Physics 390: Homework 5

For full credit, show all your working.

1. **The infinite square well:** The wavelength of light emitted by a ruby laser is 694.3 nm. Assuming that the emission of a photon of this wavelength accompanies the transition of an electron from the $n = 2$ level to the $n = 1$ level of an infinite square well, compute the width L of the well.

2. **Uncertainty relation:** Suppose that we measure the uncertainty in the position and momentum of a particle by the standard deviations:

$$\sigma_x = \sqrt{\langle x^2 \rangle - \langle x \rangle^2}, \qquad \sigma_p = \sqrt{\langle p^2 \rangle - \langle p \rangle^2},$$

   where $\langle \ldots \rangle$ denotes an average value.

   (a) Find σ_x and σ_p for the ground state of the 1D infinite square well in terms of the length L of the well.

   (b) Find $\sigma_x \sigma_p$.

3. **The simple harmonic oscillator:** In class we showed that if $\psi(x)$ is a solution of the time-independent Schrödinger equation $H\psi = E\psi$ for the simple harmonic oscillator Hamiltonian

$$H = \frac{p^2}{2m} + \tfrac{1}{2} m\omega^2 x^2,$$

   then the function

$$\psi_- = \left(\frac{\mathrm{d}}{\mathrm{d}y} + y \right) \psi,$$

   is a solution of the same equation with energy $\hbar\omega$ lower, i.e., of the equation $H\psi_- = (E - \hbar\omega)\psi_-$.

   (a) Show that

$$\psi_+ = \left(\frac{\mathrm{d}}{\mathrm{d}y} - y \right) \psi$$

   is also a solution of the Schrödinger equation, but with energy $\hbar\omega$ higher than ψ.

   (b) By repeated application of the operator $\mathrm{d}/\mathrm{d}y - y$ we can therefore make a ladder of states of higher and higher energies. The corresponding ladder of lower and lower energies for $\mathrm{d}/\mathrm{d}y + y$ stopped when we got to the ground state energy of $\frac{1}{2}\hbar\omega$. Does the up-going ladder also stop, or does it go up to infinite energy, and why?

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4. **Mixture of states:**

(a) A particle of mass m sits in a one-dimensional square well with infinitely high walls and width L. The particle is in a 50–50 mixture of states, half in the ground state and half in the first excited state. Derive a formula for the complete, time-dependent wavefunction of the particle. Make sure that your wavefunction is correctly normalized—the probability of finding the particle *somewhere* must equal 1 at all times, which sets the overall normalization constant in your expression.

(b) Hence, show that the probability of finding the particle between positions $\frac{1}{4}L$ and $\frac{1}{4}L + \mathrm{d}x$, measured from the left-hand side of the well, as a function of time, is $\left[\frac{3}{2} + \sqrt{2}\cos(3E_1 t/\hbar)\right]\mathrm{d}x/L$ with $E_1 = h^2/8mL$.

(c) What are the highest and lowest values of this probability?

(d) What is the frequency with which the probability varies if the particle is an electron and the well is 1 nm wide?