Physics 390: Homework 3

For full credit, show all your working.

1. **De Broglie waves:** As we discussed in class, the typical kinetic energy of particles such as the molecules in a gas, at temperature T in Kelvin, is kT, where k is Boltzmann's constant.

   (a) What is the typical kinetic energy of such particles at room temperature, measured in electron volts?

   (b) Estimate the de Broglie wavelength of nitrogen molecules in air at room temperature.

2. Problem 5-20 in Tipler & Llewellyn.

3. **Wave function:** An electron is constrained to move in one dimension only and has wave function at time $t = 0$ given by

$$\Psi = A\,\mathrm{e}^{-x^2/4\sigma^2}.$$

   (a) Assuming the particle is allowed to be anywhere between $x = -\infty$ and $x = +\infty$, what is the value of the constant A? (You will have to do an integral to work out the answer: you can either look up the value of the integral in tables or on-line, or, if you know how, you can work it out for yourself.)

   (b) If $\sigma = 2$, give the probability of finding the electron in a small interval of width $\mathrm{d}x$ centered at (i) $x = 0$, (ii) $x = 2$, (iii) $x = 4$.

4. Problem 5-40 in Tipler & Llewellyn. In part (c) be careful about the velocity of the electron—it's getting close to the speed of light, so you need to allow for relativity in your calculations.

5. **Phase velocity:** We have seen that the de Broglie frequency can be defined using either the relativistic or non-relativistic energy.

   (a) If we use the relativistic energy $E^2 = p^2c^2 + m^2c^4$, show that the resulting phase velocity for the de Broglie wave of an electron is greater than the speed of light.

   (b) Given that relativity says no particle can travel faster than the speed of light, is this a problem?