Complex Systems 535/Physics 508: Homework 3

1. What is the time complexity, as a function of the number n of vertices and m of edges, of the following network operations if the network in question is stored in adjacency list format?

   (i) Calculating the mean degree.

   (ii) Calculating the median degree.

   (iii) Calculating the air-travel route between two airports that has the shortest total flying time, assuming the flying time of each individual flight is known.

2. Consider the following centrality measure, which I just made up off the top of my head.

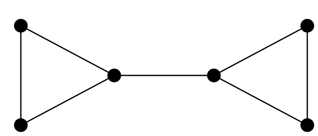
   With ordinary degree centrality, you get points for each person you are connected to. But perhaps you could get points for people you are two steps away from in the network, or three, or more, although probably you should get fewer points the further away someone is. So let us define the centrality x_i of vertex i to be a sum of contributions as follows: 1 for yourself, α for each person at distance 1 in the network, α^2 for each person at distance 2, and so forth.

   (i) Write an expression for x_i in terms of α and the geodesic distances d_{ij} between vertex pairs.

   (ii) Describe briefly an algorithm for calculating this centrality measure. What is the time complexity for calculating x_i for all i?

   (iii) Suppose individuals in a network have about c connections on average, so that a person typically has about c first neighbors, c^2 second neighbors, and so on (ignoring the effects of transitivity). What happens to the contributions to the centrality when $\alpha \gtrsim 1/c$?

3. Using your favorite numerical software for finding eigenvectors of matrices, construct the Laplacian and the modularity matrix for this small network:

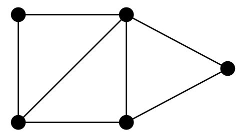


   (i) Find the eigenvector of the Laplacian corresponding to the second smallest eigenvalue and hence perform a spectral bisection of the network into two equally sized parts.

   (ii) Find the eigenvector of the modularity matrix corresponding to the largest eigenvalue and hence divide the network into two communities.

   You should find that the division of the network generated by the two methods is, in this case, the same.

4. Consider this small network with five vertices:



(i) Calculate the cosine similarity for each of the $\binom{5}{2} = 10$ pairs of vertices.

(ii) Using the values of the ten similarities construct the dendrogram for the single-linkage hierarchical clustering of the network according to cosine similarity.