Physics 390: Homework 4

For full credit, show all your working.

1. Problem 6-16 in Tipler & Llewellyn.

2. **Uncertainty relation:** Suppose that we measure the uncertainty in position and momentum by their standard deviations:

$$\sigma_x = \sqrt{\langle x^2\rangle - \langle x\rangle^2}, \qquad \sigma_p = \sqrt{\langle p^2\rangle - \langle p\rangle^2}.$$

   (a) Find σ_x and σ_p for the ground state of the 1D infinite square well in terms of the length L of the well and the mass m of the particle.

   (b) Find $\sigma_x\sigma_p$.

3. **The simple harmonic oscillator:** In class we showed that if $\psi(x)$ is a solution of the time-independent Schrödinger equation $H\psi = E\psi$ for the simple harmonic oscillator Hamiltonian

$$H = \frac{p^2}{2m} + \tfrac{1}{2}m\omega^2 x^2,$$

   then the function

$$\psi_- = \left(\frac{\mathrm{d}}{\mathrm{d}y} + y\right)\psi,$$

   is a solution of the same equation with energy $\hbar\omega$ lower, i.e., of the equation $H\psi_- = (E - \hbar\omega)\psi_-$.

   (a) Show that

$$\psi_+ = \left(\frac{\mathrm{d}}{\mathrm{d}y} - y\right)\psi$$

   is also a solution of the Schrödinger equation, but with energy $\hbar\omega$ higher than ψ.

   (b) By repeated application of the operator $\mathrm{d}/\mathrm{d}y - y$ we can therefore make a ladder of states of higher and higher energies. The corresponding ladder of lower and lower energies for $\mathrm{d}/\mathrm{d}y + y$ stopped when we got to the ground state energy of $\frac{1}{2}\hbar\omega$. Does the up-going ladder also stop, or does it go up to infinite energy, and why?

4. Problem 6-57 in Tipler & Llewellyn.