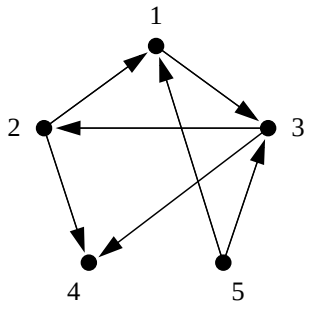
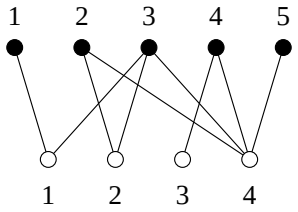
Complex Systems 535/Physics 508: Homework 1

1. Consider the following two networks:



(a)



(b)

Network (a) is a directed network. Network (b) is undirected but bipartite. Write down:

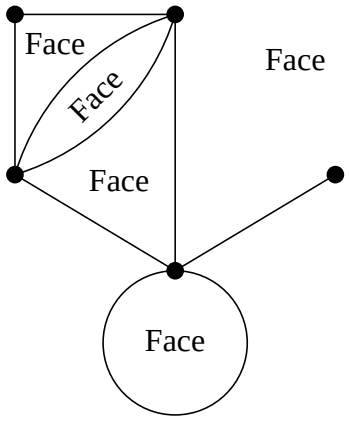
(i) the adjacency matrix of network (a);

(ii) the cocitation matrix of network (a);

(iii) the incidence matrix of network (b);

(iv) the adjacency matrix for the network created when we project network (b) onto its black vertices.

2. Consider a connected planar network with n vertices and m edges. Let f be the number of "faces" of the network, i.e., areas bounded by edges when the network is drawn in planar form. The outside of the network, the area extending to infinity on all sides, is also considered a face. The network can have multiedges and self-edges:



(i) How do n, m, and f change when we add a single vertex to such a network along with a single edge attaching it to an existing vertex?

(ii) How do n, m, and f change when we add a single edge between two existing vertices (or a self-edge attached to just one vertex), in such a way as to maintain planarity of the network?

(iii) What are the values of n, m, and f for a network with a single vertex and no edges?

(iv) Hence by induction prove a general relation between n, m, and f for all connected planar networks.

(v) Now suppose that our network is simple (i.e., it contains no multiedges or self-edges). Show that the mean degree c of a simple, connected, planar network is strictly less than six.

3. Here are the results of the in-class social network questionnaire:

Number	Name	People they know
0	Adorf, Carl	
1	Ai, Wei	23, 24, 25
2	Arzu, Leo	11
3	Cheng, Frank	
4	Coke, Reed	1, 6, 14, 24
5	Dai, Chengyu	8, 10, 13, 25
6	Datta, Srayan	4, 14
7	Fishelson, James	3
8	Geng, Yina	1, 5, 23, 25
9	Hayashi, Michael	
10	Hu, Fengmin	1, 5
11	Israel, Uriah	2, 18, 21
12	Karslake, Jason	
13	Khatib-Damavandi, Ojan	5
14	Lee, Saerom	4, 6
15	Malinas, Rebecca	20
16	Mani, Soroosh	
17	Nayar, Himanshu	
18	Rodriguez, Garrett	11, 21
19	Schudel, Joshua	
20	Sena, Dylan	15
21	Snider, Nic	11, 18
22	Wang, Chris	
23	Wei, Xinzhu	1, 8, 25
24	Yan, Shiyan	1, 4
25	Zhang, Leyou	1, 5, 8, 23

You can also download the same data as a computer file from:

`http://www.umich.edu/~mejn/courses/2014/cscs535/classnet.txt`

or in GML (Graph Mark-up Language) format from:

`http://www.umich.edu/~mejn/courses/2014/cscs535/classnet.gml`

Note that the network is directed, e.g., node 4 knows node 1 but not *vice versa*. We can define an undirected *symmetrized network*, in which there is an undirected edge between nodes i and j if there is a directed edge either from i to j *or* from j to i.

(i) Draw a picture of the symmetrized network using any means you like (pencil and paper, graph drawing software, paper clips and rubber bands, whatever).

(ii) Find the numbers n and m of vertices and edges in the directed and undirected networks.

(iii) Find the degree of each vertex in the symmetrized network and the in-degree of each vertex in the directed network. List the top individuals for each (say the top four degrees and the top three in-degrees). Which do you think is a better measure of influence and why?

(iv) List the vertices in each component of the undirected network.

(v) Calculate the diameter of each component.