Physics 390: Homework 8

For full credit, show all your working.

1. **Quantum mechanics of many particles:** The neutron and the neutral pion are two subatomic particles with no charge, so that they neither electrically repel nor attract other particles. Here are their properties:

particle	charge	spin	mass
neutron	0	$\frac{1}{2}$	$1836m_e$
pion	0	0	$264m_e$

   where m_e is the electron mass.

   (a) Calculate the lowest energy that ten noninteracting neutrons can have if they are trapped in a three-dimensional cubic quantum well of size 10 fm on a side.

   (b) Calculate the corresponding energy for ten pions.

   For both (a) and (b), explain the steps in your calculations and the effects of the spins of the particles.

2. **The quantum harmonic oscillator at finite temperature:** We previously studied the states of a single particle in a quadratic energy well—the quantum simple harmonic oscillator—finding that it has states of energy $E_n = (n + \frac{1}{2})\hbar\omega$, where $n = 0, 1, 2\ldots$ is a non-negative integer and ω is the angular frequency of oscillation of a classical particle in the same potential.

   Suppose the well and the particle in it are at a finite temperature T. The partition function Z of the system is given by a sum over states s thus:

$$Z = \sum_s \mathrm{e}^{-E_s/kT}.$$

   (a) Show that the partition function for the simple harmonic oscillator is

$$Z = \tfrac{1}{2}\,\mathrm{cosech}\,\frac{\hbar\omega}{2kT}.$$

   (b) Hence find the thermal average energy $\langle E\rangle$ of the particle.

   (c) If $\omega = 10^{14}\,\mathrm{s}^{-1}$ and $T = 300\,\mathrm{K}$, what is the probability that the particle will be in the ground state?

3. **Thermal occupation probabilities:** From observations of the spectrum of a certain star it is determined that about one in a million of the hydrogen atoms in the star is in its first excited state, the rest being in the ground state. (Other excited states can, to a good approximation, be ignored.) Allowing for that fact that the first excited state has a degeneracy of 8 (spin-up and spin-down for each of the three $\ell = 1$ states and one $\ell = 0$ state) while the ground state has a degeneracy of only 2 (spin-up and spin-down), what is the temperature of the star?