Primordial non-Gaussianity & the Galaxy Bispectrum

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Matter correlators

a bit of Perturbation Theory ...

\[ P = P_0 + P_G^{\text{loop}}[P_0] + P_{NG}^{\text{loop}}[P_0, B_0] \]

Linear power spectrum

Gravity-induced contributions (depending on \( P_0 \) alone)

Additional gravity-induced contributions present only for NG initial conditions (\( B_0 \))
Matter correlators

a bit of Perturbation Theory ...

\[ P = P_0 + P^{\text{loop}}_G[P_0] + P^{\text{loop}}_{NG}[P_0, B_0] \]

Linear power spectrum

Gravity-induced contributions (depending on \( P_0 \) alone)

Additional gravity-induced contributions present only for NG initial conditions (\( B_0 \))

Few percent effect at small scales for allowed values of \( f_{\text{NL}} \)

Ratio of the non-Gaussian to the Gaussian power spectrum for \( f_{\text{NL}} = \pm 100 \) (local) at \( z = 1 \)

Smith, Desjacques & Marian (2010)
Matter correlators

a bit of Perturbation Theory ...

\[ P = P_0 + P_{\text{loop}}^G [P_0] + P_{\text{loop}}^{NG} [P_0, B_0] \]

Linear power spectrum

Gravity-induced contributions (depending on \( P_0 \) alone)

Additional gravity-induced contributions present only for NG initial conditions (\( B_0 \))

\[ B = B_0 + B_{\text{tree}}^G [P_0] + B_{\text{loop}}^G [P_0] + B_{\text{loop}}^{NG} [P_0, B_0] \]

Primordial component

& bispectrum
At large scales

\[ B(k_1, k_2, k_3) \simeq B_0 + B^{tree}_G[P_0] \]

**Primordial component**

**Gravity-induced component**

Equilateral configurations of the matter bispectrum

\[ \frac{B_0(k, k, k)}{B^{tree}_G(k, k, k)} \xrightarrow{k \to 0} \frac{f_{NL}}{D(z)k^2} \]

The primordial component has a different dependence on scale than the gravity-induced one

This is true for almost all models (local, equilateral, orthogonal ...)

Sunday, May 15, 2011
The matter bispectrum and PNG: large scales

At large scales

\[ B(k_1, k_2, k_3) \simeq B_0 + B_{\text{tree}}^{G}[P_0] \]

**Primordial component**

**Gravity-induced component**

\[ Q = \frac{B}{P(k_1)P(k_2) + \text{cyc.}} \]

- \( k_1 = 0.01 \, h \, \text{Mpc}^{-1}, \) \( k_2 = 1.5 \, k_1 \)

The primordial component has a different shape dependence

(each model has its own, of course)
Current CMB constraints for different models of non-Gaussianity as uncertainties on generic configurations of the matter bispectrum, \( B \simeq B_0 + B_{tree}^G [P_0] \)
Matter correlators

a bit of Perturbation Theory ...

\[ P = P_0 + P_{G}^{\text{loop}}[P_0] + P_{NG}^{\text{loop}}[P_0, B_0] \]

- **Linear power spectrum**
- **Gravity-induced contributions** (depending on \( P_0 \) alone)
- **Additional** gravity-induced contributions present *only* for NG initial conditions (\( B_0 \))

\[ B = B_0 + B_{\text{tree}}^{G}[P_0] + B_{G}^{\text{loop}}[P_0] + B_{NG}^{\text{loop}}[P_0, B_0] \]

- **Primordial component**

If \( B_0 \) was the *only effect* of NG initial conditions on the LSS then future, large volume surveys (~100 Gpc\(^3\)) could provide:

\[ \Delta f_{\text{NL}}^{\text{local}} < 5 \text{ and } \Delta f_{\text{NL}}^{\text{eq}} < 10 \]

ES & Komatsu (2007)
Matter correlators

a bit of Perturbation Theory ...

\[ P = P_0 + P_{G}^{\text{loop}}[P_0] + P_{NG}^{\text{loop}}[P_0, B_0] \]

Linear power spectrum

Gravity-induced contributions (depending on \( P_0 \) alone)

Additional gravity-induced contributions present only for NG initial conditions (\( B_0 \))

\[ B = B_0 + B_G^{\text{tree}}[P_0] + B_{G}^{\text{loop}}[P_0] + B_{NG}^{\text{loop}}[P_0, B_0] \]

& bispectrum

Nonlinear corrections are also affected by the initial conditions!

There is a significant effect of NG initial conditions of about 5-15% on all triangles, at small scales and at late times for \( f_{NL} = 100 \).
The matter bispectrum and PNG: small scales

\[ B = B_0 + B_G^{tree}[P_0] + B_G^{loop}[P_0] + B_{NG}^{loop}[P_0, B_0] \]

- **Primordial component**
- **Gravity-induced contributions**
- **Additional gravity-induced contributions present for NG initial conditions \( B_0 \)**

**Generic configurations \( B(k_1, k_2, \theta) \) as a function of \( \theta \)**

with \( k_1 = 0.1 \ h/Mpc \), \( k_2 = 1.5 \ k_1 \)

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**Difference \( B(f_{NL} = 100) - B(f_{NL} = 0) \)**

\[ z = 1 \]

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**Ratio \( B(f_{NL} = 100) / B(f_{NL} = 0) \)**

\[ z = 1 \]
The matter bispectrum and PNG: small scales

\[ B = B_0 + B_G^{tree}[P_0] + B_G^{loop}[P_0] + B_{NG}^{loop}[P_0, B_0] \]

- Primordial component
- Gravity-induced contributions
- **Additional** gravity-induced contributions present for NG initial conditions \((B_0)\)

Squeezed configurations \(B(\Delta k, k, k)\)
as a function of \(k\) with \(\Delta k = 0.01 \, h/\text{Mpc}\)

- **Difference** \(B(f_{NL}=100) - B(f_{NL}=0)\)
- **Ratio** \(B(f_{NL} = 100) / B(f_{NL} = 0)\)

ES (2009)
ES, Crocce & Desjacques (2010)
**Matter Power Spectrum vs Matter Bispectrum**

Cumulative signal-to-noise for the effect of NG initial conditions.

**Sum of all configurations up to** $k_{\text{max}}$

\[
\left( \frac{S}{N} \right)_P^2 = \sum_{k}^{k_{\text{max}}} \frac{(P_{\text{NG}} - P_G)^2}{\Delta P^2}
\]

\[
\left( \frac{S}{N} \right)_B^2 = \sum_{k_1, k_2, k_3}^{k_{\text{max}}} \frac{(B_{\text{NG}} - B_G)^2}{\Delta B^2}
\]

$z = 0.5$

\[V = 4 \, h^{-1} \text{Gpc}^3\]

\[f_{\text{NL}} = 100\]

**ES, Crocce & Desjacques (in preparation)**
Effects of PNG on the galaxy power spectrum

Dalal et al. (2008):

\[
\delta_g(\vec{k}) = [b_1 + \Delta b_1(f_{NL}, k)] \delta(\vec{k}) + \ldots 
\]

\[
P_g(k) = [b_1 + \Delta b_1(f_{NL}, k)]^2 P(k)
\]

\[
\Delta b_{1,NG}(f_{NL}, k) \sim \frac{f_{NL}}{D(z) k^2}
\]
Clearly, the effect on galaxy bias affects as well the galaxy bispectrum

\[ B_g(k_1, k_2, k_3) = b_1^3 B(k_1, k_2, k_3) + b_1^2 b_2 P(k_1) P(k_2) + 2 \text{ perm.} + \ldots \]
Effects of PNG on the galaxy bispectrum

Clearly, the effect on galaxy bias affects as well the galaxy bispectrum

\[ B_g(k_1, k_2, k_3) = b_1^3 B(k_1, k_2, k_3) + b_1^2 b_2 P(k_1) P(k_2) + 2 \text{ perm.} + ... \]

\[ \Delta b_{1,NG}(f_{NL}, \vec{k}) = \Delta b_{1,si}(f_{NL}) + \Delta b_{1,sd}(f_{NL}, b_1, G, \vec{k}) \]

\[ \Delta b_{2,NG}(f_{NL}, \vec{k}_1, \vec{k}_2) = \Delta b_{2,si}(f_{NL}) + \Delta b_{2,sd}(f_{NL}, b_1, G, b_2, G, \vec{k}_1, \vec{k}_2) \]
Effects of PNG on the galaxy bispectrum

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\[ B_g(k_1, k_2, k_3) = b_1^3 B(k_1, k_2, k_3) + b_1^2 b_2 P(k_1) P(k_2) + 2 \text{ perm.} + \ldots \]

\[ B = B_0 + B_{G}^{\text{tree}}[P_0] + B_{G}^{\text{loop}}[P_0] + B_{NG}^{\text{loop}}[P_0, B_0] \]

Primordial component
(large scales)

\[ \Delta b_{1,NG}(f_{NL}, \vec{k}) = \Delta b_{1,si}(f_{NL}) + \Delta b_{1,sa}(f_{NL}, b_{1,G}, \vec{k}) \]
\[ \Delta b_{2,NG}(f_{NL}, \vec{k}_1, \vec{k}_2) = \Delta b_{2,si}(f_{NL}) + \Delta b_{2,sa}(f_{NL}, b_{1,G}, b_{2,G}, \vec{k}_1, \vec{k}_2) \]

Giannantonio & Porciani (2010)
Baldauf, Seljak & Senatore (2010)
Effects of PNG on the galaxy bispectrum

Clearly, the effect on galaxy bias affects as well the galaxy bispectrum

\[ B_g(k_1, k_2, k_3) = b_1^3 B(k_1, k_2, k_3) + b_1^2 b_2 P(k_1) P(k_2) + 2 \text{ perm.} + \ldots \]

\[ P = P_0 + P_{G}^{\text{loop}}[P_0] + P_{NG}^{\text{loop}}[P_0, B_0] \]

\[ B = B_0 + B_{G}^{\text{tree}}[P_0] + B_{G}^{\text{loop}}[P_0] + B_{NG}^{\text{loop}}[P_0, B_0] \]

Scale-dependent bias corrections

Primordial component (large scales)

\[ \Delta b_{1,NG}(f_{NL}, \vec{k}) = \Delta b_{1,si}(f_{NL}) + \Delta b_{1,sd}(f_{NL}, b_1, G, \vec{k}) \]

\[ \Delta b_{2,NG}(f_{NL}, \vec{k}_1, \vec{k}_2) = \Delta b_{2,si}(f_{NL}) + \Delta b_{2,sd}(f_{NL}, b_1, G, b_2, G, \vec{k}_1, \vec{k}_2) \]

Effect on nonlinear evolution (small scales)

Giannantonio & Porciani (2010)
Baldauf, Seljak & Senatore (2010)
Effects of PNG on the galaxy bispectrum

Clearly, the effect on galaxy bias affects as well the galaxy bispectrum

\[ B_g(k_1, k_2, k_3) = b_1^3 B(k_1, k_2, k_3) + b_1^2 b_2 P(k_1) P(k_2) + 2 \text{ perm.} + \ldots \]

\[ b_{1,G} + \Delta b_{1,NG}(f_{NL}, k) \quad b_{2,G} + \Delta b_{2,NG}(f_{NL}, \bar{k}_1, \bar{k}_2) \]

Scale-dependent bias corrections

Primordial component
(large scales)

\[ \Delta b_{1,NG}(f_{NL}, \bar{k}) = \Delta b_{1,si}(f_{NL}) + \Delta b_{1,sd}(f_{NL}, b_{1,G}, \bar{k}) \]
\[ \Delta b_{2,NG}(f_{NL}, \bar{k}_1, \bar{k}_2) = \Delta b_{2,si}(f_{NL}) + \Delta b_{2,sd}(f_{NL}, b_{1,G}, b_{2,G}, \bar{k}_1, \bar{k}_2) \]

Effect on nonlinear evolution (small scales)

\[ \langle \delta \delta \delta_h \rangle = \delta_D(\vec{k}_{123}) B_{mmh} \]
\[ \langle \delta_h \delta_h \delta_h \rangle = \delta_D(\vec{k}_{123}) B_h \]

- We test this model in N-body simulations with local NG initial conditions

- We fit all triangular configurations up to \( k = 0.07 \) h/Mpc

for \( b_{1,G}, b_{2,G}, \Delta b_{1,G} \) and \( \Delta b_{2,G} \)

\[ P_h \rightarrow b_{1,G}, \Delta b_{1,si} \]
\[ B_{h,G} \rightarrow b_{2,G} \]
\[ \Delta B_{h,NG} \rightarrow \Delta b_{2,si} \]
Effects of PNG on the galaxy bispectrum

**Matter-matter-halo** bispectrum:

\[ B_{mmh}(k_1, k_2; k_3) = b_1(f_{NL}, k) B(k_1, k_2, k_3) + b_2(f_{NL}, k_1, k_2) P(k_1) P(k_2) \]

**Generic configurations** \( B(k_1, k_2, \theta) \)

as a function of \( \theta \)

with \( k_1 = 0.1 \, h/Mpc, k_2 = 1.5 \, k_1 \)

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**Figure:**

- **High mass bin**
  - \( \Delta B_{mmh}(k_1, k_2, \theta) \)
  - total
  - Gauss. bias
  - \( \Delta b_1 \)
  - \( \Delta b_2 \)

- **High mass bin**
  - **Ratio** \( B_{NG} / B_G \)
  - total
  - Gauss. bias
  - \( \Delta b_1 \)
  - \( \Delta b_2 \)

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ES, Crocce & Desjacques (*in preparation*)
Effects of PNG on the galaxy bispectrum

Matter-matter-halo bispectrum:

\[ B_{mmh}(k_1, k_2; k_3) = b_1(f_{NL}, k) B(k_1, k_2, k_3) + b_2(f_{NL}, k_1, k_2) P(k_1) P(k_2) \]

Squeezed configurations \( B(\Delta k, k, k) \)
as a function of \( k \) with \( \Delta k = 0.01 \, h/\text{Mpc} \)

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ES, Crocce & Desjacques (in preparation)
Effects of PNG on the galaxy bispectrum

**Halo** bispectrum:

\[ B_h(k_1, k_2, k_3) = b_1^3(f_{NL}, k) B(k_1, k_2, k_3) \]

\[ + b_1(f_{NL}, k_1) b_1(f_{NL}, k_2) b_2(f_{NL}, k_1, k_2) P(k_1) P(k_2) + \text{cyc.} \]

Squeezed configurations \( B(\Delta k, k, k) \)
as a function of \( k \) with \( \Delta k = 0.01 \ h/\text{Mpc} \)

ES, Crocce & Desjacques (in preparation)
Cumulative signal-to-noise for the effect of NG initial conditions on matter and galaxy correlators ($P$ & $B$)

Sum of all configurations up to $k_{\text{max}}$

$$\left( \frac{S}{N} \right)^2_P = \sum_{k}^{k_{\text{max}}} \frac{(P_{\text{NG}} - P_G)^2}{\Delta P^2}$$

$$\left( \frac{S}{N} \right)^2_B = \sum_{k_1,k_2,k_3}^{k_{\text{max}}} \frac{(B_{\text{NG}} - B_G)^2}{\Delta B^2}$$

Power Spectrum vs. Bispectrum

$z = 0.5$

$V = 4 \, h^{-1}\text{Gpc}^3$

$f_{\text{NL}} = 100$

$S/N (<k)$

$k [h\text{ Mpc}^{-1}]$

$P_m$, matter

$B_m$, matter

$P_h$, halos

$B_h$, halos
An unrealistic Fisher matrix analysis

Assuming perfect knowledge of a complete galaxy population in a 10 Gpc$^3$ volume at redshift $z$ with density $10^{-3}$ Mpc$^{-3}$

Ignoring any complication no matter how relevant and pertinent (covariance, redshift distortions, selection function, degeneracies, etc ...)

We can estimate the uncertainty on $f_{NL}$ (local) from Power Spectrum & Bispectrum (& both)

![Graph showing $\Delta f_{NL}$ vs. $k_{\max}$ and $k_{\min}$ with various lines for P, B, and P+B.](image)

- $V = 10 \, h^{-3} \text{Gpc}^3$, $z = 1$
- $k_{\min} = 0.009 \, h \, \text{Mpc}^{-1}$
- $b_1 = 2$, $b_2 = 0.8$
An unrealistic Fisher matrix analysis

Assuming perfect knowledge of a complete galaxy population in a 10 Gpc$^3$ volume at redshift z with density $10^{-3}$ Mpc$^{-3}$

Ignoring any complication no matter how relevant and pertinent (covariance, redshift distortions, selection function, degeneracies, etc ...)

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We can estimate the uncertainty on $f_{NL}$ (local) from Power Spectrum & Bispectrum (& both)

$V = 10 \, h^{-3}\text{Gpc}^3, \, z = 1$

$k_{\text{min}} = 0.009 \, h \, \text{Mpc}^{-1}$

$b_1 = 2, \, b_2 = 0.8$

no effect on bias

ES & Komatsu (2007)

marginalized $(b_1, b_2)$

unmarginalized
Conclusions

- We have a (relatively simple) model for the large scales galaxy bispectrum with local NG initial conditions

- Bispectrum measurements in LSS surveys can confirm and improve constraints on $f_{NL}$ from the power spectrum (particularly for non-local models ...)

![Graphs and equations showing the relationship between $f_{NL}$ and various parameters.](image-url)
The matter bispectrum and PNG: large scales

Current CMB constraints for different models of non-Gaussianity as uncertainties on the equilateral configurations of the matter bispectrum

\[ B \simeq B_0 + B_{\text{tree}}^G[P_0] \]
The Matter Bispectrum induced by Gravity

\[ B_G = B_{G \text{tree}}[P_0] + B_{G \text{loop}}[P_0] \]

\[ B_{G \text{tree}}(k_1, k_2, k_3) = 2 F_2(\vec{k}_1, \vec{k}_2) P_0(k_1) P_0(k_2) + 2 \text{ perm.} \]

The bispectrum induced by gravity has a well defined dependence on scale and on the shape

The equilateral configurations of the matter bispectrum:

\[ B(k, k, k) \text{ vs. } k \]

Numerical simulations and PT predictions

Tree-level approximation valid at large scales

1-loop approximation

E.S., M. Crocce, & V. Desjacques (2010)
Non-Gaussianity from Gravitational Instability

At large scales fluctuations are small, $\sigma_\delta \ll 1$, even at low redshift we can study their evolution in terms of Perturbation Theory.

Equations of motion for matter density and velocity:

- Continuity eq. \[ \frac{\partial \delta}{\partial \tau} + \nabla \cdot [(1 + \delta) \vec{v}] = 0 \]
- Euler eq. \[ \frac{\partial \vec{v}}{\partial \tau} + \mathcal{H} \vec{v} + (\vec{v} \cdot \nabla) \vec{v} = -\nabla \phi \]
- Poisson eq. \[ \nabla^2 \phi = \frac{3}{2} \mathcal{H}^2 \Omega_m \delta \]

Perturbative solution for the matter density, in Fourier space

\[ \delta_{\vec{k}} = \delta^{(1)}_{\vec{k}} + \delta^{(2)}_{\vec{k}} + \ldots \]

Linear solution \[ \delta^{(2)}_{\vec{k}} = \int d^3 q F_2(\vec{k} - \vec{q}, \vec{q}) \delta^{(1)}_{\vec{k} - \vec{q}} \delta^{(1)}_{\vec{q}} \]

Quadratic nonlinear correction

Initial conditions

- $B_0$ and $T_0$ vanish for Gaussian initial conditions!

Perturbative solution for the matter 3-point function

\[ \langle \delta^{(1)}_{\vec{k}_1} \delta^{(1)}_{\vec{k}_2} \rangle = \delta_D(\vec{k}_1 + \vec{k}_2) P_0(k_1) \]
\[ \langle \delta^{(1)}_{\vec{k}_1} \delta^{(1)}_{\vec{k}_2} \delta^{(1)}_{\vec{k}_3} \rangle = \delta_D(\vec{k}_1 + \vec{k}_2 + \vec{k}_3) B_0(k_1, k_2, k_3) \]
\[ \langle \delta^{(1)}_{\vec{k}_1} \delta^{(1)}_{\vec{k}_2} \delta^{(1)}_{\vec{k}_3} \delta^{(1)}_{\vec{k}_4} \rangle = \delta_D(\vec{k}_1 + \ldots + \vec{k}_4) T_0(\vec{k}_1, \vec{k}_2, \vec{k}_3, \vec{k}_4) \]

\[ \langle \delta \delta \delta \rangle = \langle \delta^{(1)} \delta^{(1)} \delta^{(1)} \rangle + \langle \delta^{(1)} \delta^{(1)} \delta^{(2)} \rangle + \ldots \]

= $B_0 = 0$ for Gaussian initial conditions

non-zero bispectrum induced by gravity!
Non-Gaussianity from **Galaxy Bias** (more problems?)

Additional non-Gaussianity in the **galaxy distribution** is induced by **nonlinear galaxy bias**

The relation between the **observed galaxy overdensity** and the matter density is nonlinear

At large scales, we expand it in a Taylor series

\[
\delta_g(x) = b_1 \delta(x) + \frac{1}{2} b_2 \delta^2(x) + \ldots
\]

**Linear bias**

**Quadratic bias correction**

Perturbative solution for the **galaxy** 3-point function

\[
\langle \delta_g \delta_g \delta_g \rangle = b_1^3 \langle \delta \delta \delta \rangle + b_1^2 b_2 \langle \delta \delta \delta^2 \rangle + \ldots
\]

**matter bispectrum**

**bispectrum induced by nonlinear bias**

\[
B_g(k_1, k_2, k_3) = b_1^3 B(k_1, k_2, k_3) + b_1^2 b_2 P(k_1) P(k_2) + 2 \text{ perm.} + \ldots
\]

**The component induced by bias has a different dependence on the shape of the triangle**
Effects of PNG on the galaxy bispectrum

Clearly, the effect on galaxy bias affects as well the galaxy bispectrum

\[ B_g(k_1, k_2, k_3) = b_1^3 B(k_1, k_2, k_3) + b_1^2 b_2 P(k_1) P(k_2) + 2 \text{ perm.} + \ldots \]

Scale-dependent bias corrections

ES, Crocce & Desjacques (in preparation)
The matter bispectrum and PNG: small scales

\[ B_{NG} - B_G \]

\( k [h \text{ Mpc}^{-1}] \)

\( f_{NL} = 100 \)

\( f_{NL} = 0 \)

\[ \text{Difference } B(f_{NL}=100) - B(f_{NL}=0) \]