Physics 390: Homework 3

1. **Muonic hydrogen:** As we will discuss later in the semester, a *muon* (symbol μ^-), is an elementary particle similar to an electron (it is a point particle with electric charge $-e$), but it is more massive, having mass 1.884×10^{-28} kg—about a hundred times the electron mass.

   A standard hydrogen atom has a single proton in the nucleus and a single electron in orbit around it. *Muonic hydrogen* is the same thing but with a muon instead of an electron. Calculate the ground state energy of muonic hydrogen in electron volts to three significant figures using the Bohr model. It is important to include the reduced mass correction, which is very significant in this case.

2. **Reduced mass and the deuterium shift:** The first spectral line in the Balmer series for hydrogen is emitted when an electron hops from the energy level with quantum number $n_i = 3$ to the one with quantum number $n_f = 2$.

   (a) If we assume that the nucleus of the atom has infinite mass, what is the wavelength λ of this spectral line in nm to three significant figures? Is the line visible? If so, what color is it?

   (b) What is the wavelength (also to three figures) if we include the "reduced mass" correction for the finite mass of the nucleus, given that the proton has 1836 times the mass of the electron?

   (c) Suppose the reduced mass μ changes by a small amount $\Delta\mu$. Show that the wavelength changes by a corresponding amount $\Delta\lambda$ that approximately satisfies

   $$\frac{\Delta\lambda}{\lambda} = -\frac{\Delta\mu}{\mu}.$$

   (You can assume $\Delta\mu$ is small in comparison with μ.)

   (d) Hence find the wavelength shift in the first Balmer line between hydrogen (whose nucleus contains just a single proton) and deuterium (whose nucleus contains a proton and a neutron, the neutron having the same mass as the proton).

   It was precisely this small wavelength shift that led to the discovery of deuterium in 1931.

3. Problem 4-27 in Tipler & Llewellyn.

4. **De Broglie waves:** At temperature T (in Kelvin) the typical kinetic energy of particles such as the molecules in a gas is kT, where k is Boltzmann's constant.

   (a) What is the typical kinetic energy of such particles at room temperature, measured in electron volts?

   (b) Estimate the de Broglie wavelength of nitrogen molecules in air at room temperature.

5. Problem 5-14 in Tipler & Llewellyn.