Complex Systems 535/Physics 508: Homework 4

	species	is preyed on by		species	is preyed on by
0	phytoplankton	6, 7, 10, 11, 12, 21, 22, 33	17	crustacean deposit feeder	18, 24, 25, 26, 28, 31
1	bacteria in suspended poc	6, 7, 8, 10, 11, 12, 21, 22,	18	blue crab	32
2	bacteria in sediment poc	13, 14, 15, 16, 17	19	fish larvae	
3	benthic diatoms	16	20	alewife and blue herring	32
4	free bacteria	5	21	bay anchovy	26, 27, 29, 30, 31, 32
5	heterotrophic microflagel	6	22	menhaden	29, 31, 32
6	ciliates	7, 8, 10, 11, 12	23	shad	
7	zooplankton	8, 9, 19, 20, 21, 22, 23	24	croaker	
8	ctenophores	9	25	hogchoker	
9	sea nettle		26	spot	29
10	other suspension feeders	18	27	white perch	
11	mya arenaria	18, 25	28	catfish	
12	oysters		29	bluefish	
13	other polychaetes	24, 25, 26, 27, 28	30	weakfish	31
14	nereis	18, 24, 25, 26, 27, 28	31	summer flounder	
15	macoma	18, 26	32	striped bass	
16	meiofauna		33	dissolved organic carbon	4

1. **Food webs and trophic levels:** The table above gives the adjacency list for the food web of predator-prey interactions between marine (i.e., salt-water) species in the Chesapeake Bay, a large tidal bay on the Eastern Seaboard of the United States (D. Baird and R. E. Ulanowicz, The seasonal dynamics of the Chesapeake Bay ecosystem. *Ecological Monographs* **59**, 329–364 (1989)). You can also download the same data from here:

   `http://www-personal.umich.edu/~mejn/courses/2005/cscs535/chesapeake.adj`

   which might save you some typing time.

   (i) Calculate the eigenvalues of the adjacency matrix. Hence make a statement about whether this food web is cyclic or acyclic.

   (ii) The standard way of calculating trophic levels (i.e., positions in the food chain) is to assign to each species a trophic level equal to the mean of the trophic levels of their prey, plus 1. Derive an expression for the vector $\mathbf{x}$ of trophic levels in terms of the adjacency matrix $\mathbf{A}$. The expression you get should involve the **directed graph Laplacian** $\mathbf{D} - \mathbf{A}$, where $\mathbf{D}$ is the diagonal matrix of in-degrees. This expression does not work for autotrophs—species with no prey. Such species are usually given trophic level 1. Suggest a modification of your method that will correctly assign trophic levels to these species, and hence to all species.

   (iii) Apply your formula to the Chesapeake Bay network and calculate the trophic levels. Which species is at the top of the food chain (i.e., has the highest trophic level)?

2. **Graph with a specified degree sequence (again):** Two weeks ago we studied the graph that has a specified degree sequence $\{k_i\}$, with vertices connected at random subject to this degree sequence. Let us consider that graph again.

   (i) Using (if you wish) some of your results from Problem Set 2, show that in the limit of large n the clustering coefficient of the network is

$$C = \frac{1}{n}\frac{\left[\langle k^2\rangle - \langle k\rangle\right]^2}{\langle k\rangle^3}.$$

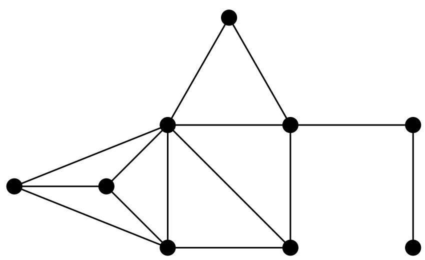
(ii) The widely studied freshwater food web of Little Rock Lake, WI (N. D. Martinez, Constant connectance in community food webs. *American Naturalist* **139**, 1208–1218 (1992)) has 92 vertices and a degree sequence (when symmetrized) that you can find in this file:

`http://www-personal.umich.edu/~mejn/courses/2005/cscs535/lrock.deg`

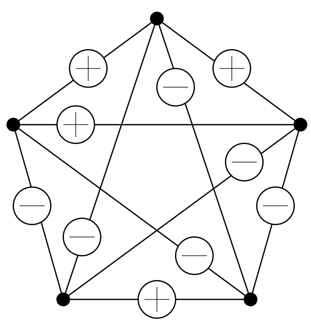
Calculate the expected clustering coefficient for this network using the formula above. The actual measured value of the clustering coefficient for the network is 0.40.

3. **Network structure measures:**

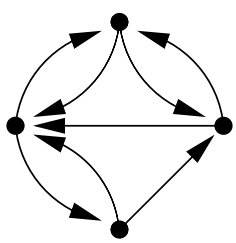
(i) Find a 3-core of this network:



(ii) In this network + and − indicate pairs of people that like each other or don't, respectively. Is the network structurally balanced and why? Is it clusterable and if so, what are the clusters?



(iii) What is the reciprocity of this graph?



(iv) What is the cosine similarity of vertices *A* and *B* in this graph?