



PROPOSITION 2: TEAM ROUND

Names: _____

Team ID: _____

INSTRUCTIONS

1. Do not begin until instructed to by the proctor.
2. You will have 30 minutes to solve 10 problems.
3. Your score will be the number of correct answers. There is no penalty for guessing or incorrect answers.
4. **Only the official team answers will be graded.** If you are submitting the official answer sheet for your team, indicate this by writing “(OFFICIAL)” next to your team name. Do not submit any unofficial answer sheets.
5. No calculators or electronic devices are allowed.
6. All submitted work must be the work of your own team. You may collaborate with your team members, but no one else.
7. When time is called, please put your pencil down and hold your paper in the air. **Do not continue to write.** If you continue writing, your score may be disqualified.
8. Do not discuss the problems with anyone outside of your team until all papers have been collected.
9. If you have a question or need to leave the room for any reason, please raise your hand quietly.
10. Good luck!



ACCEPTABLE ANSWERS

1. All answers must be simplified as much as reasonably possible. For example, acceptable answers include $\sin(1^\circ)$, $\sqrt{43}$, or π^2 . Unacceptable answers include $\sin(30^\circ)$, $\sqrt{64}$, or 3^2 .
2. All answers must be exact. For example, π is acceptable, but 3.14 or $22/7$ is not.
3. All rational, non-integer numbers must be expressed in reduced form $\pm\frac{p}{q}$, where p and q are relatively prime positive integers and $q \neq 0$. For example, $\frac{2}{3}$ is acceptable, but $\frac{4}{6}$ is not.
4. All radicals must be fully reduced. For example, $\sqrt{24}$ is not acceptable, and should be written as $2\sqrt{6}$. Additionally, rational expressions cannot contain radicals in the denominator. For example, $\frac{1}{\sqrt{2}}$ is not acceptable, and should be written as $\frac{\sqrt{2}}{2}$.
5. Answers should be expressed in base 10 unless otherwise specified.
6. Complex numbers should be expressed in the form $a + bi$, where both a and b are written in a form compliant with the rules above. In particular, no complex denominators are allowed. For example, $\frac{1+2i}{1-2i}$ should be written as $-\frac{3}{5} + \frac{4}{5}i$ or $\frac{-3+4i}{5}$.
7. If a problem asks for all solutions, you may give the answers in any order. However, no credit will be given if any solution is missing or any solution is given but not correct.
8. Angle measurements should be given in radians unless otherwise specified.
9. Answers must be written legibly to receive credit. Ambiguous answers may be marked incorrect, even if one of the possible interpretations is correct.

PROPOSITION 2: TEAM ROUND

1. Stephen has an unfair coin where both the probability of flipping heads and the probability of flipping tails is nonzero. Stephen flips the unfair coin three times. The probability that Stephen flips three heads is equal to the probability that Stephen flips two heads and one tails, in any order. Determine the probability that in three flips, Stephen flips three tails.

1. _____

2. Triangle $\triangle ABC$ has sides \overline{AB} , \overline{AC} , and \overline{BC} of length 8, 15, and 17, respectively. A point P is randomly chosen inside of $\triangle ABC$. What is the probability that \overline{BP} is longer than \overline{CP} ?

2. _____

3. We can write $\sqrt{20 + 2\sqrt{91}} + \sqrt{20 - 2\sqrt{91}}$ in the form $x + \sqrt{y}$, where x and y are integers. Determine $x + y$.

3. _____

4. Consider a regular 2020-gon. Pick three of its vertices A , B , and C at random, without replacement. What is the probability that there is a fourth vertex D such that the points A , B , C , and D are the corners of some rectangle?

4. _____

5. The slope of the common tangent line(s) to the parabolas $y = -x^2 + 4x - 3$ and $y = x^2 - 2x + 3$ are expressible in the form $m = a \pm \sqrt{b}$. Determine $a + b$.

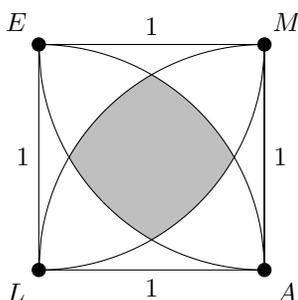
5. _____

6. Determine the number of ordered triples of sets (A, B, C) where the sets A, B, C satisfy

$$A \subseteq B \subseteq C \subseteq \{1, 2, \dots, 7\}.$$

6. _____

7. Consider the following square $LEMA$:



Each of the arcs is a quarter circle centered at one of L , E , M , and A . What is the area of the shaded region?

7. _____

8. Compute

$$\sec\left(\frac{\pi}{9}\right)\sec\left(\frac{2\pi}{9}\right)\sec\left(\frac{4\pi}{9}\right).$$

8. _____

9. The polynomial $p(x) = x^3 + 20x^2 + 20x - 20$ has roots p, q, r . Determine $(p^2 - 1)(q^2 - 1)(r^2 - 1)$.

9. _____

10. Define a function $f : \mathbb{Z}_{>0} \rightarrow \mathbb{Z}$ such that

- for any prime number p , we have $f(p) = p$;
- for all positive integers a and b , we have $f(ab) = f(a) + f(b) + 1$.

Let

$$m = \min\{f(1020), f(1021), \dots, f(2020)\},$$

$$M = \max\{f(1020), f(1021), \dots, f(2020)\}.$$

What is $M - m$?

10. _____