A modal bispectrum estimator for the CMB bispectrum

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Fergusson, Liguori and Shellard (2010)
Outline

• Summary of the technique
  1. Polynomial modes
  2. What we measure:
     \( f_{\text{NL}} \), mode spectrum, shape reconstruction

• Results from WMAP-5
  1. Model independent: mode spectrum and bispectrum reconstr.
  2. Scale independent shapes
  3. Running shape: feature in the inflaton potential

• Extension to Planck

• Summary
Bispectrum domain

\[ B_{\ell_1 \ell_2 \ell_3} = \sum \left( \begin{array}{ccc} \ell_1 & \ell_2 & \ell_3 \\ m_1 & m_2 & m_3 \end{array} \right) \langle \alpha_{\ell_1}^{m_1} \alpha_{\ell_2}^{m_2} \alpha_{\ell_3}^{m_3} \rangle ; \quad b_{\ell_1 \ell_2 \ell_3} = h_{\ell_1 \ell_2 \ell_3} B_{\ell_1 \ell_2 \ell_3} \]
Mode expansion

We define the scalar product:

\[ \langle f, g \rangle = \sum_{\ell_1, \ell_2, \ell_3} w_3(\ell_1, \ell_2, \ell_3) f(\ell_1, \ell_2, \ell_3) g(\ell_1, \ell_2, \ell_3) \]

We expand the bispectrum in terms of separable orthonormal functions defined in the shaded domain (tetrapyd) with scalar product above.
Mode estimation

\[ = \beta_0 + \beta_1 + \beta_2 + \ldots \]

Goal: for a given dataset, extract best-fit \( \beta_i \), \( i=1,\ldots,p \)

- The basis elements pictured on the right are by construction factorizable.

- Apply position space cubic statistics by Komatsu, Spergel and Wandelt (2003) to each separable template on the right to estimate the amplitudes \( \beta_i \).

- Orthonormal basis \( \beta_i \) uncorrelated (in first approx.)
$f_{NL}$ estimation

- Expand theoretical shape until a good level of correlation is achieved
- Extract mode amplitude from the data up to the highest mode in the shapes under study
- Correlate to get $f_{NL}$

\[
\hat{f}_{NL} = \frac{\vec{\alpha}_{shape} \cdot \vec{\beta}_{obs.}}{N}
\]
WMAP (5-year) mode reconstruction
Fergusson, Liguori and Shellard 2010

- $l_{\text{max}} = 500$
- 31 modes
- V+W coadded map
- Inverse variance pre-filtering
WMAP shape reconstruction using first 31 modes (distance ordering).
Comparison with local and equilateral
WMAP $f_{NL}$ estimation

Scale invariant shapes:
- Constant
- Local
- Warm
- Flat
- Equilateral family
  - Separable ansatz
  - DBI
  - Ghost inflation
  - Single field
- Orthogonal

Running shapes:
- Sharp feature in the inflaton potential
Local
\(f_{NL} = 39 \pm 20\)

\(f_{NL} = 54.4 \pm 29.4\)

Equilateral
\(f_{NL} = 155 \pm 140\)

\(f_{NL} = 143.5 \pm 151.2\) (sep.)
\(f_{NL} = 146.0 \pm 144.5\) (DBI)
\(f_{NL} = 138.7 \pm 165.4\) (ghost)
\(f_{NL} = 142.1 \pm 131.2\) (single)

Orthogonal
\(f_{NL} = -214 \pm 110\)

\(f_{NL} = -79.4 \pm 133.3\)
Flat

\[ f_{NL} = 18.1 \pm 14.9 \]

Constant

\[ f_{NL} = 149.4 \pm 116.8 \]
Scale invariance breaking feature

A step in the inflaton potential breaks slow-roll

- Glitches in the power spectrum
- A large NG can be generated
- Scale invariance is broken

*Sinusoidal running* in the shape (parameters: amplitude, period, phase)

\[ S \sim f_{NL}^{\text{feat}} \sin \left( \frac{K}{k_*} + \phi \right) \]

(Chen et al. 2006)
Amplitude computed for all phases and several scales between $l=150$ and $l=700$

- 64 combinations of scale and phase
WMAP

$I_*= 150$

$\phi = 0$
Mode decomposition at *Planck* resolution

• Extend mode decomposition to much higher $l_{\text{max}}$. That requires many more modes.

\[
\begin{align*}
l_{\text{max}} &: 500 \rightarrow 2000 & n_{\text{side}} &: 512 \rightarrow 2048 \\
p_{\text{max}} &: 7 \rightarrow 18 & n_{\text{modes}} &: 31 \rightarrow 274
\end{align*}
\]

• Nothing conceptually different, but numerical stability and optimization issues required several technical modifications to the original WMAP pipeline.

• Currently being used on *Planck* data.
Summary

✓ We introduced an estimator of primordial NG based on a separable modal expansion of the bispectrum.

Nice features of the modal estimator:

1. It allows to separate any shape in a general clear mathematical framework.
2. It allows *model independent* reconstruction of the 3-point function.
3. It makes multi-shape studies faster and simpler.
4. Through mode spectrum and shape reconstruction it allows a better monitoring of potential contaminants.

✓ We applied our estimators to WMAP 5-yr data and constrained a large number of models, *including first constraints on feature models*.

✓ We extended our pipeline to high angular resolutions and we are now applying it to Planck data (as well as WMAP7).
Bispectrum estimator

A ML bispectrum estimator of $f_{NL}$ has been shown to be optimal

$$\hat{f}_{NL} = \frac{1}{N} \sum_{\ell_i m_i} B^{m_1 m_2 m_3}_{\ell_1 \ell_2 \ell_3} \frac{a^{m_1}_{\ell_1}}{C_{\ell_1}} \frac{a^{m_2}_{\ell_2}}{C_{\ell_2}} \frac{a^{m_3}_{\ell_3}}{C_{\ell_3}}$$

In presence of rotational invariance breaking terms

$$\hat{f}_{NL} = \frac{1}{N} \sum B^{m_1 m_2 m_3}_{\ell_1 \ell_2 \ell_3} (C^{-1}a)_{\ell_1}^{m_1} (C^{-1}a)_{\ell_2}^{m_2} (C^{-1}a)_{\ell_3}^{m_3} - 3C^{-1}_{\ell_1 m_1 \ell_2 m_2} (C^{-1}a)_{\ell_3}^{m_3}$$
# WMAP constraints

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<thead>
<tr>
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<th>WMAP 7-yrs</th>
<th>WMAP 5-yrs</th>
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<tbody>
<tr>
<td>Local</td>
<td>$-10 &lt; f_{\text{NL}} &lt; 74$</td>
<td>$-4 &lt; f_{\text{NL}} &lt; 80$</td>
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<tr>
<td>Equilateral</td>
<td>$-214 &lt; f_{\text{NL}} &lt; 266$</td>
<td>$-125 &lt; f_{\text{NL}} &lt; 435$</td>
</tr>
<tr>
<td>Orthogonal</td>
<td>$-410 &lt; f_{\text{NL}} &lt; 6$</td>
<td>$-369 &lt; f_{\text{NL}} &lt; 71$</td>
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(95% c.l.)