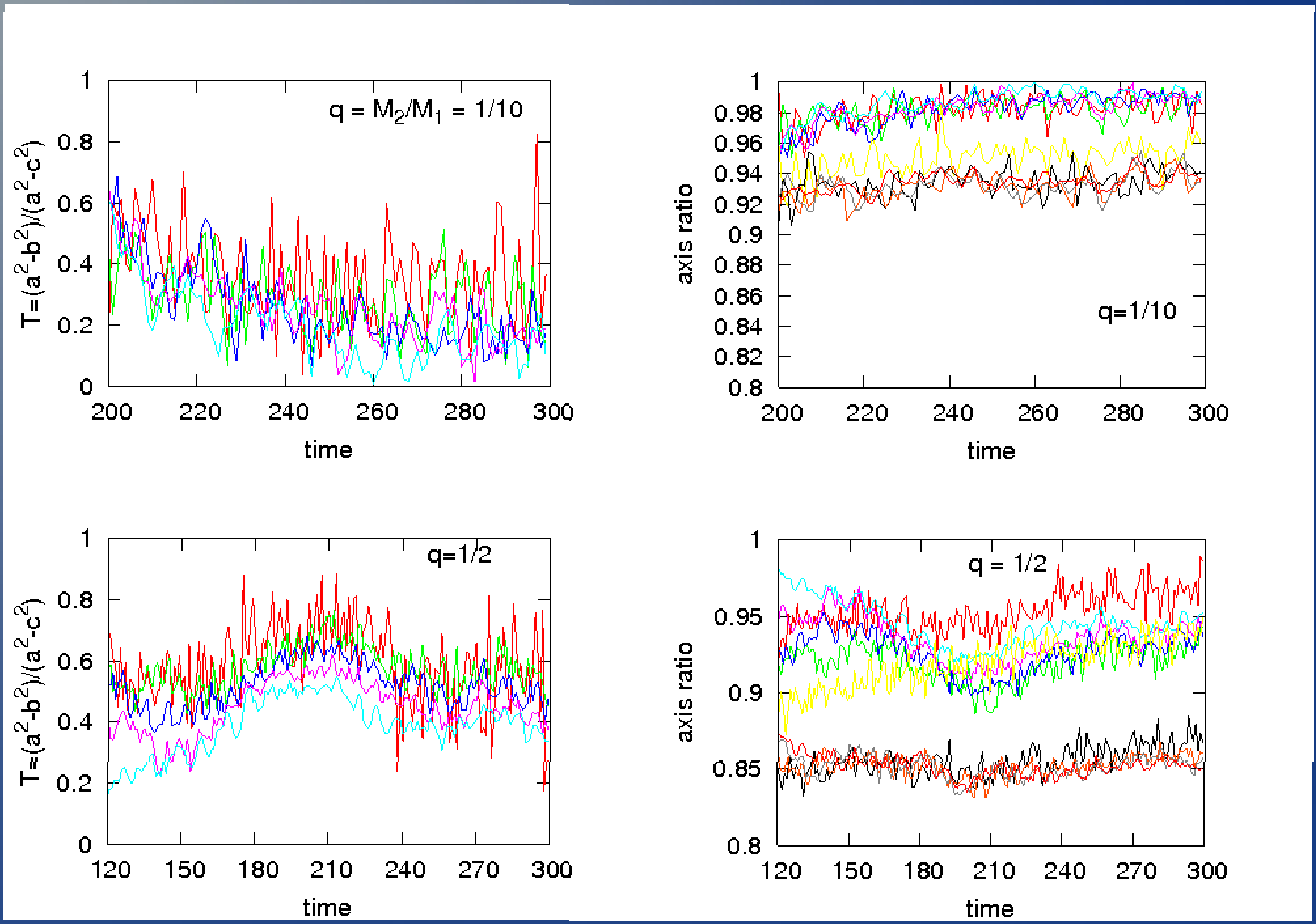
Merging Nuclei



$$\frac{d}{dt}\left(\frac{1}{a}\right)=\frac{8\pi G\rho_h}{\sigma_h}\frac{M_{12}}{4\mu}=\frac{2\pi G\rho_h}{\sigma_h}\frac{(1+q)^2}{q}$$

Khan, MP et al -- in prep

Merger Induced Triaxiality decreases with q



Khan, MP et al – in prep

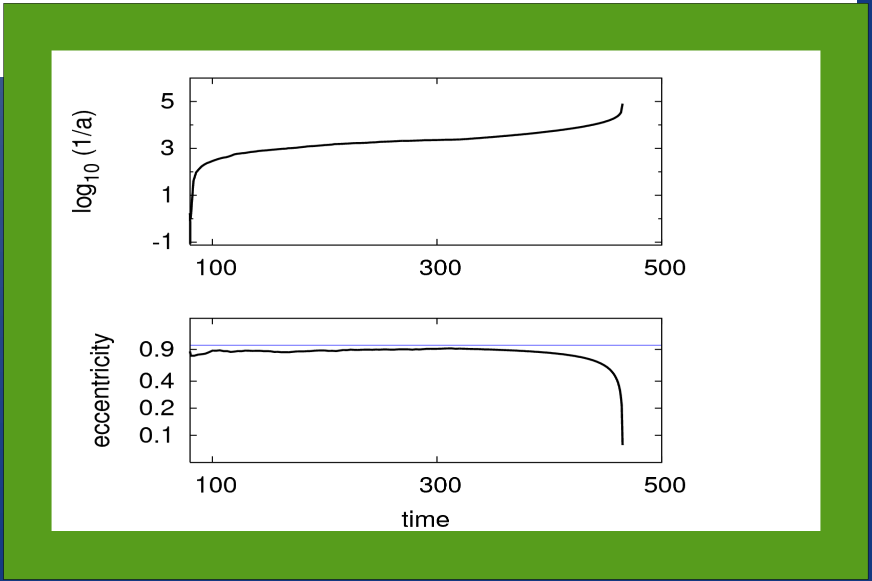
PN equations of motion in the NB simulation

$$\frac{dv^i}{dt} = -\frac{Gm}{r^2}\left[(1+\mathcal{A})\,n^i + \mathcal{B}\,v^i\right] + \mathcal{O}\left(\frac{1}{c^8}\right), \qquad (181)$$

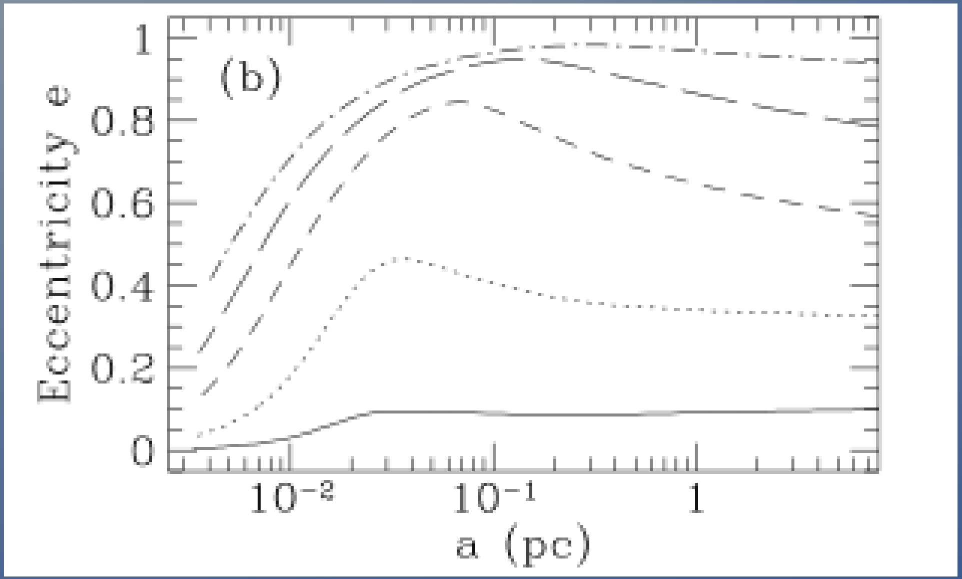
and find [43] that the coefficients $\mathcal{A}$ and $\mathcal{B}$ are

$$\mathcal{A} = \frac{1}{c^2}\left\{-\frac{3\dot{r}^2\nu}{2} + v^2 + 3\nu v^2 - \frac{Gm}{r}(4+2\nu)\right\}$$
$$+\frac{1}{c^4}\left\{\frac{15\dot{r}^4\nu}{8} - \frac{45\dot{r}^4\nu^2}{8} - \frac{9\dot{r}^2\nu v^2}{2} + 6\dot{r}^2\nu^2 v^2 + 3\nu v^4 - 4\nu^2 v^4\right.$$
$$\left.+\frac{Gm}{r}\left(-2\dot{r}^2 - 25\dot{r}^2\nu - 2\dot{r}^2\nu^2 - \frac{13\nu v^2}{2} + 2\nu^2 v^2\right) + \frac{G^2m^2}{r^2}\left(9 + \frac{87\nu}{4}\right)\right\}$$
$$+\frac{1}{c^5}\left\{-\frac{24\dot{r}\nu v^2}{5}\frac{Gm}{r} - \frac{136\dot{r}\nu}{15}\frac{G^2m^2}{r^2}\right\}$$

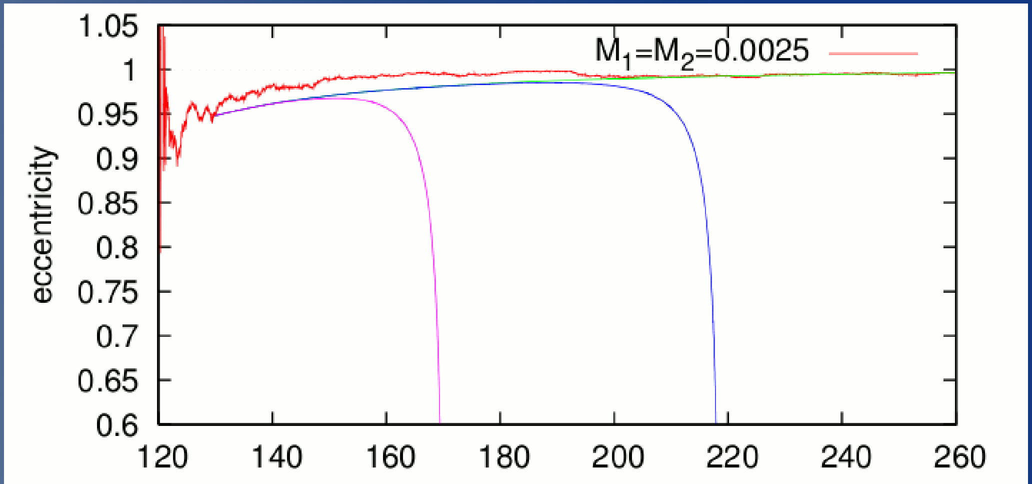
Blanchet 2006

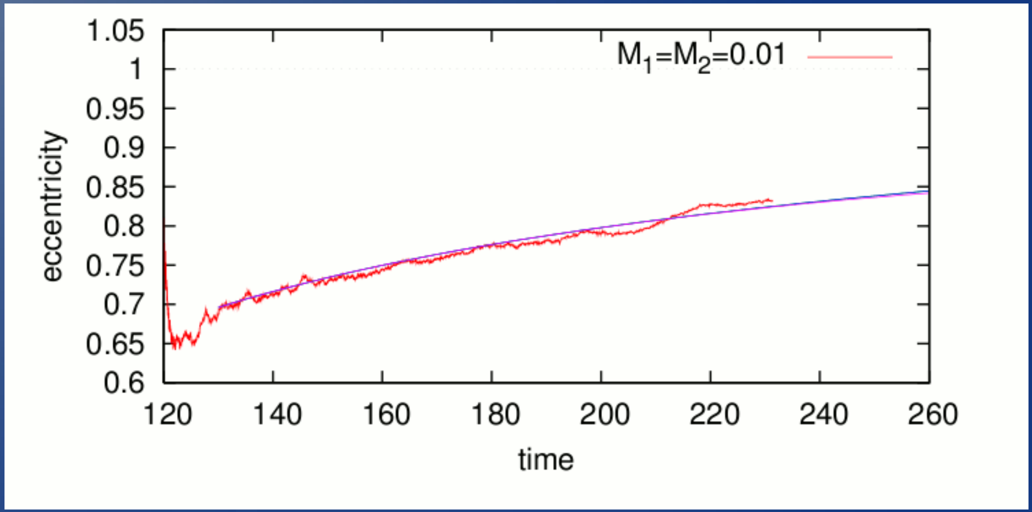


Eccentricity evolution due to unbound stars



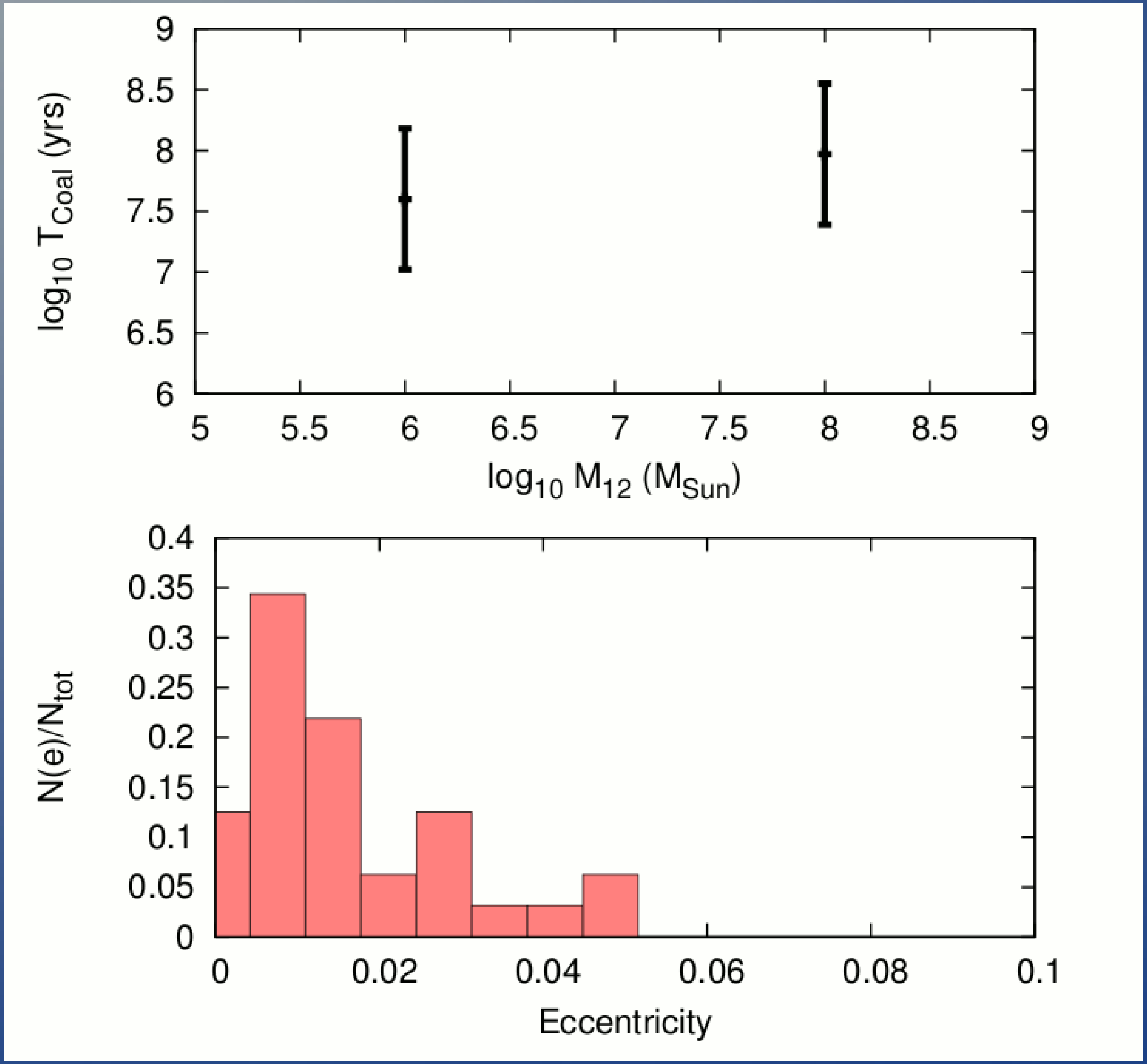
Quinlan 1996





MP et al 2011

Equal Mass Mergers: Coalescence Time and Eccentricity Evolution



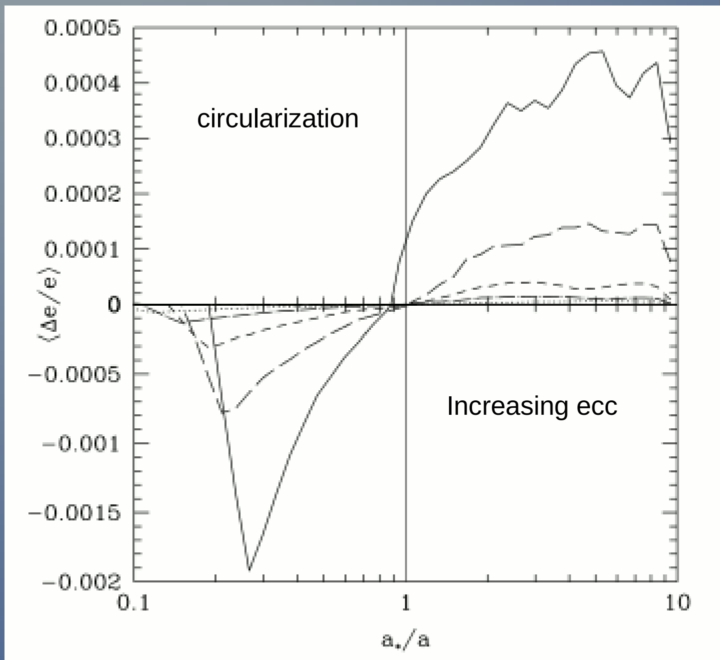
Coalescence times are well below a Hubble time.

Binary of $M_{12} = 10^6\ M_{sun}$ at separation of 100 R_S

Binaries will reach the LISA band with some residual eccentricity

MP et al 2011

Eccentricity evolution due to bound stars when q < 1



Sesana et al 2008

-- Spherical model with isotropic velocities

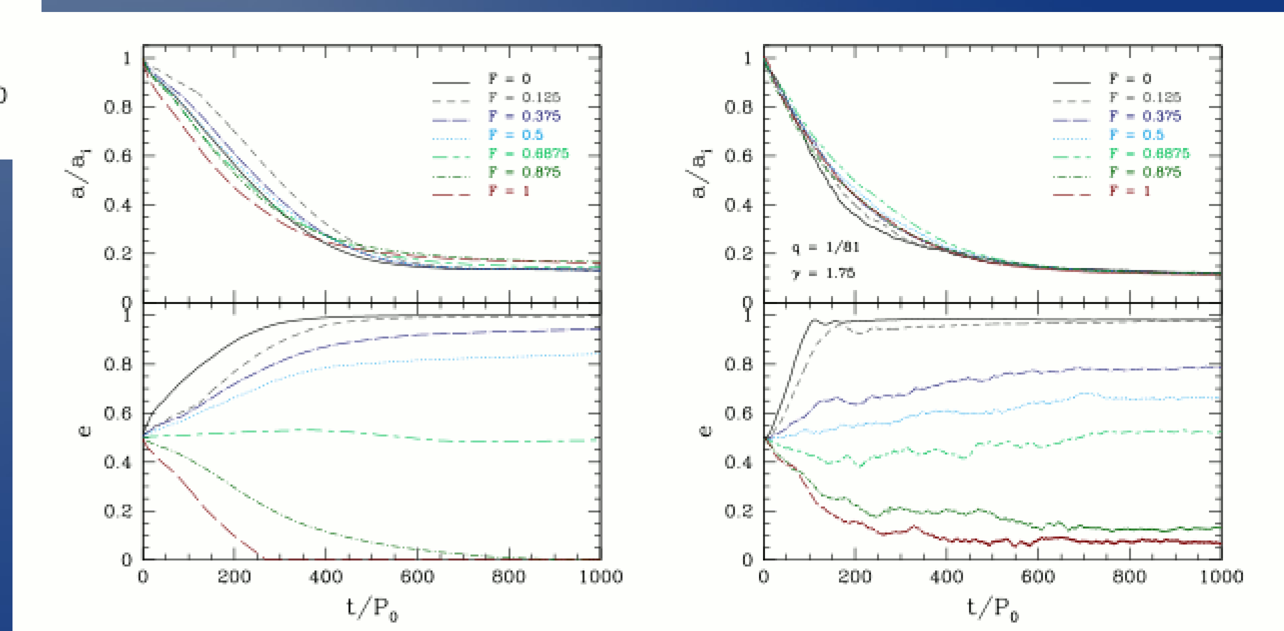
-- Reverse the sign of all (v_x, v_y, v_z) for a fraction F of stars:

$$F\,N_*(\text{co-rot}) + (1-F)\,N_*(\text{counter-rot})$$

-- Small mass ratio, q=1/81

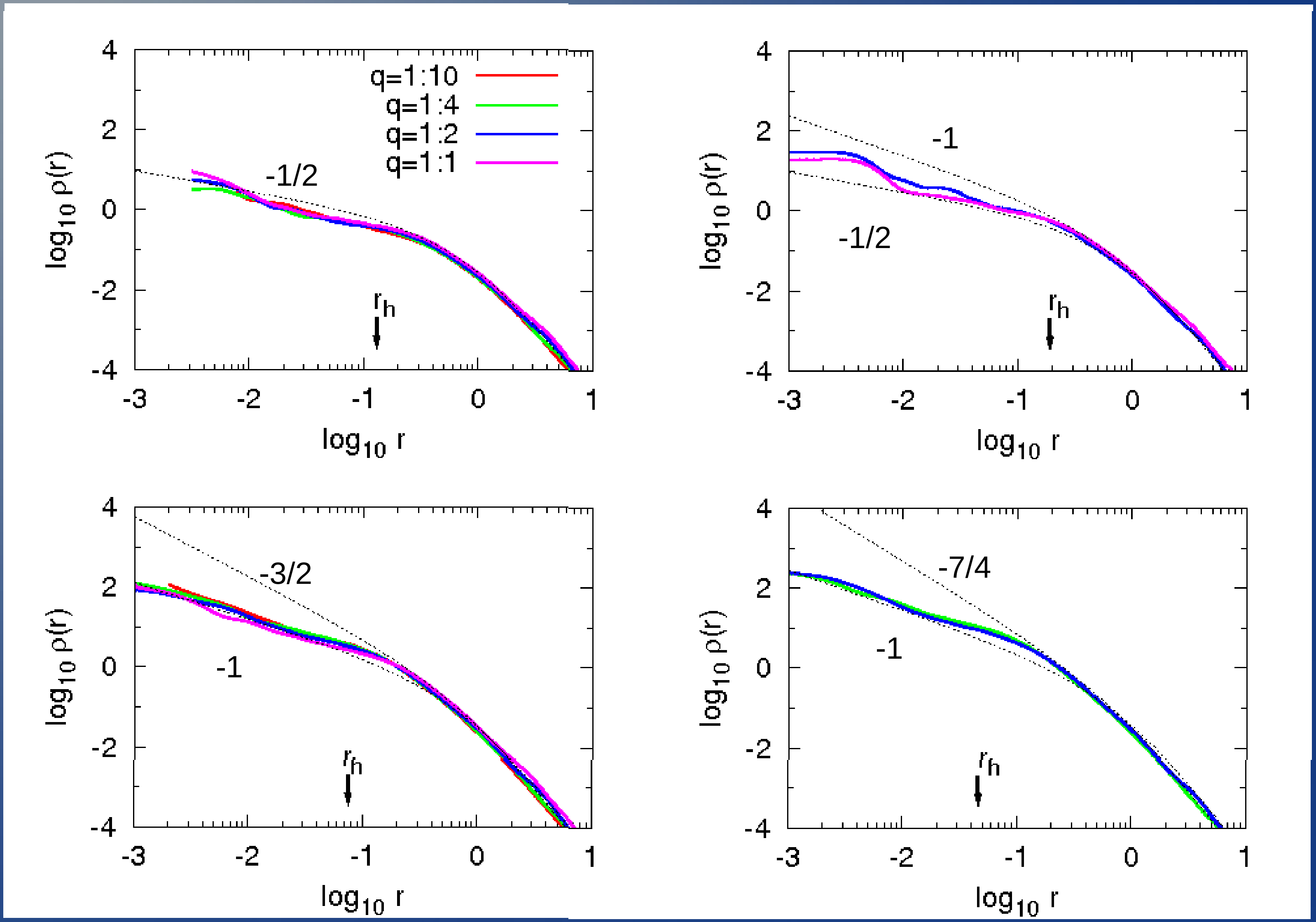
3-body scattering

N-body



Sesana et al 2011

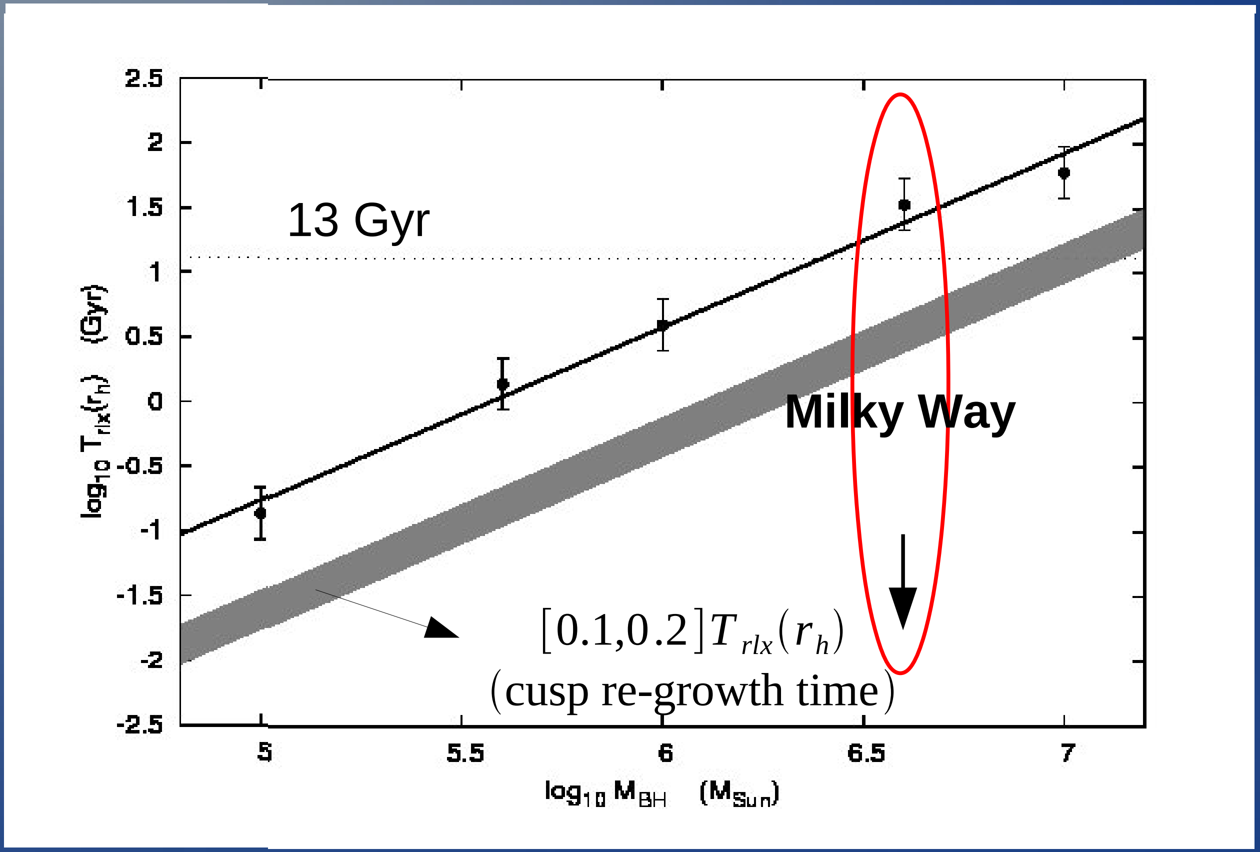
Cusp destruction?



Khan, MP et al 2011 -- in prep

Relaxed vs Unrelaxed Nuclei

$T_{rlx} > T_{hubble}$ for $M_{BH} > 10^7\ M_{Sun}$



Preto & Amaro-Seoane 2010

SUMMARY

-- We are moving closer towards a consistent solution to the Final Parsec Problem, with stellar dynamics alone supporting prompt coalescences in major mergers (q >= 1/4)

-- For lower q, the merger-induced triaxiality becomes small, and may be insufficient to overcome FPP (but this assumes spherical galaxy progenitors)

-- MBHBs tend to become bound with high eccentricity:
(i) boosting coalescence rates;
(ii) giving substantial eccentricity in PTA band
and non-negligible in LISA band

-- Non-axisymmetries, massive perturbers (Perets & Alexander), starbursts may help to coalesce

OPEN QUESTIONS

1. Is the Final Parsec Problem still a problem?

   What is the critical MBHB mass ratio q below which the remnant's triaxiality is too weak to drive the binary to coalescence?

   Is such q lower than the minimum value that will likely lead to the formation of a bound pair?

   What is the structure of a merger remnant when the binary is “close” to become bound? Does it depend on redshift?

   Zoom-in: from full galaxy merger calculations to sub-parsec binary evolution

2. How likely is the formation of triplets?

3. Eccentricity evolution of the inspiralling binaries (LISA and PTA bands) : mass ratio q, amount of rotation, density cusps. Is it necessary to consider the combined effect of stars and gas: how to make the first steps towards such calculation?

4. How do MBHB inspirals and GW recoils affect stellar distributions; how do they affect the event rates of extreme mass ratio inspirals?