Physics 390: Homework 5

For full credit, show all your working.

1. **The simple harmonic oscillator:** In class we showed that if $\psi(x)$ is a solution of the time-independent Schrödinger equation $H\psi = E\psi$ for the simple harmonic oscillator Hamiltonian
$$H = \frac{p^2}{2m} + \tfrac{1}{2}m\omega^2 x^2,$$
then the function
$$\psi_- = \left(\frac{\mathrm{d}}{\mathrm{d}y} + y\right)\psi,$$
is a solution of the same equation with energy $\hbar\omega$ lower, i.e., of the equation $H\psi_- = (E - \hbar\omega)\psi_-$.

   (a) Show that
$$\psi_+ = \left(\frac{\mathrm{d}}{\mathrm{d}y} - y\right)\psi$$
   is also a solution of the Schrödinger equation, but with energy $\hbar\omega$ higher than ψ.

   (b) By repeated application of the operator $\mathrm{d}/\mathrm{d}y - y$ we can therefore make a ladder of states of higher and higher energies. The corresponding ladder of lower and lower energies for $\mathrm{d}/\mathrm{d}y + y$ stopped when we got to the ground state energy of $\frac{1}{2}\hbar\omega$. Does the up-going ladder also stop, or does it go up to infinite energy, and why?

2. Problem 6-43 in Tipler & Llewellyn.

3. **Proton beam:** A new particle accelerator, the Large Hadron Collider, has (after considerable delays) just started operation at the CERN particle physics laboratory in Geneva, Switzerland. The accelerator creates beams of protons that are used to test the quantum theory of subatomic particles. The beams of particles that we considered in class were made of electrons, but the theory we developed can be used for beams of protons too.

   Consider a beam of protons, each having kinetic energy 30 MeV. The particles approach a potential step of 20 MeV.

   (a) What fraction of the beam is reflected and what fraction is transmitted?

   (b) How would the answer change if the particles had been electrons?

4. Problem 6-57 in Tipler & Llewellyn.