Physics 390: Homework 8

For full credit, show all your working.

1. Problem 10-4 in Tipler & Llewellyn.

2. **Madelung constant for a 1D system:** Consider a one-dimensional model of an ionic crystal consisting of an infinite line of ions spaced r apart, with alternating positive and negative charges, $+e$ and $-e$.

   (a) Show that the electrostatic energy of a single ion in such a system, as a result of its interactions with all the others, is

   $$E = -\frac{2ke^2}{r}\left[1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \ldots\right].$$

   (b) Using the fact that

   $$\ln(1 + x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \ldots$$

   find the value of the Madelung constant.

3. **Mean free paths:**

   (a) Starting from the Drude formula for the conductivity that we derived in class, find an expression for the mean free path of the electrons in a metal in terms of the conductivity, the Fermi energy E_F, and the number density $n = N/V$ of electrons.

   (b) The conductivities of sodium, gold, and tin are $2.38 \times 10^7\,\mathrm{S\,m^{-1}}$, $4.90 \times 10^7\,\mathrm{S\,m^{-1}}$, and $9.43 \times 10^6\,\mathrm{S\,m^{-1}}$, respectively. The Fermi energies and number densities for electrons in these materials are given in Table 10-3, on page 429 of the book. Combine these figures to find the mean free paths for conduction electrons in each of these elements.

4. **Transparency:** Conductors are (almost) always opaque to light, but insulators and semiconductors may or may not be transparent.

   (a) Germanium, for example, has a band gap of 0.72 eV between its valence band and its empty conduction band. What range of wavelengths of visible light will it be transparent to?

   (b) What would the answer be if the band gap were 3.6 eV?

5. **Extra credit:** By any means you like, create a (reasonably accurate) plot of M as a function of T for the Curie magnetization equation

   $$M = \tanh \frac{\mu_B J M}{kT},$$

   when $\mu_B J/k = 1$. (We've also set $\rho\mu_B = 1$ for convenience.) Remember to include both positive and negative solutions for M.