Physics 390: Relativity review

You should be familiar with the material on special relativity in Chapters 1 and 2 of Tipler & Llewellyn. This sheet summarizes the most important results. In the following formulas

$$\beta = \frac{v}{c} \qquad \text{and} \qquad \gamma = \frac{1}{\sqrt{1-\beta^2}} = \frac{1}{\sqrt{1-v^2/c^2}}$$

where c is the speed of light. Note that as v approaches the speed of light, $\beta \to 1$ and $\gamma \to \infty$.

1. **Lorentz contraction:** An object of length L moving with velocity v relative to an observer is shortened or *contracted* in the direction of motion, having reduced length

   $$L' = L/\gamma.$$

   Note that as v approaches the speed of light $L' \to 0$.

2. **Time dilation:** Two events that occur a time interval τ apart at the same location in a reference frame moving at velocity v relative to an observer are separated by a longer (or *dilated*) interval τ' for that observer, where

   $$\tau' = \gamma\tau.$$

   Note that $\tau' \to \infty$ as $v \to c$, so rapidly moving things appear to be in slo-mo. Make sure you understand the "twin paradox" and its resolution (Section 1.6).

3. **Doppler shift:** An approaching or receding light source of frequency f is observed to have a shifted frequency f' given by

   $$\text{Approaching:} \quad f' = \sqrt{\frac{1+\beta}{1-\beta}}\, f, \qquad\qquad \text{Receding:} \quad f' = \sqrt{\frac{1-\beta}{1+\beta}}\, f.$$

4. **Relativistic mass:** A body with rest mass m moving with velocity v has total relativistic mass γm. The kinetic energy gives rise to an increase in mass.

5. **Energy:** A body of mass m at rest has energy mc^2. If one could convert mass completely into energy (for example by annihilating with an identical body of antimatter), this is the amount of energy that would be produced. A moving body, which has relativistic mass γm, has total energy γmc^2 and kinetic energy

   $$E_k = \gamma mc^2 - mc^2 = (\gamma - 1)mc^2.$$

   You should check that this gives the familiar non-relativistic expression $E_k = \frac{1}{2}mv^2$ in the limit where $v \ll c$.

   If energy is added to a body in *any* form (not just kinetic), it increases the mass of the body by an amount $\Delta E/c^2$, where ΔE is the energy added. For instance, chemical energy stored in chemical compounds gives them a total mass slightly in excess of the combined masses of their constituent parts. And a charged battery weighs slightly more than the same battery uncharged.

6. **Momentum:** Relativistic momentum is given by $p = \gamma mv$.

7. **Invariant mass:** Energy, momentum, and mass are related by

   $$E^2 = (pc)^2 + (mc^2)^2.$$