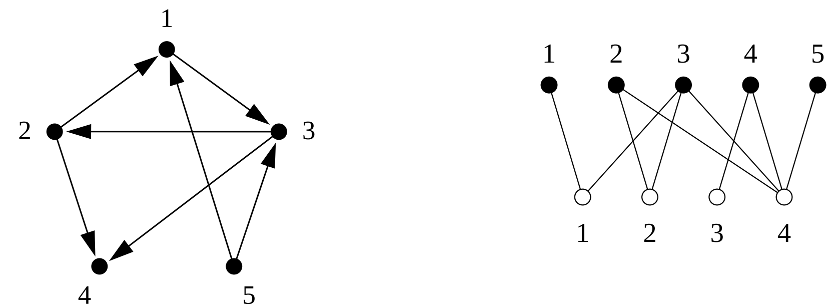
Complex Systems 535/Physics 508: Homework 1

1. Consider the following two networks:



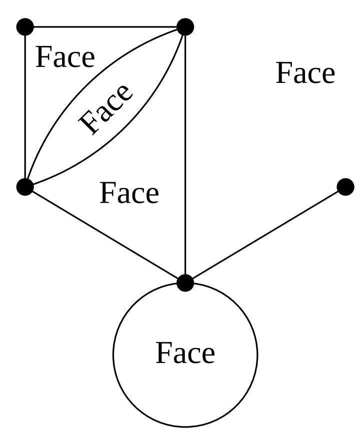
(a) (b)

Network (a) is a directed network. Network (b) is undirected but bipartite. Write down:

(i) the adjacency matrix of network (a);
(ii) the cocitation matrix of network (a);
(iii) the incidence matrix of network (b);
(iv) the adjacency matrix for the network created when we project network (b) onto its black vertices.

2. Consider a bipartite network, with its two types of vertices, and suppose there are n_1 vertices of type 1 and n_2 vertices of type 2. Show that the mean degrees c_1 and c_2 of the two types are related by
$$c_2 = \frac{n_1}{n_2} c_1.$$

3. Consider a connected planar network with n vertices and m edges. Let f be the number of "faces" of the network, i.e., areas bounded by edges when the network is drawn in planar form. The "outside" of the network, the area extending to infinity on all sides, is also considered a face. The network can have multiedges and self-edges:



(i) Write down the values of n, m, and f for a network with a single vertex and no edges.

(ii) How do n, m, and f change when we add a single vertex to the network along with a single edge attaching it to another vertex?

(iii) How do n, m, and f change when we add a single edge between two extant vertices (or a self-edge attached to just one vertex), in such a way as to maintain the planarity of the network?

(iv) Hence by induction prove a general relation between n, m, and f for all connected planar networks.

(v) Now suppose that our network is simple (i.e., it contains no multiedges or self-edges). Show that the mean degree c of a simple, connected, planar network is strictly less than six.

4. One possible definition of the trophic level x_i of a species in a (directed) food web is given in Section 5.3.1 of the book as the mean of the trophic levels of the species' prey, plus one.

(i) Show that x_i, when defined in this way, satisfies

$$x_i = 1 + \frac{1}{k_i^{\text{in}}} \sum_j A_{ij} x_j.$$

(ii) This expression does not work for autotrophs—species with no prey—because the corresponding vector element is undefined. Such species are usually given a trophic level of one. Suggest a modification of the calculation that will correctly assign trophic levels to these species, and hence to all species. Thus, show that x_i can be calculated as the ith element of a vector

$$\mathbf{x} = (\mathbf{D} - \mathbf{A})^{-1} \mathbf{D} \cdot \mathbf{1},$$

and specify how the matrix $\mathbf{D}$ is defined.