Physics 390: Homework 1

For full credit, show all your working.

1. Problem 3-8 from Tipler & Llewellyn.

2. **Planck's law:** As we saw in class, the average energy of a mode in Planck's quantized theory of the radiation in a cavity is given by

$$\overline{E} = \frac{hf\sum_{n=0}^{\infty} n\mathrm{e}^{-nhf/kT}}{\sum_{n=0}^{\infty} \mathrm{e}^{-nhf/kT}}.$$

   (a) Calculate the value of the sum in the denominator by rewriting it as a geometric series $\sum_{n=0}^{\infty} a^n$ for some value of a (to be determined) and then performing the sum using the standard formula for a geometric series.

   (b) Show that

$$-\frac{kT}{h}\frac{\partial}{\partial f}\sum_{n=0}^{\infty} \mathrm{e}^{-nhf/kT} = \sum_{n=0}^{\infty} n\mathrm{e}^{-nhf/kT}.$$

   Hence, using the answer to part (a), show that

$$\sum_{n=0}^{\infty} n\mathrm{e}^{-nhf/kT} = \frac{\mathrm{e}^{hf/kT}}{(\mathrm{e}^{hf/kT} - 1)^2}.$$

   (c) Hence derive Planck's expression for $\overline{E}$, Eq. (3-17) in the book.

3. **Wien's law:** Given Planck's radiation law,

$$u(\lambda) = \frac{8\pi hc\lambda^{-5}}{\mathrm{e}^{hc/\lambda kT} - 1},$$

   we can derive Wien's law.

   (a) Differentiate to show that the wavelength of maximum radiation λ_m depends on temperature as $\lambda_m = b/T$ for some constant b.

   (b) Find the constant b to two significant figures and state its units. You will probably need to know that the solution to the equation $5\mathrm{e}^{-x} + x = 5$ is $4.965\ldots$

4. Problem 3-15 from Tipler & Llewellyn, parts (a) and (c) only.

5. Problem 3-45 from Tipler & Llewellyn.