Physics 390: Homework 3

For full credit, show all your working.

1. **De Broglie waves:** As we discussed in class, the typical kinetic energy of particles such as the molecules in a gas, at temperature T in Kelvin, is kT, where k is Boltzmann's constant.
   (a) What is the typical kinetic energy of such particles at room temperature, measured in electron volts?
   (b) Estimate the de Broglie wavelength of nitrogen molecules in air at room temperature.
2. Problem 5-20 in Tipler & Llewellyn.
3. **Width of a spectral line:** We have seen that hydrogen and other atoms have states of various energies and that the higher energy "excited" states spontaneous decay to lower energy states, emitting photons in the process. We can calculate the wavelength of these photons from the difference in energies of the states and hence accurately predict the lines in the emission spectrum of hydrogen and other elements.

   Excited states do not in general live very long before they decay. Suppose a certain excited state lives just 10^{-7} s. In that case we know the *time* when the atom is in the excited state quite accurately—to within $\Delta t = 10^{-7}$ s. That means there is a significant inaccuracy ΔE in the energy of the state.
   (a) Estimate the size of this inaccuracy.
   (b) Hence estimate the range Δf of frequencies that will be measured for the photons given off when the state decays. This results in a "broadened" spectral line that, when inspected closely, does not consist of just a single frequency but a narrow range of frequencies.
   (c) We can also turn the calculation around. A particular spectral line in hydrogen has wavelength $\lambda = 121.5$ nm and is observed to have a line width (in terms of wavelength) of about $10^{-7}\lambda$. Estimate the lifetime of the corresponding excited state.
4. Problem 5-40 in Tipler & Llewellyn. In part (c) be careful about the velocity of the electron—it's getting close to the speed of light, so you need to allow for relativity in your calculations.
5. **Phase velocity:** We have seen that the de Broglie frequency can be defined using either the relativistic or non-relativistic energy.
   (a) If we use the relativistic energy $E^2 = p^2c^2 + m^2c^4$, show that the resulting phase velocity for the de Broglie wave of an electron is greater than the speed of light.
   (b) Given that relativity says no particle can travel faster than the speed of light, is this a problem?