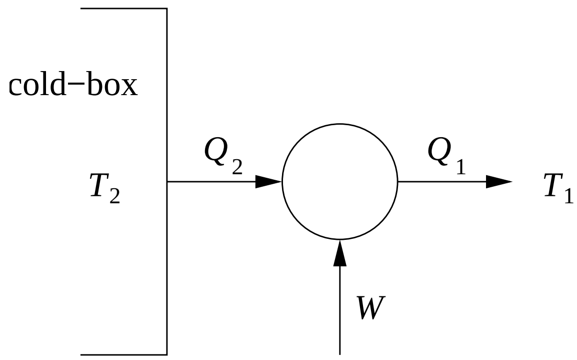
Physics 406: Homework 2

1. **Refrigerator:** An improbable domestic refrigerator, which contains a quart of milk, a bowl of petunias, and an unidentified green object left there by the previous tenant, is perfectly efficient. A refrigerator's efficiency is the ratio between the amount of heat Q_2 extracted from the cold-box and the amount of work W done: $\eta = Q_2/W$.



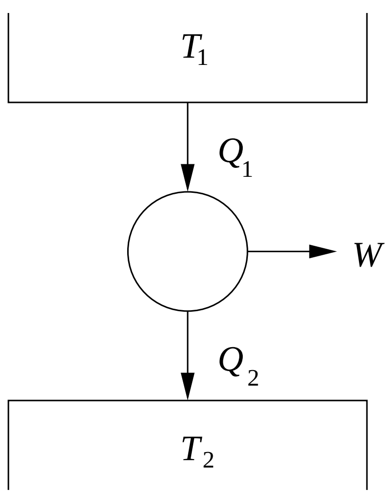
   (a) What is the efficiency of the refrigerator in terms of the temperatures T_1 and T_2 of the kitchen and the cold-box?

   (b) If the cold-box is at $-4°$C and the kitchen is at 23°C, what is the efficiency?

   (c) If the refrigerator consumes 100 Watts when running, at what rate does heat come out the back of the refrigerator?

   (d) A real refrigerator is of course not perfectly efficient. If a refrigerator with only 20% the efficiency is used to cool the same selection of objects to the same temperature, how much power will the fridge consume when running? And how much heat will come out the back?

2. **Heat engine:** A perfectly efficient heat engine operates between two very large tanks of water forming thermal reservoirs. One of them is at temperature T_1, which is about hot enough to boil an egg, and the other is at a brutally cold temperature of T_2:



   (a) The efficiency of the engine is defined as the ratio of the work it produces W to the heat that it takes in Q_1 thus: $\eta = W/Q_1$. If the heat engine is reversible, what is the value of η in terms of the heats Q_1 and Q_2 entering and leaving the reservoirs? And what is it in terms of the temperatures of the two reservoirs?

(b) If the hot reservoir is at 100°C and the cold one at 10°C, what is the value of η? Hence, show that we need to take 82.9 Joules of heat from the hot reservoir to generate just 20 Joules of work. How much heat is rejected into the cold reservoir as a result?

(c) Now suppose that the tanks of water are not that large after all. In fact, each of them contains just a few liters of water, the same amount in each, with heat capacity C, which we assume to be constant over the temperature range we are looking at. If small amounts of heat $đQ_1$ and $đQ_2$ enter and leave the two tanks at temperatures T_1 and T_2, write down again the expressions for the efficiency η in terms of the temperatures, and in terms of $đQ_1$ and $đQ_2$. Equate these two and prove that

$$\frac{đQ_1}{T_1} = \frac{đQ_2}{T_2}.$$

What is the change in temperature dT_1 of the hot reservoir, in terms of $đQ_1$ and the heat capacity? And the change dT_2 for the cold reservoir, in terms of $đQ_2$? (Hint: be careful about the signs of the temperature changes; remember that the hot reservoir will get colder when heat leaves it.) Combine all these expressions to show that

$$\frac{dT_1}{T_1} = -\frac{dT_2}{T_2}. \qquad (1)$$

(d) Now suppose that the hot reservoir starts off at temperature $T_1 = T_h$ and the cold one at $T_2 = T_c$. By running our heat engine, we cool the hot one and heat the cold one until they are both at the same final temperature T_f. Integrate Eq. (1) above and derive an expression for T_f in terms of T_h and T_c.

(e) If $T_h = 100°C$ and $T_c = 10°C$ as before, show that the final temperature of the system is 51.9°C, regardless of the heat capacity of the tanks.

(f) What is the change in entropy of the system from the beginning to the end?

3. **Entropy:** Suppose we took the two tanks from the previous question, again at temperatures T_h and T_c and simply put them in contact with one another, so that heat can flow from the hot one to the cold one.

(a) What are the changes of temperature dT_1 and dT_2 when heat $đQ$ flows from hot to cold? Hence show that $dT_1 = -dT_2$.

(b) What is the final temperature T_f' of both tanks when they come to equilibrium? For $T_h = 100°C$ and $T_c = 10°C$ as before, what is this final temperature?

(c) The entropy change of the hot tank is $dS_1 = -đQ/T_1$ and of the cold one is $dS_2 = đQ/T_2$. What is the total entropy change of the hot and cold tanks as they come to equilibrium at temperature T_f'?

(d) What is the total change in entropy of the system? If $T_h = 100°C$ and $T_c = 10°C$ and the heat capacity of the tanks are both $C = 10000\,\mathrm{J\,K^{-1}}$, how much does the entropy change?