

Physics 406: Summary of important results

This is a list of important equations and other results that you should know. You may take this list into the final exam with you.

Partial derivatives:

$$df = \left. \frac{\partial f}{\partial x} \right|_y dx + \left. \frac{\partial f}{\partial y} \right|_x dy, \quad \left. \frac{\partial x}{\partial y} \right|_z = \left[\left. \frac{\partial y}{\partial x} \right|_z \right]^{-1}, \quad \left. \frac{\partial x}{\partial y} \right|_z = - \left. \frac{\partial x}{\partial z} \right|_y \left. \frac{\partial z}{\partial y} \right|_x.$$

Internal energy: $dU = \delta Q + \delta W$ with $\delta Q = T dS$ and:

$$\begin{aligned} \delta W &= -p dV && \text{(fluid pressure/volume system)} \\ \delta W &= f dL && \text{(spring or wire with force } f \text{ and length } L) \\ \delta W &= V dq && \text{(capacitor with voltage } V \text{ and charge } q) \\ \delta W &= \gamma dA && \text{(surface with surface tension } \gamma \text{ and area } A) \end{aligned}$$

Thus for example, in a pressure/volume system $dU = T dS - p dV$. This applies for irreversible as well as reversible changes, but the individual equalities $\delta Q = T dS$ and $\delta W = -p dV$ only apply for reversible ones. Heat capacity at constant x (where x is any variable) is in general given by

$$C_x = T \left. \frac{\partial S}{\partial T} \right|_x, \quad \text{e.g.,} \quad C_V = T \left. \frac{\partial S}{\partial T} \right|_V \quad \text{and} \quad C_p = T \left. \frac{\partial S}{\partial T} \right|_p.$$

Potential functions and Maxwell relations: For a pressure/volume system

$$H = U + pV \quad (\text{enthalpy}), \quad F = U - TS \quad (\text{free energy}), \quad G = U + pV - TS \quad (\text{Gibbs energy}).$$

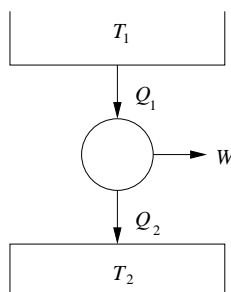
Similar expressions apply for other types of systems (non-pressure/volume systems). There is one Maxwell relation for each potential function, derived by equating partial second derivatives. For instance

$$\frac{\partial^2 U}{\partial V \partial S} = \frac{\partial^2 U}{\partial S \partial V} \quad \Rightarrow \quad \left. \frac{\partial T}{\partial V} \right|_S = - \left. \frac{\partial p}{\partial S} \right|_V.$$

Each heat capacity is the derivative of the corresponding potential function:

$$C_V = \left. \frac{\partial U}{\partial T} \right|_V, \quad C_p = \left. \frac{\partial H}{\partial T} \right|_p.$$

Heat engines:



Efficiency of a reversible engine ($T_1 > T_2$):

$$\eta = \frac{W}{Q_1} = 1 - \frac{T_2}{T_1}, \quad \eta_R = \frac{Q_2}{W} \quad (\text{refrigerator}), \quad \eta_H = \frac{Q_1}{W} \quad (\text{heat pump}).$$

Isolated systems: All microstates equally likely. Most likely macrostate maximizes the Boltzmann entropy $S = k \ln g$, where g is the multiplicity. When non-interacting systems are combined, entropy is additive (i.e., extensive); multiplicity is multiplicative.

Fixed temperature systems: States s appear with Boltzmann probability

$$p(s) = \frac{e^{-\epsilon_s/\tau}}{Z}, \quad Z = \sum_s e^{-\epsilon_s/\tau},$$

where $\tau = kT$ and $k = 1.38 \times 10^{-23} \text{ JK}^{-1}$. Macroscopic thermodynamic quantities are then given by

$$U = \tau^2 \frac{\partial \ln Z}{\partial \tau} \Big|_V, \quad F = -\tau \ln Z, \quad C = \frac{\partial U}{\partial \tau} \Big|_V, \quad \sigma = -\frac{\partial F}{\partial \tau} \Big|_V, \quad p = -\frac{\partial F}{\partial V} \Big|_\tau.$$

Sterling's approximation: $\ln k! \simeq k \ln k - k$.

Ideal gas: Density of states in three dimensions is

$$n(\epsilon) = \frac{V(2I+1)}{4\pi^2} \left(\frac{2m}{\hbar^2} \right)^{3/2} \epsilon^{1/2},$$

where I is the spin of the particles. ($I = \frac{1}{2}$ for fermions.)

$$Z = \frac{1}{N!} Z_1^N, \quad Z_1 = \frac{V}{(2\pi\hbar^2/m\tau)^{3/2}}, \quad pV = N\tau, \quad \sigma = N\left(\frac{5}{2} - \ln[(2\pi\hbar^2/m\tau)^{3/2}\rho]\right),$$

where $\rho = N/V$ is the number density.

Photons and phonons: Density of states is

$$n(\omega) = \frac{V}{\pi^2 c^3} \omega^2 d\omega \quad (\text{photons, } c \text{ is speed of light}), \quad n(\omega) = \frac{3V}{2\pi^2 v^3} \omega^2 d\omega \quad (\text{phonons, } v \text{ is speed of sound}).$$

Systems with variable numbers of particles: Grand ensemble:

$$Z = \sum_s e^{-(\epsilon_s - \mu N_s)/\tau}, \quad U = \tau^2 \frac{\partial \ln Z}{\partial \tau}, \quad \Omega = -\tau \ln Z, \\ \sigma = -\frac{\partial \Omega}{\partial \tau} \Big|_{V, \mu}, \quad p = -\frac{\partial \Omega}{\partial V} \Big|_{\tau, \mu}, \quad N = -\frac{\partial \Omega}{\partial \mu} \Big|_{V, \tau}.$$

Quantum gases: Number of particles in single-particle state with energy ϵ is

$$f(\epsilon) = \frac{1}{e^{(\epsilon - \mu)/\tau} + 1} \quad (\text{fermions}), \quad f(\epsilon) = \frac{1}{e^{(\epsilon - \mu)/\tau} - 1} \quad (\text{bosons}).$$