Enabling Large-scale Multidisciplinary Design Optimization through Adjoint Sensitivity Analysis

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This paper is written to honor Rafael T. Haftka's seminal contributions to the field of multidisciplinary design optimization. We focus on those contributions that had a direct impact on our research, namely: the adjoint method for computing derivatives, wing aerostructural design optimization, and architectures for multidisciplinary design optimization. For each of these topics, we describe Haftka's contributions, how they impacted our research, and examples of what they enabled us to do. The overarching theme of the contributions and developments described in this paper is the efficient computation of derivatives, which together with gradient-based optimizers enables the optimization with respect to large numbers of design variables, even when using costly high-fidelity models.

I. Introduction

Rafael T. Haftka is a pioneer in structural optimization and multidisciplinary design optimization (MDO) who has continued to relentlessly make new contributions in this area of research. Although neither of us (the authors) have collaborated with Haftka, his work laid down the foundation upon which we have built our research. In this paper, we review the contributions from Haftka that have been influencing our work, which has been aimed towards enabling large-scale MDO. We by no means intend to cover *all* of Haftka's contributions, but only those that we have benefited from. The key enabler for large-scale MDO has been the efficient computation of derivatives, and in particular, *coupled* derivatives. Haftka identified the need for computing derivatives (sensitivity analysis) efficiently and made key contributions on this topic. In addition, his seminal work on wing design optimization considering both structures and aerodynamics was visionary and inspired our work on high-fidelity aerostructural optimization.

We start this paper by reviewing Haftka's contributions in structural sensitivity analysis and structural optimization, which were the basis for the development of our general purpose finite-element analysis and optimization framework. Then, we look at Haftka's seminal contribution towards considering multiple disciplines in wing design optimization, which established the birth of MDO and laid the groundwork for our own work in high-fidelity aerostructural optimization. Finally, we look at the broader contributions that Haftka made to methods for solving MDO problems, and connect them to the latest work in coupled system sensitivities, which ended up being basis for the latest version of the OpenMDAO framework.

II. Structural Sensitivity Analysis and Optimization

The advent of the finite-element method (FEM) enabled the structural analysis of arbitrary geometries. As soon as FEM techniques were available, researchers proposed coupling them with numerical optimization algorithms to perform structural sizing [1, 2]. While there have been persistent efforts that use gradient-free optimization algorithms in structural design applications, gradient-based methods are still the only viable choice if we want to optimize design problems parameterized with hundreds of variables or more. Gradient-based algorithms require the derivatives of the objective and constraint functions with respect to the design variables, and as Adelman and Haftka [3] pointed out,

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"Early attempts to use formal optimization for large structural systems resulted in excessively long and expensive computer runs. Examination of the optimization procedures indicated that the predominant contributor to the cost and time was the calculation of derivatives."

This motivated researchers to seek efficient methods for computing derivatives [4]. In the rest of this section we introduce these methods for computing derivatives and how we implemented them in a parallel structural finite-element solver.

There are various options for computing derivatives, as covered in detail in two papers coauthored by Haftka [4, 5]. Finite differences were used to compute the derivatives required for gradient-based optimization early on because they are the most straightforward approach. However, the cost of the finite-difference method is proportional to the number of design variables and the method suffers from serious accuracy issues—the step-size dilemma [6].

Analytic methods address both accuracy and efficiency issues. These methods consist in differentiating the governing equations and result in linear equations whose solution yields the desired derivatives. The vector of unknowns in these linear equations is of the same size as the vector of unknowns for the governing equations.

There are two approaches to the differentiation of the governing equations. The *continuous* approach consists in differentiating the continuous governing equations and then discretizing those equations for numerical solution. This approach was used by Haftka and Mroz [7] to compute both first- and second-order derivatives. Instead of differentiating and then discretizing, the *discrete* approach differentiates the discretized governing equations [3]. Over time, the discrete approach has become dominant. In the discrete approach, the matrix in the linear system is the Jacobian of the governing equations, i.e., the derivatives of the residuals of the discretized governing equations with respect to the states of those equations. In structural analysis, this Jacobian corresponds to the stiffness matrix.

Another major distinction among the analytic methods has to do with the form of the linear equations derived from the governing equations and yields two options: the direct method and the adjoint method. Both of these methods share the same Jacobian matrix in the linear system, which is thus of the same size; the difference is in the vector of unknowns and the right-hand size. More significantly, the cost of these two methods scales differently with the number of variables and the number of functions to be differentiated. The cost of either method is dominated by the number of times the linear system needs to be solved. The direct method requires the solution of the linear system *for each design variable*, while the adjoint method requires the solution of the linear system *for each function*. Thus, the adjoint method is particularly advantageous for optimization problems with large numbers of variables, as illustrated in Figure 1. However, such problems typically also involve a large number of constraints, in which case neither the adjoint or the direct method would have an advantage. We will see later, however, that constraint aggregation mitigates this issue [8].

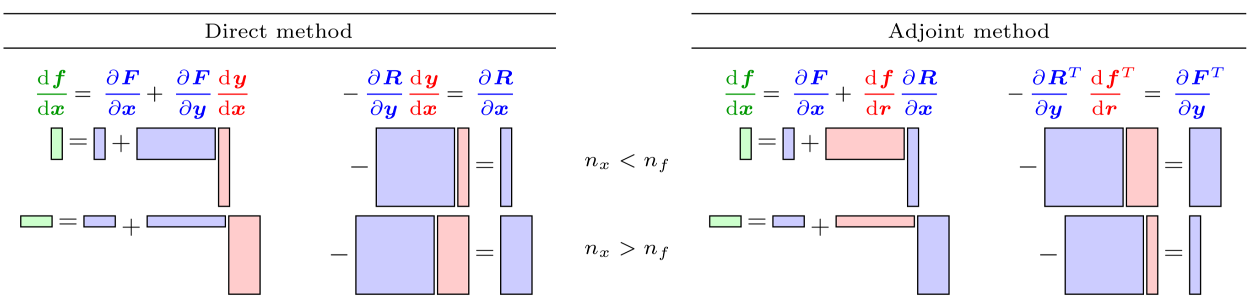


Fig. 1 Direct and adjoint equations, with diagrams illustrating the matrix and vector sizes for a problem with a large number of functions of interest (top), versus a problem with a large number of design variables (bottom) (figure from Martins and Hwang [9])

Adelman and Haftka [3] detail the early progress of methods for computing derivatives. Adjoint methods originate from optimal control theory [10] and were soon thereafter transferred to the structural design community [11, 12]. Both adjoint and direct methods were applied to compute derivatives with respect to sizing variables [13, 14] and shape variables [15, 16]. A more recent review coauthored by Haftka [5] includes the developments in the two decades after Adelman and Haftka [3]'s earlier review.

Our own quest for large-scale structural design optimization resulted in the development of TACS [17], an open source structural finite-element tool specifically designed for both stand-alone structural optimization problems, and multidisciplinary analysis and design problems where the structures discipline is an integral component. The development of TACS was driven in large part by Haftka's findings. Venkatamaran and Haftka [18] wrote an insightful paper on the progress in structural optimization in light of the increasing computational power available to engineers. A key observation made in their paper was that structural optimization complexity can be measured along three axes of increasing complexity: model complexity, analysis complexity, and design complexity (see Figure 2).

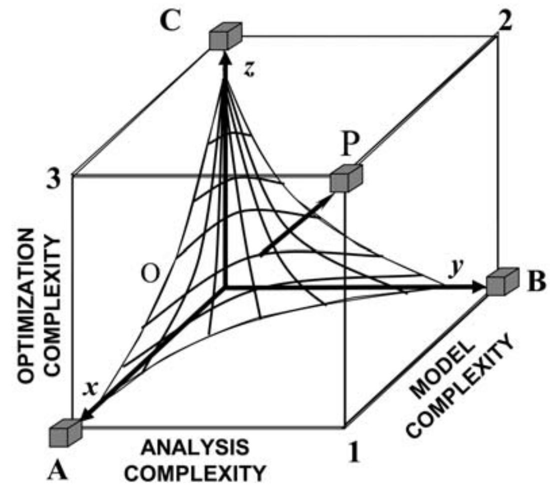


Fig. 2 Venkatamaran and Haftka [18] classified the complexity of structural optimization problems in three distinct axes: analysis, model, and design complexity.

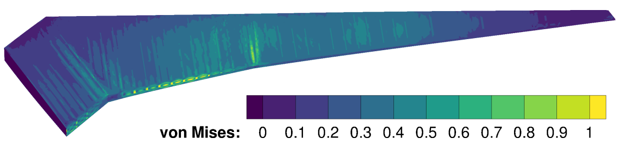
Model complexity is measured not only by the number of degrees of freedom in a structural model, but also in the type of modeling elements. Analysis complexity proceeds from static linear analysis, to natural frequency and buckling, to transient and nonlinear analyses. Design and optimization complexity consists in adding additional load cases and using more sophisticated optimization algorithms. In their paper, Venkatamaran and Haftka note that

> "The desire of the structural optimization specialist is to reach point P where all three levels of complexity are maximal. This appears a difficult if not impossible target as disciplinary specialists constantly extend the limits of points A, B, and C. Therefore state-of-the-art optimization problems will have to form a surface in the box-shaped region shown above, on which moving further along any one direction will require reduction in the other two directions."

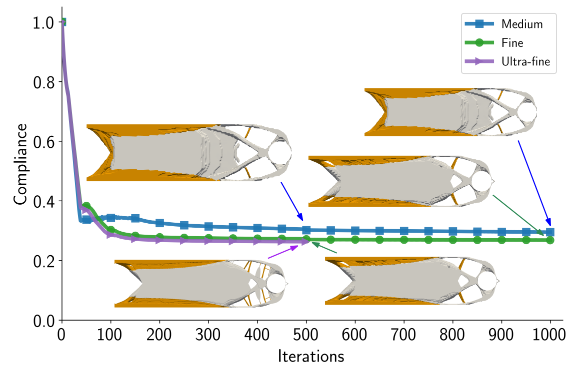
Understanding and addressing these optimization challenges through the development of new tools has motivated us to ensure that TACS achieves good parallel scalability, scalability in the number of design variables, and scalability in the number of functions of interest. Model complexity is addressed through parallel scalability, while optimization complexity is addressed through efficient derivative computation for multiple functions of interest.

In addition, the survey paper by Sobieszczanski-Sobieski and Haftka [19] highlighted that tight integration of analysis and design is almost always superior to loosely integrated architectures, which was also observed by Tedford and Martins [20]. As will be further discussed in the next section, this steered us towards developing disciplinary analysis tools that can be efficiently integrated within a monolithic MDO architecture, and towards satisfying the requirement that this tight integration be performed in a modular way. These two requirements are not in conflict if sound software engineering principles are consistently applied. Our approach has therefore been to develop TACS and other disciplinary tools such that they are designed from the ground up to be tightly integrated with other disciplines, both for analysis and adjoint derivative computation. These motivations can be traced back directly to the work of Haftka and his coauthors.

Efficient and modular adjoint implementations require a concerted and sustained code development effort. As a result of its modularity, TACS has been utilized in numerous research projects on a broad spectrum of applications. The application programming interface (API) has remained the same, which has enabled its use by multiple researchers across our research groups. We have found that a key attribute for usability has is the Python API, which supports almost all high-level operations that are available from the underlying C++ API. The use of Python has also facilitated the use by new students, for which a compiled language is more challenging to work with when implementing new analysis and optimization problems. The Python API was also used in the integration with the high-fidelity aerostructural optimization framework discussed in the next section. Overall, we have found that Python is far more valuable than a solver-specific scripting language.



(a) Wingbox optimization of the uCRM configuration (figure from Kiviaho and Kennedy [21]).



(b) Large-scale topology optimization (figure from Chin et al. [22]).

Fig. 3 Examples of TACS applications.

TACS has been applied to a wide range of structural design problems, from structural sizing problems with hundreds of design variables [23] to topology optimization problems with hundreds of millions of design variables [22]. The first applications of TACS focused on the design of built-up wingbox structures for transport aircraft wings [24, 25]. An application of TACS for stress-constrained mass minimization of the uCRM wingbox [26] is shown in Figure 3a. More recently, TACS has been applied to large-scale topology optimization applications [22, 27]. Figure 3b shows the application of TACS to a multimaterial compliance minimization problem using meshes with up to 329 million nodes and 125 million design variables [22]. In these topology optimization applications, TACS scaled up to 460 processors. While the primary mode of operation is static or dynamic analysis, TACS also supports a wide range of additional structural design problems. These include eigenvalue problems for natural frequency and buckling analysis [27] and time-dependent geometrically nonlinear flexible multibody dynamics [28]. In addition, efficient Hessian-vector products have been used to accelerate certain types of optimization problems using second-order information [29].

Haftka and his coauthors were among the first to recognize the challenge of structural design optimization with stress constraints [30, 31]. Stress constraints pose a challenge because the number of discrete stress constraints can easily exceed the dimension of the design variable vector. This means that the number of functions that need to be differentiated and the number of design variables is equally large, making neither the adjoint or the direct method particularly efficient [30]. As a result, Haftka and coauthors [31] proposed the use of the Kreisselmeier–Steinhauser (KS) aggregation function [32], which smoothly approximates an upper bound on the stress within a structure. By aggregating multiple constraint functions, the adjoint method becomes advantageous, enabling optimization with respect to large numbers of design variables *and* constraints.

In our own research, we have pursued refinements on stress aggregation functions. Poon and Martins [33] proposed a refinement on the original KS function, where the selection of the KS parameter controlling aggregation accuracy is selected adaptively, essentially removing it as a parameter. Later, Kennedy and Hicken [34] performed an analysis of aggregation functions and proposed a family of functions called induced aggregates, which exhibit better accuracy than the KS function but are non-conservative. Kennedy [35] proposed an adaptive optimization strategy integrated with an interior point method to adaptively increment the KS parameter over the course of an optimization. Finally, Lambe et al. [36] studied the application of different aggregation strategies to structural wingbox optimization. Overall, constraint aggregation has enabled us to leverage the adjoint method to solve problems that would be impossible to solve otherwise.

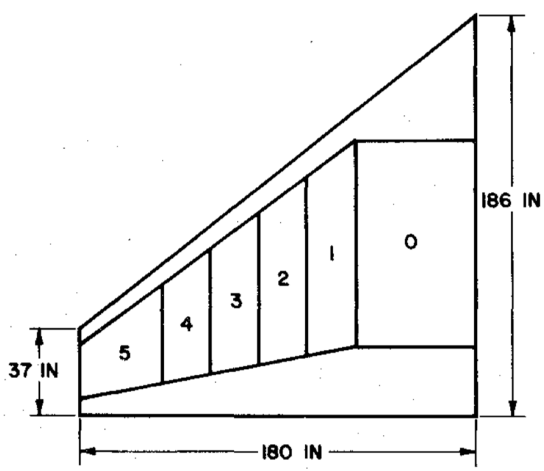
Haftka has also contributed significantly to the design optimization of structures made with composite materials.

Composite layup design problems are often discrete in nature because the number of plies is an integer and the ply angles themselves are often limited to a fixed number of discrete selections [37]. There are numerous methods to handle the combinatorial laminate design space that essential fall into three categories: (1) methods that treat the discrete variables directly, (2) relaxation methods that transform the discrete problem into a continuous problem either through lamination parameters or SIMP-like parametrization, or (3) hybrid methods that combine categories (1) and (2), either sequentially or hierarchically. Haftka and coauthors have written numerous papers that encompass all three of these categories.

Haftka first demonstrated that a certain class of stacking sequence optimization problem for buckling can be treated as a mixed integer linear optimization problem [38]. While linear mixed integer problems can be solved exactly, given sufficient computing time, global optimality for general nonlinear mixed integer problems is not guaranteed. To address this, Haftka and his coauthors pioneered the application of genetic algorithms (GAs) for composite layup design [39–43]. For large-scale problems, GAs are computationally challenging. To address this, Haftka and his coauthors pursued bi-level methods [44], and methods that leverage lamination parameters [45, 46]. Haftka's work on bi-level optimization strategies for composite structures leveraged his work on multi-level MDO architectures [47–49].

III. Wing Design Optimization

Haftka's initial contributions to structural optimization broadened significantly when he identified the need for considering aerodynamic performance in wing design, and the possibility of performing wing MDO. His 1977 paper entitled "Optimization Flexible Wing Structures Subject Strength Induced Drag Constraints" was well ahead of its time, and laid down the roadmap for decades of research on this topic [50]. In that work, Haftka performs gradient-based numerical optimization of a fighter wing using a Newton method with up to 10 structural design variables. Figure 4 is a reproduction of two illustrations from that paper. The approach was seminal in that it proposed a way to perform optimal tradeoffs between weight and drag for the first time, as shown in Figure 4. He compared the optimized results for aluminum and composite wings, showing that the composite wing achieved lower drag due to the additional tailoring of the structural properties.



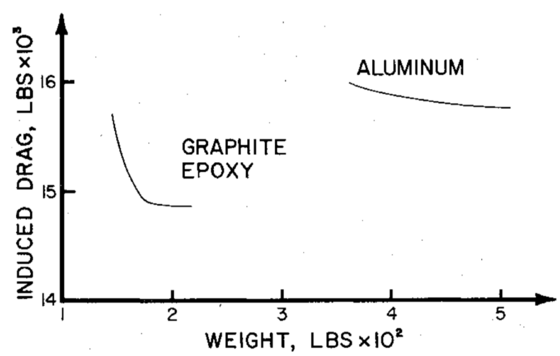


Fig. 4 Haftka's 1977 fighter wing MDO was well ahead of its time. Model and sizing variables (left) and Pareto front showing tradeoff between induced drag and structural weight (right) (figure from Haftka [50]).

Haftka followed up this work with colleagues a decade later in a landmark paper that demonstrated wing optimization with respect to structural sizing, wing twist, and wing planform of a glider [51]. The wing was modeled by coupling an aerodynamic lifting-line model to a beam model. Significantly, they compared sequential optimization of the aerodynamics and structures to the integrated design approach (MDO), showing that MDO achieved a better design.

Haftka et al. [52] followed up these efforts with work towards approaches for reducing the "enormous computational cost" of performing aerostructural optimization. In this paper, they also cited composite materials as requiring a "more integrated multidisciplinary design process" to take full advantage of tailoring. The fidelity of the models was increased by replacing the lifting-line with a panel method for the aerodynamic loads on the wing and the beam model with finite-element analysis of the wing structure. They also identified the need for *coupled derivatives* and the computational bottleneck that they represent. Together with the costly coupled analysis using the limited computer hardware of the time, they concluded that approximations methods would have to be employed.

Just one year later, Haftka tackled the aerostructural design optimization of a forward-swept transport wing following the same vision of integrated design, in collaboration with several colleagues [53]. The major contribution in that work was the formulation of a modular way to compute the derivatives of the coupled system, which was first proposed by Sobieszczanski-Sobieski [54], who was one of the coauthors in that paper. Their design problem maximized range, which represented a truly multidisciplinary objective function that was able to quantify the tradeoffs between drag and weight. The coupled derivative computation informed not just the gradient-based solver, but also provided the Jacobian of the system for a coupled Newton solver.

The work of Haftka and his colleagues cited above provided the motivation for our own work in high-fidelity aerostructural design optimization because it pointed out the need for MDO in wing design and identified the computational bottleneck of computing the coupled derivatives. In addition, the work provided guidance on ways to compute these derivatives.

Sobieszczanski-Sobieski [54] had derived two different versions of what they called the global sensitivity equations (GSE), which amounts to a *direct* coupled derivative computation approach. As mentioned above, however, the cost of direct methods is proportional to the number of design variables, which is not desirable. To address this issue, we developed the *coupled adjoint method* and implemented it in a framework consisting of an Euler computational fluid dynamics (CFD) solver coupled to a finite-element structural solver [55]. In this work, we showed that it was possible to compute the derivatives of the coupled aerostructural problem required to solve a wing design optimization problem. The first problem that we tackled was the aerostructural design optimization of a supersonic business jet [56], where the range was maximized with respect to wingbox sizing variables, wing twist, airfoil shapes, and fuselage camber. While this enabled the optimization with respect to almost one hundred design variables using models that were considered high-fidelity at the time, the coupled derivative computations showed some dependence on the number of variables, and did not scale well with the number of wing surface displacements. Because the coupled-adjoint equations include the Jacobian of all governing equations with respect to all the state variables, there are off-diagonal terms that represent the derivatives of the governing equations of one discipline with respect to the states of the other. These off-diagonal terms were the ones responsible for the lack of scalability because they were computed using finite differences.

Over the next several years, we applied the lessons learned in the development of the aerostructural framework above to develop an all-new framework that was more modular, more efficient, and that used higher-fidelity models. The end result was the MACH framework (MDO for aircraft configurations with high fidelity) [57], which we first applied to the aerostructural optimization of a transonic transport configuration [25]. Again, the key enabler in this framework is the coupled adjoint approach for efficiently computing the derivatives of the objective and constraints with respect to large numbers of design variables from both disciplines.

The modularity of the framework was achieved by wrapping the various modules with Python. The structural finite-element solver used in MACH is TACS, which was already described above, and its parallel adjoint solver is a key component. The CFD solver is ADflow, an open-source code that solves the Reynolds-averaged Navier–Stokes equations on structured overset meshes. Like TACS, ADflow is a parallel code with an efficient adjoint implementation for computing derivatives. To compute coupled derivatives, however, it is not sufficient to have the derivatives of each discipline. This is because, as previously mentioned, the Jacobian of the coupled system involves off-diagonal blocks. We developed efficient ways of computing the derivatives in these off-diagonal blocks, which resulted in a truly scalable coupled derivative computation [57].

In addition to the CFD module, we also developed a 3D panel code with estimates for profile and wave drag that provides a less costly alternative for the aerodynamic model in MACH. The rapid aerostructural optimization capability enabled by this lower fidelity model and its coupled adjoint enabled us to investigate wing planform optimization and tradeoffs between takeoff gross weight (TOGW) and fuel burn, much in the same spirit of Haftka's 1977 paper [58]. The resulting Pareto front from Kennedy et al. [58] is reproduced in Figure 5, which shows fronts for both metallic and composite wings. The fuel burn and TOGW are surrogates for two conflicting objectives in aircraft design: direct operating cost and acquisition cost, respectively. The minimum fuel burn extremes ($\beta = 1$) are biased towards drag minimization and result in higher aspect ratio wings that reduce the lift-induced drag. The minimum TOGW extremes

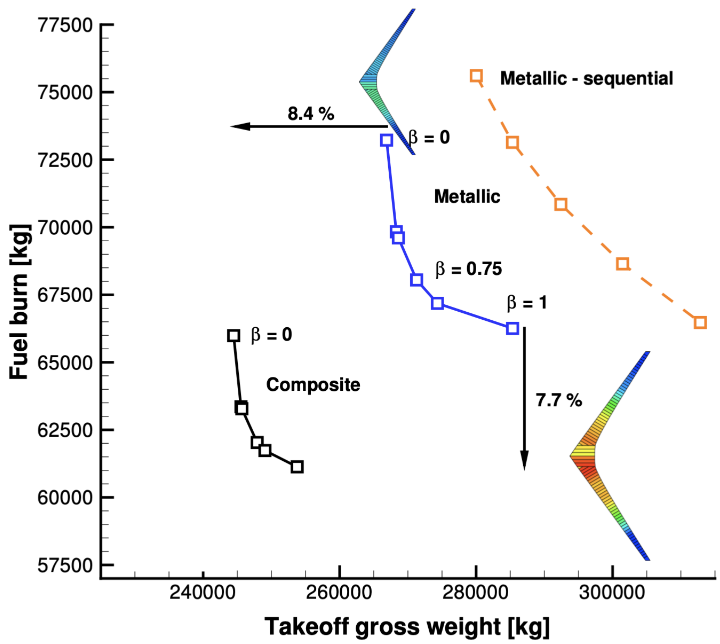


Fig. 5 Pareto front showing tradeoff between fuel burn and weight for a transonic twin-aisle aircraft (adapted from Kennedy et al. [58]).

($\beta = 0$) result in lighter wings that burn more fuel because this objective is biased towards structural weight. Similarly to Haftka, we were curious to compare the results of a sequential optimization process versus and integrated one. The integrated approach was shown to produce better results for all points in the Pareto front, as shown in Figure 5, consistent with Haftka's original findings. The overall conclusion of this study was that while airframe and material technologies can be leveraged to increase aircraft performance, the design optimization approach is equally important.

Since the first application cited above [24], we have been using RANS CFD for the aerodynamic model, which is much more realistic, especially in the transonic regime. The RANS-based version of MACH has been used in a variety of wing aerostructural design studies [26, 59–63]. An example of such an aerostructural optimization is shown in Figures 6 and 7. The optimization is performed with respect to 972 design variables that consist in structural sizing, wing shape, angle of attack, and tail rotation angle, as shown on the right in Figure 6. The wing shape is linked to the wingbox shape, and consists in airfoil shapes at eight spanwise stations, twist distribution, span, and sweep.

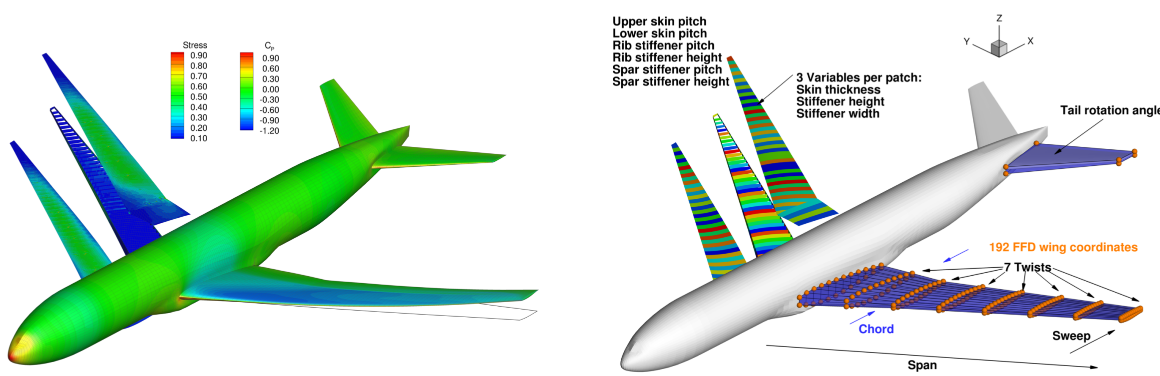


Fig. 6 High-fidelity aerostructural optimization model (left) and design variables (right) (figure from Kenway and Martins [64]).

The aircraft performance is computed at multiple cruise flight conditions, and stress and buckling constraints are enforced at three maneuver conditions. There are no constraints on span, but since we are considering aerodynamics and structures simultaneously, spans that are too large are penalized by the increase in structural weight required to satisfy the structural constraints. This can be seen in Figure 7, where we show the results of two optimizations. Both optimization problems consider the same flight conditions and constraints, and the set of design variables is also the identical. The only difference between the two problems is the objective function. The wing on the left is optimized for minimum TOGW, while the wing on the right is the result of a fuel burn minimization. This corresponds to the two extremes in the Pareto front we described earlier.

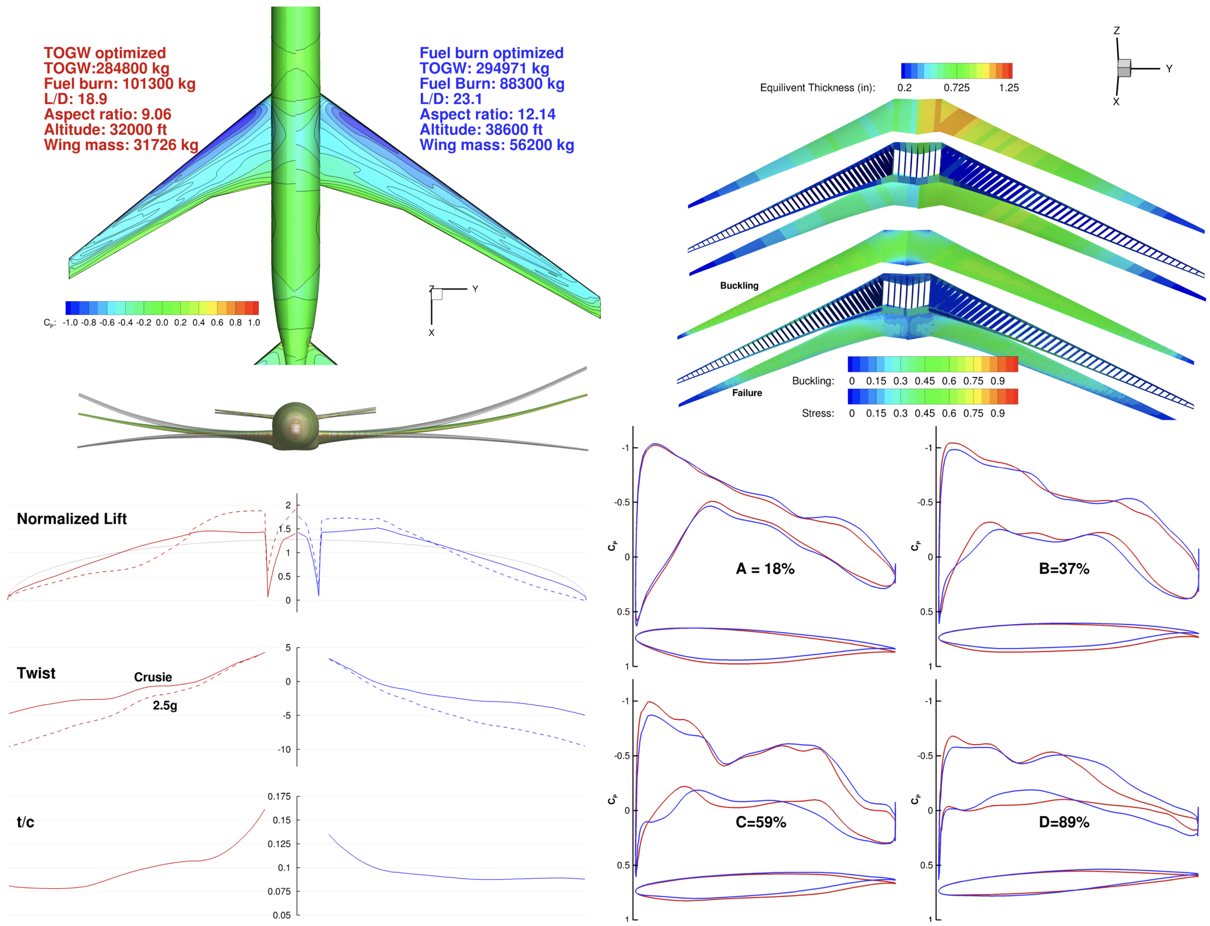


Fig. 7 High-fidelity aerostructural optimization results for minimum TOGW (left wing) and minimum fuel burn (right wing) (figure from Kenway and Martins [64]).

Another study similar to the above resulted in the development of open high-fidelity aerostructural models based on NASA's Common Research Model (CRM). These models consist of a configuration with the same flying shape as the original one, the uCRM-9, and a higher aspect ratio version, the uCRM-13.5 [26], which was obtained through full aerostructural optimization.

In other recent work, we developed methods in MACH for performing the aerostructural optimization of wings manufactured with tow-steered composites [63]. The project was funded by NASA and led by Aurora Flight Sciences, who manufactured a 1/3 scale model of the wingbox of one of our optimized high aspect ratio wings using an automatic fiber placement machine. The model underwent structural testing at NASA Armstrong Research Center earlier in the Summer of 2018.

Another validation or our optimization approaches took place recently for a marine application. Garg et al. [65]

performed the hydrostructural optimization of an hydrofoil with a linear taper planform using MACH. The optimization problem consisted in minimizing drag with respect to angle of attack, airfoil shapes, and twist distribution. Constraints were enforced on lift coefficient, stress, and cavitation. One major difference in this problem relative to the aircraft wing problems cited above is that structure is solid (as opposed to thin walled) due to the much higher density of the fluid, which results in higher loads. In a follow-up article, Garg et al. [66] built and tested the baseline and optimized hydrofoils in a water tunnel, obtaining good agreement.

After his first contributions, Haftka continued to be involved in the MDO of aircraft configurations as a member in the multidisciplinary analysis and design (MAD) center at Virginia Tech. He was involved in the first MDO study of the strut-braced wing configuration [67]. The study used a framework that coupled CFD to a detailed finite-element model of the wingbox and considered flutter constraints, engine sizing, and mission analysis. The blended-wing body is another unconventional aircraft configuration that Haftka and his team studied using MDO [68]. In that work, they focused on the distributed propulsion aspect of the configuration, while considering aerodynamics, structural weight, and stability and control.

IV. MDO Methodologies

The pioneering work of Haftka in MDO started with the first aerostructural wing design optimization mentioned in the previous section [50]. Thus, MDO emerged through an application first and only later it was formalized and generalized, where Haftka also made key contributions.

Haftka et al. [69] published one of the first papers reviewing the options for MDO at that time. They covered methods for converging multidisciplinary analysis, computing coupled sensitivities, and optimization problem formulation. Soon after that, they also discussed and analyzed a decomposition algorithm [70].

In their timely review on MDO of aerospace systems, Sobieszczanski-Sobieski and Haftka [19] define MDO as a "methodology for the design of systems in which strong interaction between disciplines motivates designers to simultaneously manipulate variables in several disciplines." To clarify this definition, they state an example that some might consider to be MDO: "structural optimization of aircraft wings to prevent flutter is not MDO. In this case, the interaction of structures and aerodynamics is analyzed; however, the aerodynamic shape of the wing is not optimized." On the methodologies side, in addition to approximation concepts, system decomposition, and human interfacing, Sobieszczanski-Sobieski and Haftka [19] devoted a section to "system sensitivity analysis", where they mention the progress made in computing coupled derivatives and how it had been used in design optimization thus far. On the applications side, the review includes two major areas: "simultaneous aerodynamic and structural optimization" (now called aerostructural optimization) and "simultaneous structures and control optimization".

Haftka showed a strong interest in MDO *architectures*, that is, the combination of the organization strategy for the various disciplines and the problem formulation in MDO. In their survey of MDO architectures, Martins and Lambe [71] cite 17 of Haftka's papers. In collaboration with computer scientists, Haftka developed a parallel MDO approach using variable-complexity modeling and multipoint surrogate models that they applied to the design optimization of a supersonic civil transport [72]. Haftka and Watson [73] developed the quasi-separable decomposition architecture, which efficiently solves problems where the system objective and constraint functions are dependent only on a subset of design variables. Haftka and Watson [49] extended this architecture to handle design variables that include both discrete and continuous types.

MDO architectures can be broadly divided into monolithic and distributed architectures (see Figure 8). Monolithic architectures involve the solution of a single optimization problem, while distributed architectures formulate separate optimization problems for each discipline, with a "system-level" optimization problem that coordinates the various discipline sub-problems. Haftka [74] proposed the ultimate monolithic architectures: simultaneous analysis and design (SAND). Although he proposed this in the context of a single discipline, demonstrating it in a structural optimization problem, the approach can be generalized for MDO problems. SAND combines the solution of the governing equations with the optimality conditions, and has been shown to be more efficient than the conventional approach not only for structural optimization [75, 76], but for aerodynamic optimization [77, 78] and MDO problems as well [20]. In a benchmarking study of various monolithic and distributed MDO architectures, Tedford and Martins [20] found that monolithic architectures vastly outperform distributed ones in terms of convergence time. In spite of several efforts, we were unable to develop new distributed architectures that exhibited good performance. For that reason, we re-focused our efforts on monolithic MDO architectures.

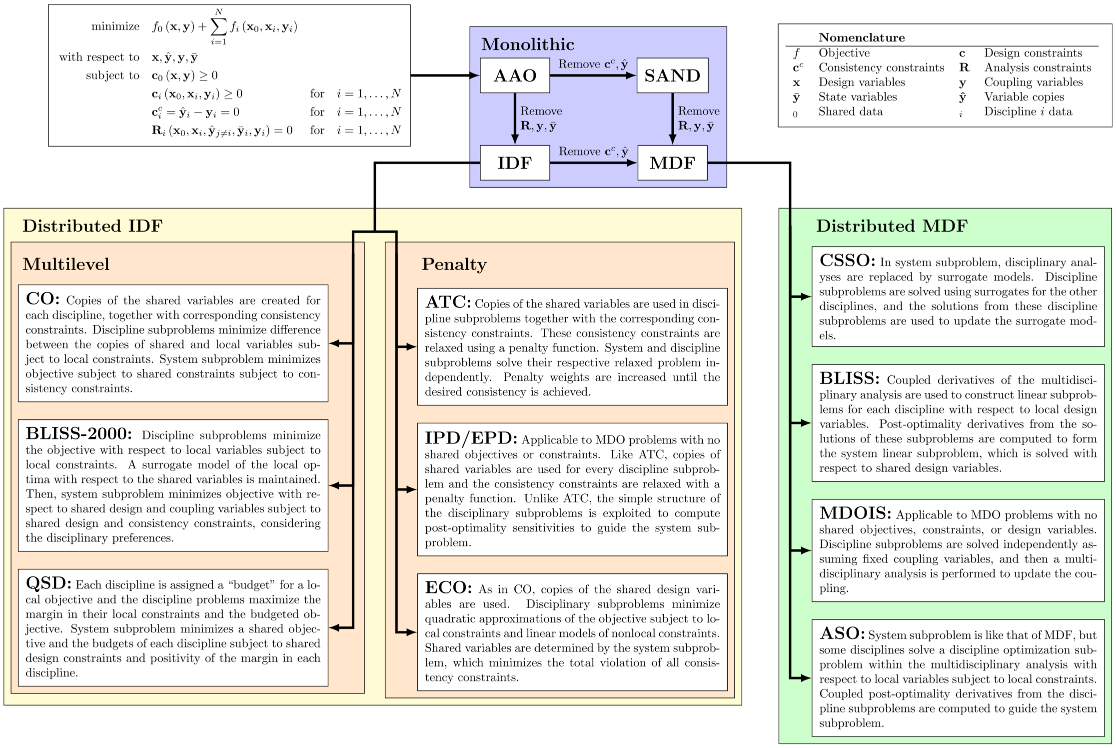


Fig. 8 Classification and summary of MDO architectures (figure from Martins and Lambe [71]).

As previously mentioned in the context of both single discipline and multidisciplinary problems, derivative computation is one of the potential bottlenecks. This is true not only in terms of computational time, but in terms of the *implementation effort* as well. Analytic derivative computation methods have historically required long development times (in the same order as the development of the solver itself). Coupled analytic methods require the analytic methods implemented for each discipline to be implemented and are further complicated by the off-diagonal blocks in the Jacobian that represent the derivatives of all disciplines with respect to all others. The number of these off-diagonal blocks grows with the square of the number of disciplines, which is an unfavorable scaling.

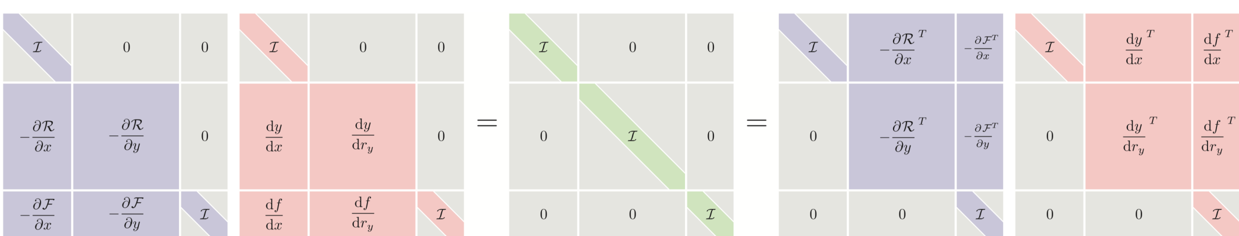


Fig. 9 Block matrix structure of the unifying derivative equation (figure from Hwang and Martins [79]).

In an effort to unify the theory behind all methods for computing derivatives, Martins and Hwang [9] derived an equation—the unifying derivatives equation—from which all methods can be derived (see Figure 9). Although this equation was just envisioned to be a curiosity, we learned through its derivation that some concepts that inspired us to develop a new monolithic MDO architecture that we called modular analysis and unified derivatives (MAUD) [79]. MAUD is essentially a generalization of the high-fidelity aerostructural optimization described above for arbitrary disciplines. MAUD can use fixed-point or Newton-type methods for the convergence of the multidisciplinary system, and it implements both direct and adjoint coupled derivative computations. It does all this through a compact API based on governing equation residuals and can handle both explicit and implicit equations [79]. In MAUD, we use the notion of *components* that encapsulate sets of governing equations instead of disciplines for the sake of generality. Components can be arranged hierarchically and parallel computing is used to speed up both the solution of the coupled system and the derivative computation. The MAUD architecture was first demonstrated in the MDO of a satellite involving 25,000 design variables and 2.2 million state variables [80].

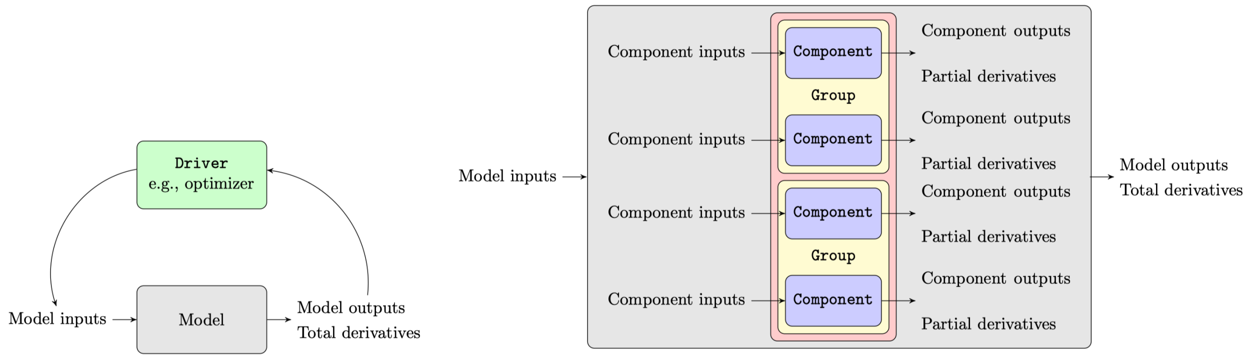


Fig. 10 Overall view of the OpenMDAO framework software design, showing the relationship between the "Driver" and "Model" classes. Models are composed of a hierarchy of Group and Component instances, each of which has its own derivatives contributing to the coupled derivatives of the complete model (figure from Gray et al. [81]).

The MAUD architecture has been recently implemented in OpenMDAO [81], and has enabled the solution of large-scale optimization problems. Figure 10 shows the overall object-oriented design of OpenMDAO. In addition to implementing MAUD, OpenMDAO has a vast array of practical convenient features * and implements new monolithic and hierarchical solution strategies. Many of these problems have focused on aircraft design, such as a problem coupling trajectory optimization, and aerodynamic performance with the objective of maximizing airline profit [82]; the simultaneous design of aerodynamic shape and propulsor sizing for boundary layer ingestions configurations [83]; a conceptual design model for aircraft electric propulsion [84]; a mission planning tool for the X-57 aircraft [85]; the design optimization of a next-generation airliner considering operations and economics [86]; the design and trajectory optimization of a morphing wing aircraft [87]; and trajectory optimization of an aircraft with a fuel thermal management system [88]. OpenMDAO is also being used extensively by the wind energy community for wind turbine design [89–92] and wind farm layout [93, 94].

Finally, circling back to Haftka's 1977 paper, we have used OpenMDAO to implement a low-fidelity aerostructural analysis and design optimization tool called OpenAeroStruct [95]. This tool couples an aerodynamic panel code to a beam finite-element model and computes coupled derivatives using the adjoint method for rapid design optimization. A sample of an optimization is shown in Figure 11. OpenAeroStruct provides an open-source platform for students to learn about MDO and wing design tradeoffs, but it is has also been used beyond the classroom in aircraft design efforts [96, 97], as well as in research into uncertainty quantification [98–106].

V. Conclusions

While the authors have not (yet!) collaborated directly with Raphael Haftka, we have greatly benefited from his contributions, which provided the foundation and inspiration for much of our work. In other words, Haftka is a giant whose shoulders we stand on, enabling us to see further in the MDO research landscape. The goal of this paper was to do justice to some of Haftka's key contributions and to explain how we used them in our own work.

Haftka established early on that gradient-based optimization was a promising approach to structural design and identified a potential bottleneck in the computation of derivatives. Together with his colleagues, he contributed to the

*http://openmdao.org

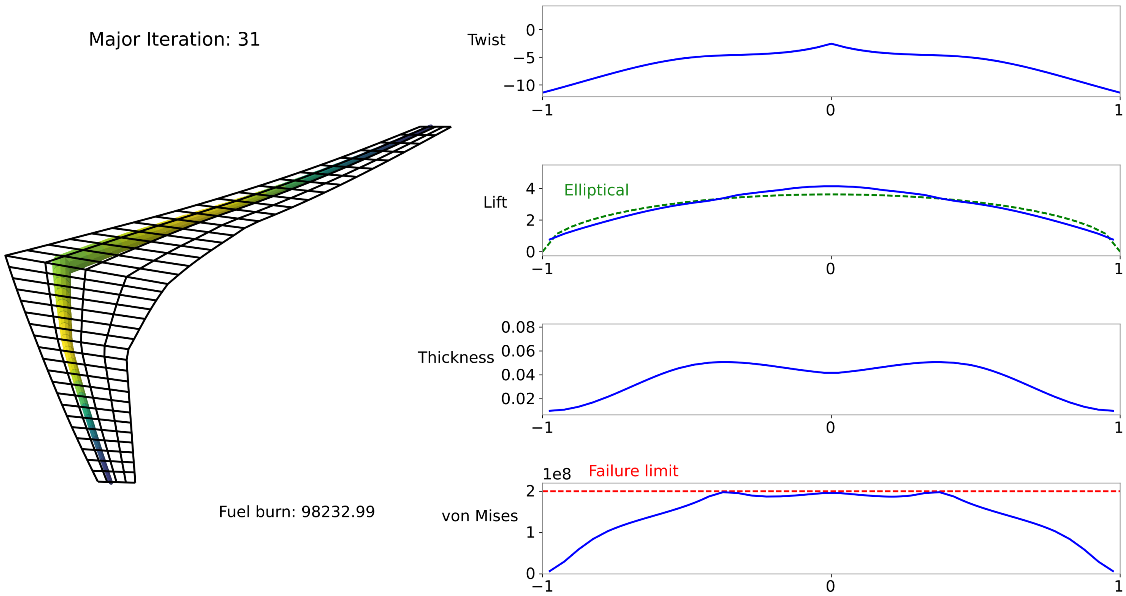


Fig. 11 Screenshot of interactive visualization tool included in OpenAeroStruct (figure from Jasa et al. [95]).

development and dissemination of the adjoint method in structural applications, a contribution that paid many dividends to the research community.

Within the topic of structural analysis and optimization, we have implemented the approaches that Haftka pioneered to develop an open-source structural finite-element analysis and design optimization framework. This framework implements state-of-the-art parallel numerical methods in the structural solver and its adjoint, which provides accurate and efficient computation of derivatives. This has enabled us to perform structural optimizations with up to hundreds of millions of design variables.

We coupled this structural analysis and design capability with CFD-based aerodynamic analysis and shape optimization to develop what amounts to a high-fidelity version of Haftka's vision of wing MDO. This high-fidelity aerostructural design optimization capability has been demonstrated with up to one thousand design variables that included both structural sizing and aerodynamic shape variables. This capability was in large part due to the coupled adjoint derivative computation, which is an extension of the contributions of Haftka and his colleagues. Finally, we tackled another issue identified by Haftka: the large effort required to implement coupled derivative computation for large-scale MDO problems. We addressed this challenge by developing an algorithmic framework that was eventually implemented in the OpenMDAO framework, which has already enabled the solution of problems of unprecedented scale.

We are thankful to Raphael Haftka for his contributions, which provided the inspiration for much of our work. We particularly appreciate Haftka's drive to publish high quality journal articles that *distill the concepts and conclusions, and communicate them clearly and succinctly*. Effective communication is often an undervalued aspect of research, and Haftka has been leading us by example in his writing, and through his relentless service as a reviewer and editor. We are inspired by Haftka's legacy, we have seen a similar positive impact on our colleagues, and we hope that future generations of researchers will find inspiration in his work and example as well.

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