Physics 390: Homework 6

For full credit, show all your working.

1. Problem 7-10 in Tipler & Llewellyn.

2. **Ground state of hydrogen:** We claimed that the ground-state wave function of the hydrogen atom is

   $$\psi_{100}(\mathbf{r}) = Ce^{-r/a_0},$$

   where a_0 is the Bohr radius and C is a normalizing constant.

   (a) A small element of space with spherical polar coordinates (r, θ, ϕ) spans the interval $(\mathrm{d}r, \mathrm{d}\theta, \mathrm{d}\phi)$ in the three coordinates. What is its volume?

   (b) In terms of a_0 and C, what is the quantum mechanical probability of the electron in a hydrogen atom being in this small volume, assuming that the nucleus is at the origin and the atom is in its ground state?

   (c) Integrate over all r, θ, ϕ and make use of the normalization condition on the wave function to calculate C.

   (d) Verify that the wave function is indeed a solution of the Schrödinger equation with energy $-13.6\,\mathrm{eV}$.

3. Problem 7-17 in Tipler & Llewellyn.

4. **Spin of the electron:** Electron "spin" angular momentum was initially given that name because it was thought that it arose because the electron was a small spinning object. When you look into it more closely, however, this cannot be true. Experiments indicate that the electron must be at least as small as the nucleus, so its radius is no larger than 10^{-15} m. Assume it is in fact a sphere of exactly this size, with uniformly distributed mass equal to the electron mass.

   (a) What is the moment of inertia I of a uniform sphere, as a function of its mass m and radius r? Hence what is the angular momentum L of a such a sphere if it is spinning with angular frequency ω? Put $L = \frac{1}{2}\hbar$ and find the corresponding ω for a sphere with the mass of the electron.

   (b) Hence calculate the linear velocity v of the fastest moving point on the sphere. Compare your result to the speed of light. What do you conclude?

   (c) If the electron were smaller than 10^{-15} m would this solve the problem?