Proposition 5: Tiebreaker Round

Name: _____

Team ID: _____

Instructions

1. Do not begin until instructed to by the proctor.

2. You will have 30 minutes to solve 3 problems.

3. When you would like to submit your answers, please inform your proctor.

4. **Your score will be the number of correct answers, with ties broken by time of submission.**

5. No calculators or electronic devices are allowed.

6. All submitted work must be your own. You may not collaborate with anyone else during the individual round.

7. When time is called, please put your pencil down and hold your paper in the air. **Do not continue to write.** If you continue writing, your score may be disqualified.

8. Do not discuss the problems until all papers have been collected.

9. If you have a question or need to leave the room for any reason, please raise your hand quietly.

10. Good luck!

Acceptable Answers

1. All answers must be simplified as much as reasonably possible. For example, acceptable answers include $\sin(1°)$, $\sqrt{43}$, or π^2. Unacceptable answers include $\sin(30°)$, $\sqrt{64}$, or 3^2.

2. All answers must be exact. For example, π is acceptable, but 3.14 or $22/7$ is not.

3. All rational, non-integer numbers must be expressed in reduced form $\pm\frac{p}{q}$, where p and q are relatively prime positive integers and $q \neq 0$. For example, $\frac{2}{3}$ is acceptable, but $\frac{4}{6}$ is not.

4. All radicals must be fully reduced. For example, $\sqrt{24}$ is not acceptable, and should be written as $2\sqrt{6}$. Additionally, rational expressions cannot contain radicals in the denominator. For example, $\frac{1}{\sqrt{2}}$ is not acceptable, and should be written as $\frac{\sqrt{2}}{2}$.

5. Answers should be expressed in base 10 unless otherwise specified.

6. Complex numbers should be expressed in the form $a + bi$, where both a and b are written in a form compliant with the rules above. In particular, no complex denominators are allowed. For example, $\frac{1+2i}{1-2i}$ should be written as $-\frac{3}{5} + \frac{4}{5}i$ or $\frac{-3+4i}{5}$.

7. If a problem asks for all solutions, you may give the answers in any order. However, no credit will be given if any solution is missing or any solution is given but not correct.

8. Angle measurements should be given in radians unless otherwise specified.

9. Answers must be written legibly to receive credit. Ambiguous answers may be marked incorrect, even if one of the possible interpretations is correct.

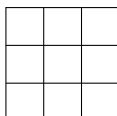## PROPOSITION 5: TIEBREAKER ROUND

1. Luke writes a rational number on each of 4 cards, denoted a_1, a_2, a_3, a_4. Luke computes all possible products of two different cards, and tells you the six products he gets are

$$\left\{-45, -\frac{9}{5}, -\frac{5}{3}, -\frac{1}{15}, 3, 1\right\}.$$

Determine all possible values of $a_1 + a_2 + a_3 + a_4$.

1. _____

2. Consider the grid below.

How many ways can we place the integers $\{1, 2, \ldots, 9\}$, without replacement, into the grid, such that the sum of each row and the sum of each column is divisible by 3?

2. _____

3. Let S_1 be a sphere of radius 1. Inscribe in S_1 a regular tetrahedron T_1. Inside T_1, inscribe a sphere S_2. Inside S_2, inscribe a regular tetrahedron T_2. Inside T_2, inscribe a sphere S_3. Inside S_3, inscribe a regular tetrahedron T_3. What is the volume of T_3?

3. _____