Complex Systems 535/Physics 508: Homework 2

1. Consider a k-regular undirected network (i.e., a network in which every vertex has degree k).

   (i) Show that the vector $\mathbf{1} = (1, 1, 1, \ldots)$ is an eigenvector of the adjacency matrix with eigenvalue k.

   (ii) By making use of the fact that eigenvectors are orthogonal (or otherwise), show that there is no other eigenvector that has all elements positive. The Perron–Frobenius theorem says that the eigenvector with all elements positive has the largest eigenvalue, and hence the eigenvector $\mathbf{1}$ gives, by definition, the eigenvector centrality of our k-regular network and the centralities are the same for every vertex.

   (iii) Find the Katz centralities of all vertices in a k-regular network.

   (iv) Name a centrality measure that could give different centralities for different vertices in a regular network.

2. A particular network is believed to have a degree distribution that follows a power law for degree greater than or equal to 10. A random sample of vertices is taken and their degrees measured. The degrees of the first twenty vertices with degrees 10 or greater are:

16	17	10	26	13
14	28	45	10	12
12	10	136	16	25
36	12	14	22	10

   Estimate the exponent α of the power law and the error on that estimate.

3. In a survey of couples in the city of San Francisco in 1992, Catania *et al.* recorded, among other things, the ethnicity of interviewees and calculated the fraction of couples whose members were from various ethnic groups. The fractions were as follows:

		Women				
		Black	Hispanic	White	Other	Total
Men	Black	0.258	0.016	0.035	0.013	0.323
	Hispanic	0.012	0.157	0.058	0.019	0.247
	White	0.013	0.023	0.306	0.035	0.377
	Other	0.005	0.007	0.024	0.016	0.053
	Total	0.289	0.204	0.423	0.084	

   Assuming the couples interviewed to be a representative sample of the edges in the undirected network of relationships for the community studied, and treating the vertices as being of four types—black, hispanic, white, and other—calculate the numbers e_{rr} and a_r that appear in Eq. (7.76) of the book, for each type. Hence calculate the modularity of the network with respect to ethnicity. What do you conclude about homophily in this community?

4. The data for the revised CmplxSys 535 class network, in GML format, can be found here:

   `http://www.umich.edu/~mejn/courses/2014/cscs535/classnet2.gml`

   Download a copy of this file. Note, as previously, that the network is directed. Also download and install the network analysis software *Gephi* (`www.gephi.org`, available for PC, Mac, or Linux). Load up the class network in Gephi. The program will give you the option to load the network as a directed network or an undirected one (i.e., ignoring edge directions). Load it as directed.

   (i) The default visualization of the network that the program generates is not very clear. Work out how to do a clear visualization. (Hint: I find the "Yifan Hu" algorithm works well.) Work out how to zoom in and out of the picture to examine different parts of the network. Work out how to turn on the node labels, so you can see which nodes correspond to which individuals. You may also need to change the size of the labels. (Hint: look at the buttons and sliders along the bottom.)

   (ii) Calculate the in-degree of every node and the PageRank with $\alpha = 0.85$. (Hint: α is called p in Gephi.) Find the nodes with the top three scores by each measure. (The "data laboratory" button at the top is useful.)

   (iii) Reload the network as undirected and calculate the eigenvector centrality of every node. Work out how to color the nodes on the screen according to eigenvector centrality so you can see which have the highest score. (Hint: the "Nodes" panel on the left is a good place to start.) Save a copy of the resulting visualization.

   (iv) Find the "communities" in the network using the modularity maximization method and make a picture of the network with node colors representing the communities. Roughly speaking, what is the computer doing when it performs this calculation? (We will look at community detection in networks in more detail later in the course.)

   (v) Do one other cool thing with this network using Gephi—calculate some interesting thing, or produce some interesting visualization. Explain what you did.

   For full credit turn in the lists of top nodes from (ii), the pictures from (iii) and (iv), your explanation from (iv), and your results and explanation from (v).