Physics 390: Homework 4

1. **Width of a spectral line:** We have seen that hydrogen and other atoms have states of various energies and that the higher energy "excited" states spontaneous decay to lower energy states, emitting photons in the process. We can calculate the wavelength of these photons from the difference in energies of the states and hence accurately predict the lines in the emission spectrum of hydrogen and other elements.

   Excited states do not in general live very long before they decay. Suppose a certain excited state lives just 10^{-7} s. In that case we know the *time* when the atom is in the excited state quite accurately—to within $\Delta t = 10^{-7}$ s. That means there is a significant uncertainty ΔE in the energy of the state.

   (a) Estimate the size of this uncertainty.

   (b) Hence estimate the range Δf of frequencies that will be measured for the photons given off when the state decays. This results in a "broadened" spectral line that, when inspected closely, does not consist of just a single frequency but a narrow range of frequencies.

   (c) We can also turn the calculation around. A particular spectral line in hydrogen has wavelength $\lambda = 121.5\,\text{nm}$ and is observed to have a line width (in terms of wavelength) of about $10^{-7}\lambda$. Estimate the lifetime of the corresponding excited state.

2. Problem 5-42 in Tipler & Llewellyn. In part (c) be careful about the velocity of the electron—it's getting close to the speed of light, so you need to allow for relativity in your calculations.

3. **Phase velocity:** We have seen that the de Broglie frequency can be defined using either the relativistic or non-relativistic energy.

   (a) If we use the relativistic energy $E^2 = p^2c^2 + m^2c^4$, show that the resulting phase velocity for the de Broglie wave of an electron is greater than the speed of light.

   (b) Given that relativity says no particle can travel faster than the speed of light, is this a problem?

4. **Relativistic quantum mechanics:** We have seen that Schrödinger correctly deduced his famous wave equation by taking the relation between frequency and wavenumber $\hbar\omega = \hbar^2k^2/2m$ and making the replacements

$$k \to -\mathrm{i}\frac{\partial}{\partial x} \qquad \text{and} \qquad \omega \to \mathrm{i}\frac{\partial}{\partial t}.$$

   (a) Starting from the relativistic relation between energy and momentum $E^2 = p^2c^2 + m^2c^4$ find the relativistic relation between angular frequency ω and wavenumber k of de Broglie waves.

   (b) Make the replacements above and hence derive a relativistic version of the Schrödinger equation.

   (c) Substitute the wave $\Psi = \Psi_0 e^{\mathrm{i}(kx-\omega t)}$ into your wave equation and show that you can recover the relation between ω and k.