Physics 406: Homework 1

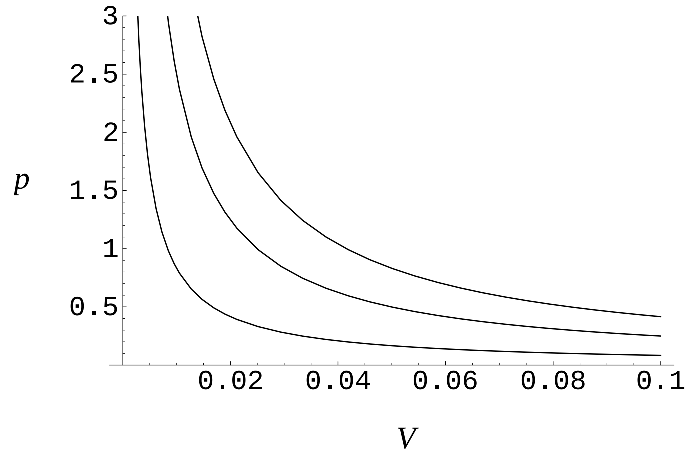
1. **Perfect and imperfect gases:** The pressure p and volume V of one (gram) mole of a perfect gas obey the equation of state $pV = RT$, where $R = 8.314\,\mathrm{J\,K^{-1}\,mol^{-1}}$.

   (a) How much work must be done to compress a mole of this gas at constant temperature T from volume V_1 to volume V_2? Verify that your answer is indeed positive when the volume of the gas gets *smaller*.

   (b) How much work must be done to compress a mole of this gas at constant temperature T from pressure p_1 to pressure p_2?

   (c) How much work must be done to compress *half* a mole of this gas at constant temperature $T = 20°\mathrm{C}$ from one atmosphere pressure to two atmospheres? (Recall that $0°\mathrm{C} \equiv 273.15\,\mathrm{K}$ at atmospheric pressure.)

   (d) The isotherms of a substance are the sets of states with a given temperature. For the perfect gas, this means lines of constant pV, which on a plot of p against V are hyperbolas thus:



   Except at low pressure this is not a very good representation of the behavior of a real gas. A better representation is the van der Waals equation of state:

$$(p + a/V^2)(V - b) = RT,$$

   where a and b are constants. What do the isotherms look like for the van der Waals gas? (Good ways to answer this question: (1) sketch the curves by hand; (2) use a graphing calculator and some reasonable parameter values, such as the ones from part (f) below, and copy the output by hand; (3) use Matlab or Mathematica or a similar computer program and print out a figure, as I did for the figure above.) Notice anything strange?

   (e) How much work is required to compress a mole of van der Waals gas from volume V_1 to volume V_2?

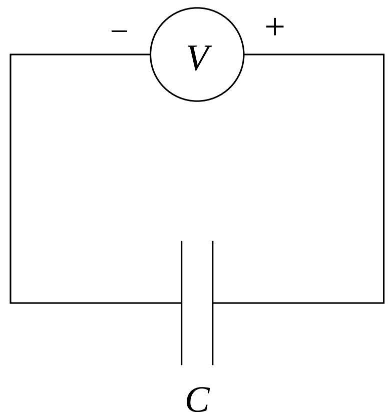
   (f) If $a = 0.136\,\mathrm{N\,m^4\,mol^{-2}}$ and $b = 3.64 \times 10^{-5}\,\mathrm{m^3\,mol^{-1}}$ (which are the values for air), how much work is required to compress 0.3 moles of the van der Waals gas from a volume of $5 \times 10^{-3}\,\mathrm{m^3}$ to a volume of $2 \times 10^{-5}\mathrm{m^3}$ at 300K? (Hint: look carefully at the units of the constants a and b.)

   (g) The **isothermal compressibility** κ_T of a gas measures the fractional change in volume of a gas as we change the pressure on it at constant temperature:

$$\kappa_T = -\frac{1}{V}\frac{\partial V}{\partial p}\bigg|_T .$$

Derive expressions for κ_T as a function of volume for one mole of a perfect and of a van der Waals gases. How would these expressions change if we had 2 moles of the gases?

2. **Electrical work:** Suppose we have a circuit in which a voltage source is connected to a capacitor with capacitance C (which may depend on temperature):



Starting with an uncharged capacitor and zero voltage, we isothermally increase the voltage, so as to charge the capacitor.

(a) If the charge across the capacitor is q at some point, what is the voltage across it?

(b) The definition of electrical work is the product of charge moved times the potential difference it moves across. So the amount of work $\text{đ}W$ that needs to be done by the voltage source in order to move an extra small amount of charge $\mathrm{d}q$ from one side of the capacitor to the other is $\text{đ}W = V\mathrm{d}q$. This is the equivalent of $\text{đ}W = -p\mathrm{d}V$ in our fluid system. If we charge the capacitor from empty up to a charge q, what is the total work done as a function of q and the capacitance C?

(c) And as a function of the voltage V?

(d) Capacitors heat up when you charge them. What is the differential form of the First Law for a change in internal energy of the capacitor (i.e., what is the expression for an infinitessimal change $\mathrm{d}U$ in the internal energy of the capacitor)? And what is the amount of heat absorbed $\text{đ}Q$ by the capacitor for a given change $\mathrm{d}U$ in internal energy and $\mathrm{d}q$ in charge?

(e) Hence write down an expression for the heat capacity c_q at constant charge of the capacitor, and another (more complicated) one for the heat capacity c_V at constant voltage. If the capacitance is independent of temperature, what is the expression for c_V?

(f) An infinitessimal change in the enthalpy of a pressure/volume system is, as we have seen, $\mathrm{d}H = \text{đ}Q + V\mathrm{d}p$. What is the corresponding expression for the enthalpy of a capacitor?

3. **Heat capacity of copper:** A 0.1 kg piece of copper metal is heated to 100°C and dropped into a thermally insulated vessel containing 0.2 liters of water at 15°C. After the system has come to equilibrium, the temperature is measured again and both copper and water are found to be at 18.8°C. Assuming the heat capacities of both water and copper to be constant over the range of temperatures involved, calculate the ratio of the specific heat capacities of copper and water.