Halo mass function with $f_{NL}$, $g_{NL}$ and $\tau_{NL}$

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Outline

- Three simple local models: $f_{NL}$, $g_{NL}$, $T_{NL}$
- Primordial non-Gaussianity in the halo mass function
- Analytic estimates & N-body results
- Conclusions
\[ \Phi(x) = \Phi_G(x) + f_{NL}(\Phi^2(x) - \langle \Phi^2 \rangle) \]

- **Gaussian** random field, \( \Phi \)
- **same variance**, **positive skewness**
- **same variance**, **negative skewness**

(\( \Phi = \) primordial gravitational potential)

- **skewness** \( \langle \Phi^3 \rangle \sim f_{NL} \langle \Phi_G^2 \rangle^2 \)
- **kurtosis** \( \langle \Phi^4 \rangle_c \sim f_{NL}^2 \langle \Phi_G^2 \rangle^3 \)

Salopek and Bond 1990; Gangui, Lucchin, Matarrese, Mollerach 1994; Komatsu and Spergel 2001

\(-10 < f_{NL} < 74\)

WMAP, Komatsu et al 2010
What about $\Phi(x) = \Phi_G(x) + g_{NL} (\Phi_G(x)^3 - 3\Phi_G(x)\langle\Phi_G^2\rangle)$?

$g_{NL} > 0$: positive kurtosis

$g_{NL} < 0$: negative kurtosis

Current constraints:

$-12 < g_{NL}/10^5 < 16$

(WMAP, Fergusson et al 2010)

(Okamoto and Hu 2002; Enqvist and Nurmi 2005)
\[ \Phi(x) = \varphi_{G,i}(x) + \varphi_{G,c}(x) + \tilde{f}_{NL} (\varphi_{G,c}^2(x) - \langle \varphi_{G,c}^2 \rangle) \]

**Gaussian**

- **Negative skewness and usual kurtosis:** \( \tau_{NL} = (6/5f_{NL})^2 \)
- **Positive skewness and larger kurtosis:** \( \tau_{NL} > (6/5f_{NL})^2 \)

**Probability**

\[ \Phi^3 \sim f_{NL} \langle \Phi^2 \rangle^2 \]

\[ \langle \Phi^4 \rangle_c \sim \tau_{NL} \langle \Phi^2 \rangle^3 \]

**Current constraints:**

\[-6000 < \tau_{NL} < 33,000 \]

(WMAP, Smidt et al 2010)

(Lyth and Wands 2002; Ichikawa, Suyama, Takahishi, Yamaguchi (2008); Tseliakhovich, Hirata, Slosar 2010; Shandera, Dalal, Huterer 2010)
A Signature: more/fewer massive halos

dark matter halos form in peaks of the density field

$\delta \rho / \rho \uparrow \delta_c$
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dark matter halos form in peaks of the density field

$\delta \rho / \rho \uparrow \quad \delta c$

non-Gaussianity changes the number density of peaks

Gaussian

$f_{NL}, \tau_{NL} = (6/5 f_{NL})^2$

$f_{NL}, \tau_{NL} = 2(6/5 f_{NL})^2$

$f_{NL}=0, g_{NL}$

number of peaks $\Leftrightarrow$ number of halos

Lucchin & Matarrese 1988; Chiu, Ostriker, Strauss 1998; Robinson, Gawiser, Silk 2000; Matarrese, Verde, Jimenez 2000
Simplest approach for analytic mass function

number of peaks \(\approx\) number of halos

number of peaks \(\approx\) area in tail of PDF

PDF for \(\delta(M)\) \(\leftrightarrow\) \# of halos of mass \(M\)

(Press & Schechter 1974)

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Simplest approach for analytic mass function

number of peaks \( \approx \) number of halos

PDF for \( \delta(M) \leftrightarrow \# \) of halos of mass \( M \)

(Press & Schechter 1974)

\[ \ln(M/\rho_0) = \ln(n/\delta \rho_{\text{rms}}) \]

Simulations

Press-Schechter

Jenkins et al 2000

PDF for \( \delta(M) \leftrightarrow \# \) of halos of mass \( M \)

number of peaks \( \approx \) area in tail of PDF

probability

\[ \delta \rho/\rho \]

\[ \delta_c \]

only qualitative agreement for Gaussian cosmology

Lucchin & Matarrese 1988; Chiu, Ostriker, Strauss 1998; Robinson, Gawiser, Silk 2000; Matarrese, Verde, Jimenez 2000
Simplest approach for analytic mass function

number of peaks ≈ number of halos

PDF for \( \delta(M) \) ↔ # of halos of mass \( M \)

(Press & Schechter 1974)

number of peaks ≈ area in tail of PDF

But seems to work OK for the non-Gaussian correction \( n_{NG}(M)/n_{G}(M) \)

Pillepich, Porciani, Hahn 2008 (and others)

Lucchin & Matarrese 1988; Chiu, Ostriker, Strauss 1998;
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Simplest approach for analytic mass function

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Of course we need simulations to trust this, and once we have them we can just fit for \(n_{NG}(M)\)

Dalal, Dore, Huterer, Shirokov 2007

Lucchin & Matarrese 1988; Chiu, Ostiker, Strauss 1998; Robinson, Gawiser, Silk 2000; Matarrese, Verde, Jimenez 2000
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Cumulants easy to compute, pretty insensitive to "shape" of polyspectra ($\tau_{NL}$ terms log-divergent w/box size)  Boubeker & Lyth 2005
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A Signature: more/fewer massive halos
N-body simulations with $f_{NL}$, $g_{NL}$, and $\tau_{NL}$

\[ \Phi(x) = \Phi_G(x) + f_{NL} (\Phi_G(x)^2 - \langle \Phi_G^2 \rangle) \]

- $f_{NL}$
- $\Phi(x)$
- $\Phi_G(x)$
- $\langle \Phi_G^2 \rangle$
- $M (h^{-1} M_\odot)$
- Non-Gaussian correction
- $\tilde{n}_{NL}(M)/\tilde{n}_{Gaussian}(M)$
- Edgeworth, $f_{NL} = \pm 500$, $\tau_{NL} = (\frac{6}{5} f_{NL})^2$
- Log Edge., $f_{NL} = \pm 500$, $\tau_{NL} = (\frac{6}{5} f_{NL})^2$
A Signature: more/fewer massive halos
N-body simulations with $f_{\text{NL}}$, $g_{\text{NL}}$, and $\tau_{\text{NL}}$

$\Phi(x) = \Phi_G(x) + g_{\text{NL}} (\Phi_G(x)^3 - 3\Phi_G(x)\Phi_G^2)$?

kurtosis can have important effects on the mass function!

(see also Desjacques and Seljak 2010)
A Signature: more/fewer massive halos

N-body simulations with $f_{NL}$, $g_{NL}$, and $\tau_{NL}$

$f_{NL}$, $\tau_{NL}$ independent

$\tau_{NL} \neq (5/6 f_{NL})^2$ is noticeable!
A Signature: more/fewer massive halos

comparison of $f_{NL}$, $g_{NL}$, and $\tau_{NL}$

$\tau_{NL}$ looks like $f_{NL}$ with larger $f_{NL}$

$g_{NL}$ looks a little different
Summary

- $f_{NL}$, $g_{NL}$ and $\tau_{NL}$ non-Gaussian initial conditions can significantly change the abundance of dark matter halos.

- We've found an analytic description for the change to the halo mass function that compares well to N-body for $f_{NL}$, $g_{NL}$ and $\tau_{NL}$ -- perhaps it works for more general forms of NG?

See also Sugiyama's talk!