Probing scale-dependent non-Gaussianities in the WMAP data using surrogates

Primordial non-Gaussianity: Theory Confronts Observations, Ann Arbor, May 2011

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References:
C. Räth et al., PRL, 102, 131301, 2009; arXiv:0810.3805
C. Räth et al., MNRAS in press; arXiv:1012.2985
Motivations

„More shapes of non-Gaussianities (from inflation) than...stars in the sky.“
(S. Matarrese, this meeting)

„I don’t see a convergence of the theories.“
(M. Rees, 2008)

„It doesn't matter how beautiful your theory is, it doesn't matter how smart you are. If it doesn't agree with experiment, it's wrong.“
(R. Feynman)

„The model is the data.“
(C. Grebogi)
⇒ Method of surrogates (Theiler et al. 1992)

Model-independent („agnostic“) test ⇒ „explorative data analysis“, which is sensitive to any NG signatures (not „just“ $f_{nl}$ - models) and any other anomalies
Scaling indices for spherical data

Transformation of the data to a 3D point distribution:
Each „sky element“ is characterised by two angles $\theta$ and $\varphi$ (on the unit sphere) and its temperature. Thus, one possible 3D representation of the WMAP data is given by:

$$
x = (R + dR) \cos \varphi \sin \theta
$$

$$
y = (R + dR) \sin \varphi \sin \theta
$$

$$
z = (R + dR) \cos \theta
$$

where:

$$
dR = a(r) \cdot (T - < T >) / \sigma_T
$$

Temperature fluctuations are transformed to variations in $R$-direction.

$R$, $r$ and $a$ are the free (scale) parameters.
SIM for spherical data

Transformation of the WMAP-data to a 3D point distribution:

Consider a point distribution $P$:

$$P = \{\tilde{p}_i\}, i = 1, \ldots, N_{\text{points}},$$

$$\tilde{p}_i = \{x_i, y_i, z_i\}$$

Local cumulative weighted density:

$$\rho(\tilde{p}_i) = \sum_{j=1}^{N} e^{-\left(\frac{d_{ij}}{r}\right)^n}, d_{ij} = \|\tilde{p}_i - \tilde{p}_j\|$$

Scaling Index:

$$\alpha(\tilde{p}_i) \equiv \frac{\partial \log(\rho(\tilde{p}_i))}{\partial \log(r)}$$

$$\Rightarrow \alpha(\tilde{p}_i) = \frac{\sum_{j=1}^{N} n \cdot \left(\frac{d_{ij}}{r}\right)^n \cdot e^{-\left(\frac{d_{ij}}{r}\right)^n}}{\sum_{j=1}^{N} e^{-\left(\frac{d_{ij}}{r}\right)^n}}$$


Generating Surrogates (I.)

Fourier Transform of the temperature map:

\[ T(n) = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} a_{lm} Y_{lm}(n) \]

with \( a_{lm} = \int T(n) Y_{lm}^* d\Omega_n \)

One can write:

\[ a_{lm} = |a_{lm}| e^{i\phi_{lm}} \]

with \( \phi_{lm} = \arctan \left( \frac{\text{Im}(a_{lm})}{\text{Re}(a_{lm})} \right) \)

Non-Gaussian Field:
Fourier Phases are correlated and/or not uniformly distributed

How to test for possible phase correlations?

Destroy (only) them (by scale-dependent shuffling) and look what happens...
Generating Surrogates (II.)

Two shuffling steps:

First order Surrogate: Shuffle outside \((l_{\text{min}}, l_{\text{max}})\)
Second order Surrogates: Shuffle inside \((l_{\text{min}}, l_{\text{max}})\)

All \(|a_{lm}|\)’s are preserved.
Generating Surrogates (III): $\Delta l$-intervals

$\Delta l = [2,20]$  $\Delta l = [20,60]$  $\Delta l = [60,120]$  $\Delta l = [120,300]$

$l(1+l)C_l^{\text{TT}}/2\pi$ [uK$^2$]

Multipole moment $l$
Generating Surrogates (IV.)

Two preprocessing steps:
Rank-ordered remapping of the Amplitudes (in real space) and Phases (in Fourier space).

original

surro1 $\Delta l = [2,20]$

surro2 $\Delta l = [2,20]$
Deviation in rotated Hemispheres

*WMAP data / 1st order surrogate*

*Simulations / 1st or 2nd order Surrogates*

\[ S(\theta, \phi) = \frac{X - \langle X \rangle}{\sigma_X}, \]

\[ X = \langle \alpha(r) \rangle, \sigma_T, \chi^2(M_i), i = 1, \ldots, 3 \]
Results

$S(X)$ in $N$ rotated hemispheres ($\Delta l = [2, 20]$):

And remember also Heike’s results:

=> Highly significant signatures of non-Gaussianity and asymmetries. “Consistent picture of inconsistencies”
Results

Probability densities for the two different foreground-cleaned maps:

- WMAP ILC 7 year map
- Needlet-based ILC 5 year map

Signature remains the same for the two maps
Results

$S(X)$ in rotated hemispheres for varying $\Delta l$ and $r$:

$\Delta l = [2,1024]$  
$\Delta l = [2,20]$  
$\Delta l = [20,60]$  
$\Delta l = [60,120]$  
$\Delta l = [120,300]$

ILC 7yr map, $X = <\alpha_{r2}>, <\alpha_{r6}>, <\alpha_{r10}>$ (from top to bottom)
Results

S(X) in rotated hemispheres for varying $\Delta l$ and $r$:

• Most significant deviations for $\Delta l = [2,20]$ and $\Delta l = [120,300]$
• Signal in $\Delta l = [2,1024]$ to be interpreted as superposition of the signals in $\Delta l = [2,20]$ and $\Delta l = [120,300]$
Results

Scale-independent NGs:

NILC map \( \Delta l = [2, 1024] \)

Full sky
Upper hemisphere
Lower hemisphere

ILC map \( \Delta l = [2, 1023] \)
Results

Scale-dependent NGs on large scales:

NILC map \quad \Delta l = [2,20]

Full sky
Upper hemisphere
Lower hemisphere
## Results

Some numbers (scale-independent $\chi^2$-measures):

### ILC 7 yr map

<table>
<thead>
<tr>
<th>$\Delta l$</th>
<th>Full Sky</th>
<th>Upper Hemisphere</th>
<th>Lower Hemisphere</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\chi^2_{(\alpha)}$</td>
<td>(S/%)</td>
<td>(S/%)</td>
<td>(S/%)</td>
</tr>
<tr>
<td>[2, 1024]</td>
<td>5.73 / &gt;99.8</td>
<td>9.35 / &gt;99.8</td>
<td>0.33 / 55.2</td>
</tr>
<tr>
<td>[20, 60]</td>
<td>0.97 / 95.0</td>
<td>4.57 / 99.6</td>
<td>4.01 / 99.2</td>
</tr>
<tr>
<td>[60, 120]</td>
<td>1.41 / 99.0</td>
<td>1.53 / 99.6</td>
<td>0.01 / 82.8</td>
</tr>
<tr>
<td>[120, 300]</td>
<td>3.17 / 92.8</td>
<td>10.53 / &gt;99.8</td>
<td>1.19 / 87.8</td>
</tr>
</tbody>
</table>

$\chi^2_{\sigma_\alpha}$:

<table>
<thead>
<tr>
<th>$\Delta l$</th>
<th>Full Sky</th>
<th>Upper Hemisphere</th>
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</tr>
</thead>
<tbody>
<tr>
<td>[2, 1024]</td>
<td>5.50 / &gt;99.8</td>
<td>11.50 / &gt;99.8</td>
<td>0.66 / 79.6</td>
</tr>
<tr>
<td>[20, 60]</td>
<td>0.32 / 52.8</td>
<td>4.03 / 98.6</td>
<td>4.04 / 99.6</td>
</tr>
<tr>
<td>[60, 120]</td>
<td>2.15 / 95.8</td>
<td>4.00 / 99.8</td>
<td>2.18 / 96.4</td>
</tr>
<tr>
<td>[120, 300]</td>
<td>1.40 / 98.2</td>
<td>3.26 / 99.4</td>
<td>2.01 / 95.6</td>
</tr>
</tbody>
</table>

$\chi^2_{(\alpha),\sigma_\alpha}$:

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<th>Lower Hemisphere</th>
</tr>
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<tbody>
<tr>
<td>[2, 1024]</td>
<td>1.89 / 94.2</td>
<td>8.38 / &gt;99.8</td>
<td>3.03 / 98.8</td>
</tr>
<tr>
<td>[20, 60]</td>
<td>0.73 / 77.4</td>
<td>5.64 / &gt;99.8</td>
<td>6.01 / 99.8</td>
</tr>
<tr>
<td>[60, 120]</td>
<td>1.60 / 92.8</td>
<td>3.42 / 99.2</td>
<td>1.49 / 91.0</td>
</tr>
<tr>
<td>[120, 300]</td>
<td>0.26 / 52.4</td>
<td>2.15 / 96.6</td>
<td>0.53 / 75.6</td>
</tr>
</tbody>
</table>

### NILC 5 yr map

<table>
<thead>
<tr>
<th>$\Delta l$</th>
<th>Full Sky</th>
<th>Upper Hemisphere</th>
<th>Lower Hemisphere</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\chi^2_{(\alpha)}$</td>
<td>(S/%)</td>
<td>(S/%)</td>
<td>(S/%)</td>
</tr>
<tr>
<td>[2, 1024]</td>
<td>27.93 / &gt;99.8</td>
<td>27.23 / &gt;99.8</td>
<td>4.47 / 99.4</td>
</tr>
<tr>
<td>[20, 60]</td>
<td>0.39 / 55.8</td>
<td>8.18 / &gt;99.8</td>
<td>9.27 / &gt;99.8</td>
</tr>
<tr>
<td>[60, 120]</td>
<td>0.61 / 69.0</td>
<td>2.02 / 96.0</td>
<td>0.74 / 83.0</td>
</tr>
<tr>
<td>[120, 300]</td>
<td>0.88 / 87.0</td>
<td>4.11 / 99.4</td>
<td>1.01 / 85.4</td>
</tr>
</tbody>
</table>

$\chi^2_{\sigma_\alpha}$:

<table>
<thead>
<tr>
<th>$\Delta l$</th>
<th>Full Sky</th>
<th>Upper Hemisphere</th>
<th>Lower Hemisphere</th>
</tr>
</thead>
<tbody>
<tr>
<td>[20, 60]</td>
<td>0.45 / 59.8</td>
<td>9.76 / &gt;99.8</td>
<td>9.17 / &gt;99.8</td>
</tr>
<tr>
<td>[60, 120]</td>
<td>0.69 / 73.4</td>
<td>1.54 / 92.2</td>
<td>0.41 / 71.6</td>
</tr>
<tr>
<td>[120, 300]</td>
<td>0.88 / 82.2</td>
<td>4.04 / 99.4</td>
<td>1.73 / 94.0</td>
</tr>
</tbody>
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$\chi^2_{(\alpha),\sigma_\alpha}$:

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<tbody>
<tr>
<td>[2, 1024]</td>
<td>9.73 / &gt;99.8</td>
<td>10.04 / &gt;99.8</td>
<td>4.03 / 99.8</td>
</tr>
<tr>
<td>[20, 60]</td>
<td>0.90 / 88.0</td>
<td>7.17 / &gt;99.8</td>
<td>6.85 / &gt;99.8</td>
</tr>
<tr>
<td>[60, 120]</td>
<td>1.21 / 94.6</td>
<td>0.70 / 77.4</td>
<td>0.64 / 70.6</td>
</tr>
<tr>
<td>[120, 300]</td>
<td>0.30 / 55.2</td>
<td>2.73 / 98.4</td>
<td>0.08 / 51.6</td>
</tr>
<tr>
<td>[120, 300]</td>
<td>0.86 / 83.6</td>
<td>6.48 / &gt;99.8</td>
<td>3.44 / 99.8</td>
</tr>
</tbody>
</table>
Results:

Robustness of results ($\Delta l = [2,20]$):

Three year Tegmark map  
Three year Tegmark map (Wiener filtered)  
Five year needlet based ILC - map  
Five year ILC - map  
Five year ILC - map without the cold spot  
Seven year ILC - map
Results:

Checks on systematics ($\Delta l = [2,20]$):

- Uncorrected ILC map
- Difference ILC map (year 7 – year 6)
- Asymmetric Beam map
- Simulated Coadded VW-band map
- Simulated ILC-like map

$=>$ No test can so far explain the low-$l$ anomalies!
Results:

Checks on systematics ($\Delta l=[120,300]$):

Uncorrected ILC map
Difference ILC map (year 7 – year 6)
Asymmetric Beam map
Simulated Coadded VW-band map
Simulated ILC-like map

$=>$ A number of 'residuals' found for the high-$l$ case
Summary

• Using surrogates and scaling indices we performed a comprehensive study of scale-dependent non-Gaussianities in full sky CMB data and find a 5.0+\sigma detection of non-Gaussianities especially at the largest scales and hemispherical asymmetries, i.e. violation of statistical isotropy.

• The signal is stable and found using different test statistics (\sigma_T, scaling indices and Minkowski-functionals (see Heike ‘s Talk)).

• All checks on systematics we performed so far revealed that no clear candidate can be found to explain the low-l signal.

⇒ The signatures at low l must so far be taken to be cosmological at high significance.

That would mean:

• Single field slow roll inflation seriously questioned,

• Anisotropic model of NGs with running f_{nl} required
Concluding Remarks

A surprising statement...:

„A detection of non-Gaussianity and/or phase correlations in the WMAP $a_{\text{im}}$ data would be of great interest. While a detection of non-Gaussianity could be indicative of an experimental systematic effect or of residual foregrounds, it could also point to new cosmological physics.“

(Bennett et al., 2011)

My immediate thoughts...:

Chiang et al. 03, Chiang et al. 06, Coles et al. 04, Naselsky et al. 05, etc.

and also CR et al. 09, CR et al. 11.

With this presentation I hope I could convince you that

it is no longer the question whether there are phase correlations (i.e. signatures of NGs) in the WMAP $a_{\text{im}}$ data.

It ‘s rather of interest what their origin is.
thank you for your attention!

attention your you for! thank
## Results

### Some numbers (small ($r_2$) and large ($r_{10}$) scaling ranges):

#### ILC 7 yr map

<table>
<thead>
<tr>
<th>$\Delta l$</th>
<th>Full Sky</th>
<th>Upper Hemisphere</th>
<th>Lower Hemisphere</th>
<th>$\langle \alpha(r_2) \rangle$</th>
<th>$\langle \alpha(r_{10}) \rangle$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$(S/%)$</td>
<td>$(S/%)$</td>
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<td></td>
<td>$(S/%)$</td>
<td>$(S/%)$</td>
</tr>
<tr>
<td>[2, 1024]</td>
<td>7.73 / &gt;99.8</td>
<td>4.53 / &gt;99.8</td>
<td>1.87 / 96.0</td>
<td>3.75 / &gt;99.8</td>
<td>3.53 / &gt;99.8</td>
</tr>
<tr>
<td>[2, 20]</td>
<td>0.14 / 56.6</td>
<td>3.54 / &gt;99.8</td>
<td>3.44 / &gt;99.8</td>
<td>0.64 / 74.2</td>
<td>3.24 / &gt;99.8</td>
</tr>
<tr>
<td>[20, 60]</td>
<td>0.88 / 80.6</td>
<td>1.84 / 96.4</td>
<td>1.08 / 85.2</td>
<td>0.67 / 74.2</td>
<td>1.41 / 91.6</td>
</tr>
<tr>
<td>[60, 120]</td>
<td>0.26 / 60.4</td>
<td>0.32 / 64.8</td>
<td>0.64 / 71.6</td>
<td>0.01 / 50.5</td>
<td>2.28 / 99.0</td>
</tr>
<tr>
<td>[120, 300]</td>
<td>6.97 / &gt;99.8</td>
<td>5.36 / &gt;99.8</td>
<td>0.92 / 83.0</td>
<td>2.45 / 99.4</td>
<td>3.58 / &gt;99.8</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$\sigma_{\alpha(r_2)}$</th>
<th>$\sigma_{\alpha(r_{10})}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>[2, 1024]</td>
<td>4.16 / &gt;99.8</td>
</tr>
<tr>
<td>[2, 20]</td>
<td>0.48 / 69.2</td>
</tr>
<tr>
<td>[20, 60]</td>
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</tr>
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<td>[60, 120]</td>
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</tr>
<tr>
<td>[120, 300]</td>
<td>3.54 / &gt;99.8</td>
</tr>
</tbody>
</table>

$\chi^2_{\langle \alpha(r_2) \rangle, \sigma_{\alpha(r_2)}}$:

| [2, 1024] | 24.55 / >99.8 | 14.44 / >99.8 | 0.94 / 84.4 |
| [2, 20]   | 0.90 / 85.2 | 7.67 / >99.8 | 8.47 / 99.8 |
| [20, 60]  | 0.82 / 83.4 | 4.03 / 99.2 | 0.31 / 50.4 |
| [60, 120] | 0.51 / 61.4 | 3.63 / 98.6 | 1.00 / 85.2 |

$\chi^2_{\langle \alpha(r_{10}) \rangle, \sigma_{\alpha(r_{10})}}$:

| [2, 1024] | 1.46 / 90.4 | 9.83 / >99.8 | 3.15 / 98.0 |
| [2, 20]   | 0.21 / 54.8 | 7.10 / >99.8 | 6.77 / 99.8 |
| [20, 60]  | 2.74 / 97.2 | 5.27 / 99.6 | 0.29 / 73.6 |
| [60, 120] | 0.38 / 50.2 | 2.09 / 94.2 | 0.43 / 75.8 |
| [120, 300] | 0.26 / 57.2 | 2.23 / 96.2 | 0.19 / 60.4 |
Probing non-Gaussianity

(CMB) data

Constrained Randomisation

Surrogate CMB maps with the same power spectrum (and partially randomised phases)

Calculation of statistical measures $M$ sensitive to higher order correlations

Statistical comparison in terms of e.g. significances, Confidence levels, etc.

$M$ derived from e.g.:
- Bispectrum
- N-point corr. Funct.
- Minkowski-functionals
- Wavelets
- Needlets
- Scaling indices
- Etc.