

The Habit Liability in Life-Cycle Consumption and Portfolio Choice*

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ABSTRACT

In this paper, the effects of habit formation on consumption and portfolio choice are examined in a life-cycle context with mortality risk, labor income and a fixed age of retirement. The investment opportunity set is assumed to be constant with both a risk free saving technology and a single asset with geometric Brownian price movements. Habit formation is recast in a way that emphasizes its forward looking property. The effect of a household's current habit stock is to create a habit liability that reduces net wealth. The habit stock carries an implicit price that reflects the cost of maintaining a unit of habit over the remaining life-cycle. The habit liability is the cost of the habit at its current price. Effective wealth is defined as the difference between net wealth and the habit liability and shown to play the same role in decision making for a habit forming household as net wealth plays for a non-habit forming household. Life-cycle simulations show the demand for the risky asset is greatly reduced by the habit liability both in terms of levels and as a share of net wealth. Households with a larger habit liability cannot afford to take on as much risk as a household without habit formation, all else equal. Aggregate data is used to estimate closed form solutions for the risky asset demand and consumption functions. Results indicate a strong role for habit formation in the determination of risky asset demand while no such evidence is found in estimation of the consumption function. An estimate of the time series of aggregate relative risk aversion is also provided.

I. Introduction

The inability of canonical models of consumption and portfolio allocation to yield empirically consistent results has led to some of the larger puzzles in the fields of macroeconomics and finance. Mehra and Prescott (1985) note that the unconditional mean of the equity premium poses a problem for additively time separable preferences since it requires implausibly high levels of risk aversion on the part of the representative agent in order to reconcile asset returns with the unconditional moments of aggregate consumption. Campbell and Deaton (1989) document that aggregate consumption is “excessively” smooth to changes in permanent income. Carroll and Weil (1994) find that high aggregate income growth is followed by high aggregate saving which is inconsistent with forward looking consumers with standard utility who should save less, not more, in a fast growing economy. Over the past decade, researches have turned to time non-separable utility as a way of explaining away these puzzles.¹ In particular, the role of habit formation seems to have come to the rescue of rational expectations in household decision making.

In this paper, an intuitively appealing interpretation of consumption habits is provided, the effects of habit formation on life-cycle consumption and portfolio allocation are examined both analytically and by way of life-cycle simulations, and the subjective force of habit is estimated structurally using a combination of aggregate data on household balance sheets, consumption and earnings. Empirical estimates suggest that habit formation is highly significant in the determination of the demand for risky assets. The marginal cost of consumption is estimated to be roughly three times the cost implied by the standard time separable model. In contrast, estimates of the aggregate total consumption function suggest a small but significant *durability* of consumption. Decomposing by the type of consumption indicates that neither habits nor durability play a role in the determination of non-durable consumption while consumption expenditures on durable goods, not surprisingly, imply a significant preference durability. Noting that time non-separability places a wedge between the standard coefficient of relative risk aversion and actual relative risk aversion toward atemporal gambles, the estimated models are used to generate a time series of relative risk aversion. This series, based on the estimated risky asset demand model, is consistent with asset market behavior with risk aversion reaching a peak of 5.74 in the second quarter of 1982 and all increases in risk version leading every recession over the past fifty years.

¹ See, for example, Sundaresan (1989), Abel (1990), Constantinides (1990), Ferson and Constantinides (1991), Heaton (1993, 1995), Campbell and Cochrane (1999, 2000), Carroll, Overland and Weil (2000).

Habit formation introduces time non-separability in the utility function in the sense that current felicity, or instantaneous utility, is generated by the current rate of consumption and reduced by an exponentially weighted average of past consumption rates. The conventional interpretation of consumption habits is that this non-separability generates a greater desire to smooth consumption over the life-cycle. An equally valid and intuitive interpretation rests on the key insight that the weighted average of past consumption – the habit stock – carries an implicit *price* which reflects the desire to maintain that habit, or standard of living, for the rest of the individual’s life. At each point in time, the cost of this habit inflicts a *liability* upon the individual’s current net wealth. As a result, decision making is not a function of net wealth but of *effective wealth*, defined as the difference between net wealth and the habit liability. The effect of human wealth in the form of future labor earnings is easily nested into this framework and provides an additional dimension to effective wealth.

This interpretation of the habit price, the resulting habit liability, and the role of effective wealth rather than net wealth is appealing for several reasons. First, time non-separable utility has the potential of greatly complicating the standard life-cycle problem. Indeed, this could explain why habit forming preferences have not seen more use in behavioral modeling despite their merits. However, it is shown that these complications mask some strong similarities between non-habit and habit forming households. Optimal consumption and portfolio allocation in a life-cycle model with time separable Hyperbolic Absolute Risk Aversion (HARA) felicity are both linear in financial (Merton, 1969, 1971) plus human (Bodie, Merton, Samuelson, 1992) wealth. The introduction of time non-separable utility from habit formation yields solutions for the optimal service flow to utility (consumption less the subjectively weighted habit stock) and portfolio allocation that are linear in effective wealth. Thus, habit forming households are shown to behave similar to non-habit forming households whose wealth is reduced by the liability of maintaining their current habit stock over their remaining lives. Thus, it is not surprising that this result also holds for all HARA felicity. In this sense, it is conceivable for researchers to bypass the seeming complications of time non-separable utility and treat habit formation in a manner similar to they way in which one jumps between models with and with out capitalized nonstochastic labor earnings. Indeed, as shown below, this is precisely how it should be treated.

The second advantage of this interpretation lies in the perspicuous intuition that can be applied both to the solution method used in Section II and the resulting optimal policy functions. The way in which researchers typically solve life-cycle problems of a complicated nature is that of “divine revelation”, i.e. guess a solution and show that it works. The approach taken in this paper follows a more pragmatic solution procedure that is based on the Symmetry Theorem of

Boyd (1990). By using the various forms of symmetry that exist in the problem, the solution can be easily developed in several minor steps. Furthermore, the recognition of these symmetries builds the intuition for the habit liability. A consumption habit can be thought of as the desire to maintain a current standard of living in terms of a subjectively weighted consumption rate. An individual determines the amount of wealth required to maintain their standard of living for the rest of their lives. This amount is the habit liability and is equal to the present value, discounted at the risk free rate of interest, of the future stream of weighted consumption. Thus, although the habit stock is backward looking, the habit liability looks toward the future.

In this paper, an emphasis is placed on the behavior of life-cycle portfolio allocation under the influence of a consumption habit. Most research regarding the equity premium puzzle has emphasized the relationship between the unconditional moments of aggregate consumption and asset returns. This is not surprising since any model of asset returns is a model of the first two moments of the stochastic discount factor which, in general equilibrium, must be related to consumption in some manner (Hansen and Jagannathan, 1991). However, the manifestation of this puzzle is that households hold too little of their wealth in the form of risk bearing assets. Using this insight, it is shown below how habits significantly reduced the demand for risky assets both in absolute terms and as a share of net wealth. Again, the intuition is clear in terms of the difference between actual and effective wealth. A higher standard of living is met by an increased habit liability. This habit liability reduces the amount of risk an individual can take on. To wit, for a given level of wealth, households with a high standard of living cannot “afford” to be as risky as households with a lower standard of living.

In the next section, a life-cycle model of consumption and portfolio allocation is developed. A similar model was developed in Constantinides (1990) who examined an infinite horizon household with no labor earnings. In contrast, the model examined in Section II considers the finite life-cycle aspects of habits for a household with mortality risk, nonstochastic labor earnings and a period of retirement. Life-cycle simulations are used to examine the various aspects of habit formation in Section III. It is shown that while the marginal cost of consumption reduces net wealth by exactly one dollar, effective wealth is reduced by as much as four dollars depending on the age of the household. This explains the significantly smaller marginal propensity to consume for households with a higher standard of living, all else equal. Also, the life-cycle course of portfolio allocation is decomposed into the parts attributable to myopic demand, changes in human wealth, and changes in the habit liability. In Section IV, an empirical model for both consumption and asset demand is developed and estimated using aggregate data. Concluding remarks are offered in Section V.

II. Life-Cycle Consumption and Portfolio Choice with a “Habit Liability”

Households are assumed to maximize over their finite lives the expected discounted sum of all future felicities which are generated by the difference between consumption (c_t) and a weighted state dependent consumption habit stock (k_t),

$$E_t \int_t^T e^{-r_t} u(c_t - \mathbf{q}k_t) dt, \quad (2.1)$$

where the felicity function is of the isoelastic form, $u(x) = x^{1-g}/(1-g)$, and \mathbf{g} is defined as the coefficient of relative risk aversion. As shown below, \mathbf{g} is not the actual level of relative risk aversion toward atemporal gambles in a model with labor earnings and habit formation but still plays an important role. Future felicity is discounted exponentially by $\mathbf{r}_t = \mathbf{r}t + \mathbf{h}(e^{ut} - 1)$. This can be interpreted as reflecting the intertemporal rate of time preference, \mathbf{r} , and a random date of death with an increasing mortality hazard rate (Yaari, 1969).²

A household’s current consumption habit can be thought of as the added desire to maintain a given standard of living once it has been achieved. The importance a household attaches to this consumption habit, or standard of living, is reflected in the weight \mathbf{q} . Although attention is focused on the case of $\mathbf{q} \geq 0$, the solution provided below is equally valid for cases in which $\mathbf{q} < 0$ whereby the felicity function reflects a positive effect from the *durability* of past consumption. The interpretation attributed to the consumption habit suggests it is effected by the accumulation of past consumption. A household’s standard of living is increased by contemporaneous consumption at rate \mathbf{d}_c while the effect of accumulated past consumption diminishes over time at rate \mathbf{d}_k . Thus, the current standard of living is given by

$$k_t = \mathbf{d}_c \int_0^t e^{-\mathbf{d}_k(t-t')} c_{t'} dt' + e^{-\mathbf{d}_k t} k_0. \quad (2.2)$$

The initial standard of living (k_0) may be greater than zero reflecting the consumption patterns passed on from the parents of a household. The consequences of this endowment may not be trivial. To the extent that consumption habits affect behavior, heterogeneity in the initial standard of living could aid in explaining heterogeneity in consumption and portfolio allocation among households, particularly among younger households. Given (2.2), k_t evolves over time according to the differential equation $dk_t = (\mathbf{d}_c c_t - \mathbf{d}_k k_t) dt$. Without loss of generality, it is assumed that

² Unlike in Yaari (1969), it is not assumed that a perfect annuity market exists. The nonexistence of annuity markets implies that a net wealth constraint preventing borrowing must be imposed at every point in time. Since net wealth does not fall below zero along the optimal solution path for the parameters examined in this paper, this constraint is not explicitly imposed.

$\mathbf{d}_c = \mathbf{d}_k = \mathbf{d}$. This normalization is made so as to keep the consumption habit in the same units as consumption itself.³

Households can allocate their existing wealth over time using two technologies. The first is in the form of a riskless asset with rate of return r . The second is in the form of shares in a risky asset (a_t) that can be purchased at price p_t . The dynamics of this price are described by the stochastic differential equation $dp_t = \mathbf{m}p_t dt + \mathbf{s} p_t dz_t$ where \mathbf{m} is the instantaneous expected rate of return and \mathbf{s} is the instantaneous standard deviation of the rate of return. Uncertainty regarding the rate of change in the price is generated by dz_t which is a Gauss-Wiener process with zero mean and unit variance rate.

Along with the income generated from savings, households earn a non-stochastic flow of labor earnings (y_t) which grows at rate g , $dy_t = gy_t dt$, until a certain age of retirement, R . Defining the demand for risky assets as $\mathbf{a}_t = p_t a_t$, the change in a household's wealth over a differential unit of time is given by

$$dw_t = [\mathbf{a}_t (\mathbf{m} - r) + rw_t + y_t - c_t] dt + \mathbf{a}_t \mathbf{s} dz_t \quad (2.3)$$

where $y_t = 0$ for $t \geq R$. Choosing the functions c_t and \mathbf{a}_t over the period $\mathbf{t} \in [t, T]$ to maximize (2.1) subject to (2.3) and the dynamic equations for labor earnings and the standard of living yields the indirect utility, or value, function

$$J(w_t, y_t, k_t, t) = \text{Max}_{\{c_t, \mathbf{a}_t\}_{t \rightarrow T}} E_t \int_t^T e^{-r_t} u(c_t - \mathbf{q} k_t) dt, \quad (2.4)$$

given $\{w_t, y_t, k_t\}$. By way of the principle of optimality, (2.4) can be reduced to maximizing with respect to two variables at time t ,

$$0 = \text{Max}_{\{c_t, \mathbf{a}_t\}} e^{-r_t} u(c_t - \mathbf{q} k_t) h + E_t \Delta J + O(h), \quad (2.5)$$

where $\Delta J = J(w_{t+h}, y_{t+h}, k_{t+h}, t+h) - J(w_t, y_t, k_t, t)$ and $O(h)$ is of order larger than h . Taking expectations of the Taylor expansion of the first term in ΔJ around $h=0$, dividing by h and taking the limit as $h \rightarrow 0$ yields the Hamilton-Jacobi-Bellman (HJB) equation for the control problem,

³ Consider rewriting the differential equation as $d\tilde{k} = \mathbf{d}_k (c_t - \tilde{k}_t) dt$ where $\tilde{k}_t \equiv (\mathbf{d}_k / \mathbf{d}_c) k_t$. Making this transformation in the felicity function, $u(c_t - \mathbf{q} k_t) = u(c_t - \mathbf{q} (\mathbf{d}_c / \mathbf{d}_k) \tilde{k}_t)$, suggests that allowing $\mathbf{d}_c \neq \mathbf{d}_k$ simply applies an alternative metric to the habit stock. In Constantinides (1990), the normalization is $\mathbf{q} = 1$ and $\mathbf{d}_c \neq \mathbf{d}_k$. The normalization chosen here is more intuitive in the sense that the habit stock is simply the weighted sum of past consumption and enters the felicity function with weight \mathbf{q} .

$$\begin{aligned}
0 = \underset{\{c_t, \mathbf{a}_t\}}{\text{Max}} & e^{-r_t} u(c_t - \mathbf{q}k_t) + J_w(\mathbf{a}_t(\mathbf{m} - r) + rw_t + y_t - c_t) \\
& + J_y g y_t + J_k \mathbf{d}(c_t - k_t) + J_t + \frac{1}{2} J_{ww} \mathbf{a}_t^2 \mathbf{s}^2,
\end{aligned} \tag{2.6}$$

where subscripts on J indicate partial derivatives of $J(w_t, y_t, k_t, t)$. The last term is a result of the multiplication rule $(dz_t)^2 = dt$.

Optimal consumption and demand for the risky asset at time t are obtained directly from the first order conditions to (2.6):

$$\begin{aligned}
c_t^* &= \left[e^{r_t} (J_w - \mathbf{d}J_k) \right]^{-1/g} + \mathbf{q}k_t \\
\mathbf{a}_t^* &= \left(\frac{\mathbf{m} - r}{\mathbf{s}^2} \right) \left(\frac{-J_w}{J_{ww}} \right).
\end{aligned} \tag{2.7}$$

Substituting these into (2.6) yields a partial differential equation for the value function. It is convenient to think of a household as maximizing with respect to the service flow from the gap between consumption and the weighted consumption habit, $s_t \equiv c_t - \mathbf{q}k_t$. Re-labeling c_t as $s_t + \mathbf{q}k_t$, the HJB equation is re-written as

$$\begin{aligned}
0 = e^{-r_t} u(s_t^*) + J_w(\mathbf{a}_t^*(\mathbf{m} - r) + rw_t + y_t - s_t^* - \mathbf{q}k_t) + J_y g y_t \\
+ J_k \mathbf{d}(s_t^* - (1 - \mathbf{q})k_t) + J_t + \frac{1}{2} J_{ww} \mathbf{a}_t^{*2} \mathbf{s}^2,
\end{aligned} \tag{2.8}$$

where $s_t^* = c_t^* - \mathbf{q}k_t$ is readily obtained from (2.7).

Rather than guess a solution for the value function, a much more pragmatic methodology utilizes the various forms of symmetry in the optimization problem to determine the functional form. A useful theorem from Boyd (1990) regarding constrained optimization notes that there are a multitude of transformations that leave the constraint set unchanged thereby unaffected the shape of the value function or doing so in a tractable manner. Upon repeated application of this result, a solution to the HJB partial differential equation becomes obvious.

The first symmetry transformation involves the capitalization of labor earnings. Consider decreasing earnings in period t by \mathbf{e} , i.e. $y_t \rightarrow y_t - \mathbf{e}$. For this to leave the budget set unchanged and thus not affect the value function, wealth must be increased. In the first period, wealth is increased by \mathbf{e} . Since income is generated by an AR(1) process, all future values of income are also affected. That is, income is given by $y_{t+t} = y_t e^{gt}$. So if $y_t \rightarrow y_t - \mathbf{e}$, then $y_{t+t} \rightarrow y_{t+t} - \mathbf{e}e^{gt}$ for $t \leq t \leq R$. Provided there is no difference between a discounted dollar in the future and a current dollar, the budget set is unaffected if current wealth is increased by the transformation

$$w_t \rightarrow w_t + \int_t^R \mathbf{e} e^{-(r-g)t} dt = w_t + \frac{\mathbf{e}}{r-g} [1 - e^{-(r-g)(R-t)}]. \quad (2.9)$$

Since there is no change to the budget set and preferences are unchanged, this transformation does not alter the shape of the value function,

$$J(w_t, y_t, k_t, t) = J\left(w_t + \frac{\mathbf{e}}{r-g} [1 - e^{-(r-g)(R-t)}], y_t - \mathbf{e}, k_t, t\right). \quad (2.10)$$

Setting $\mathbf{e} = y_t$ eliminates income as a state variable from the problem and allows the problem to be solved in terms of the sum of net wealth (w_t) and human wealth (h_t),

$$J(w_t, y_t, k_t, t) = J(w_t + h_t, 0, k_t, t), \quad (2.11)$$

where $h_t = y_t [1 - e^{-(r-g)(R-t)}] (r-g)^{-1}$ is the present discounted value of future labor income.

Now consider an increase in the consumption habit at time t holding the flow of utility services constant. That is, $k_t \rightarrow k_t + \mathbf{e}$ holding s_t constant for all time periods. For utility services to remain the same, consumption must be increased. By how much? As with income, an increase in the current habit increases it at all future dates as well. Given a fixed $s_t = s$, (2.2) implies that the change in a household's future consumption habit by a current one time increase of \mathbf{e} is $k_{t+t} \rightarrow k_{t+t} + \mathbf{e} e^{-d(1-q)t}$. Consumption must therefore change by $c_{t+t} \rightarrow c_{t+t} + \mathbf{q} \mathbf{e} e^{-d(1-q)t}$. The amount of current wealth required to finance this increase in consumption is given as

$$w_t \rightarrow w_t + \int_t^T \mathbf{q} \mathbf{e} e^{-(r+d(1-q))t} dt = w_t + \frac{\mathbf{q} \mathbf{e}}{r+d(1-q)} [1 - e^{-(r+d(1-q))(T-t)}]. \quad (2.12)$$

Again, since both the budget set and preferences are unchanged under this transformation, the value function is also unchanged. Setting $\mathbf{e} = -k_t$ eliminates the household's standard of living as a state variable and further simplifies the functional form of the value function,

$$J(w_t, y_t, k_t, t) = J(\Omega_t, 0, 0, t), \quad (2.13)$$

where $\Omega_t = w_t + h_t - b_t k_t$. The price of the habit stock is given by

$$b_t = \frac{\mathbf{q}}{r+d(1-q)} [1 - e^{-(r+d(1-q))(T-t)}] \quad (2.14)$$

and is interpreted as the amount of wealth required to maintain an extra dollar of the \mathbf{q} weighted habit stock. Thus, the habit liability is given by $b_t k_t$.

The two transformations used above involve additive symmetries in the constraint set leaving preferences unaffected. The last symmetry used to simplify the solution involves a scale symmetry in both the constraint set as well as preferences. Consider increasing all current control

and state variables by same factor \mathbf{e} , i.e. $w_t \rightarrow \mathbf{e} w_t$, $y_t \rightarrow \mathbf{e} y_t$, $\mathbf{a}_t \rightarrow \mathbf{e} \mathbf{a}_t$, $c_t \rightarrow \mathbf{e} c_t$, and $k_t \rightarrow \mathbf{e} k_t$. Note that these transformations imply $\Omega_t \rightarrow \mathbf{e} \Omega_t$ and $s_t \rightarrow \mathbf{e} s_t$. Also note that this transformation simply re-scales the budget set. However, this change affects preferences since s_t is transformed. The result is a modified value function

$$\begin{aligned}
J(\mathbf{e} w_t, \mathbf{e} y_t, \mathbf{e} k_t, t) &= J(\mathbf{e} \Omega_t, 0, 0, t) \\
&= \text{Max } E_t \int_t^T e^{-r_t} u(\mathbf{e} s_t, \mathbf{t}) dt \\
&= \mathbf{e}^{1-g} \text{Max } E_t \int_t^T e^{-r_t} u(s_t, \mathbf{t}) dt \\
&= \mathbf{e}^{1-g} J(w_t, y_t, k_t, t).
\end{aligned} \tag{2.15}$$

Setting $\mathbf{e} = \Omega_t^{-1}$ yields the solution to the partial differential equation in (2.8),

$$J(w_t, y_t, k_t, t) = \Omega_t^{1-g} J(1, 0, 0, t) \tag{2.16}$$

leaving $J(1, 0, 0, t)$ which is only a function of time. Although not necessary, it is convenient to choose a modified function of $J(1, 0, 0, t)$,

$$J(w_t, y_t, k_t, t) = \frac{e^{-r_t} A_t^{-g}}{1-g} \Omega_t^{1-g}, \tag{2.17}$$

where $A_t \equiv [(1-g)e^{r_t} J(1, 0, 0, t)]^{-1/g}$ which is a differential equation in time and solved using (2.8) yielding:

$$\begin{aligned}
A_t &= \left[\int_t^T e^{l_{1t}} \mathbf{I}_{2t} dt \right]^{-1} \\
l_{1t} &= \left(\frac{1-g}{g} \right) \left(r - \frac{r}{1-g} + \frac{1}{2} \frac{(m-r)^2}{g s^2} \right) (t-t) - \mathbf{h}(e^{ut} - e^{ut}) \\
\mathbf{I}_{2t} &= (1 + \mathbf{d}b_t)^{(g-1)/g}.
\end{aligned} \tag{2.18}$$

Verification that (2.17) is a solution to the partial differential HJB equation with A_t given by (2.18) is provided in the appendix.

The solution for the value function is similar to the intertemporal consumption/portfolio allocation problem of Merton (1969, 1971).⁴ However, rather than wealth, indirect utility is generated by *effective wealth*, Ω_t . A dollar of effective wealth is worth less than a dollar of actual wealth, either financial or human, since the amount of consumption this dollar can purchase will also be met by an increase in the household's standard of living. The degree to which actual

⁴ Setting $\mathbf{q} = 0$ and assuming no labor income yields the Merton (1969) result. Also, note that setting $\mathbf{q} = 1$, assuming $\mathbf{d}_c \neq \mathbf{d}_k$, $y_t = 0$ and $\mathbf{h} = 0$, and letting $T \rightarrow \infty$ yields the solution provided in Constantinides (1990).

wealth is effectively reduced is precisely characterized by the necessity of maintaining the current habit stock for the rest of the household's life, $b_t k_t$.

Using the value function in (2.17), the solution to the control functions follows directly from (2.7),

$$\begin{aligned} c_t^* &= \mathbf{q} k_t + A_t (1 + \mathbf{d} b_t)^{-1/g} \Omega_t \\ \mathbf{a}_t^* &= \left(\frac{\mathbf{m} - r}{g \mathbf{s}^2} \right) \Omega_t \end{aligned} \quad (2.19)$$

for $t \leq T$ given $\{w_t, y_t, k_t\}$. Consumption and the demand for risky asset are both linear functions of effective wealth which is comprised of financial and human wealth as well as the liability of maintaining the household's current standard of living. In general, for the habit forming model outlined in this section, optimal consumption and the demand for the risky asset will be linear functions of effective wealth if and only if $u(s)$ is a member of the Hyperbolic Absolute Risk Aversion (HARA) family. A proof of this follows Merton (1971) where wealth is replaced by effective wealth and is provided in the appendix.

The dynamics of (2.19) stem largely from the dynamics of effective wealth which is itself endogenous. In the appendix the growth rate of the service flow to utility is derived,

$$ds_t^* / s_t^* = \mathbf{f}_{s,1}(t) dt + \mathbf{f}_{s,2} dz_t \quad (2.20)$$

where

$$\mathbf{f}_{s,1}(t) = \mathbf{g}^{-1} \left(r - \mathbf{r} - \mathbf{h} u e^w - \frac{b_t (r + \mathbf{d}(1 - \mathbf{q})) - \mathbf{q}}{(1 + \mathbf{d} b_t)} + \left(\frac{1 + \mathbf{g}}{\mathbf{g}} \right) \frac{(\mathbf{m} - r)^2}{\mathbf{s}^2} \right) \text{ and } \mathbf{f}_{s,2} = \frac{(\mathbf{m} - r)}{g \mathbf{s}}.$$

Thus, the consumption habit can be interpreted as making the household more patient since $\dot{b}_t = b_t (r + \mathbf{d}(1 - \mathbf{q})) - \mathbf{q} < 0$. This increase in patience will partially offset the increase in impatience from mortality risk. However, this effect decreases monotonically over the life-cycle as the number of years the habit needs to be maintained falls. In the infinite horizon context examined by Constantinides (1990), $\dot{b}_t = 0$. In this case, the growth rate of consumption for non-habit forming households is identical to the growth rate of the service flow to utility of habit forming households. The growth rate of consumption follows directly from (2.20) and the definition of s_t ,

$$dc_t^* / c_t^* = \left[(\mathbf{f}_1^*(t) + \mathbf{q} \mathbf{d}) - (\mathbf{f}_1^*(t) + \mathbf{d})(\mathbf{q} k_t / c_t^*) \right] dt + \left[1 - (\mathbf{q} k_t / c_t^*) \right] \mathbf{f}_2^* dz_t. \quad (2.21)$$

Although the effect of the habit on the service flow to utility is clear, the effect on consumption is ambiguous depending on the parameters of the model as well as the current ratio of the weighted

habit stock to consumption. It is clear however that the variance of the consumption growth rate is increasing in the level of consumption and decreasing in the weighted habit stock. It is this ability of the consumption habit to reduce the variance of consumption growth has been noted as a possible explanation for the equity premium puzzle (e.g. Sundaresan, 1989; Constantinides, 1990; Campbell and Cochrane, 1999, 2000).

The growth rate for the optimal demand of the risky asset, also derived in the appendix, is given by

$$d\mathbf{a}_t^*/\mathbf{a}_t^* = \mathbf{f}_{a,1}(t)dt + \mathbf{f}_{a,2}dz_t \quad (2.22)$$

where

$$\mathbf{f}_{a,1}(t) = r + \frac{(\mathbf{m}-r)^2}{g\mathbf{s}^2} - A_t(1+db_t)^{(g-1)/g} \quad \text{and} \quad \mathbf{f}_{a,2} = \frac{(\mathbf{m}-r)}{g\mathbf{s}}.$$

Demand is generally increasing in the early stages of the life-cycle but decreasing at some point since all wealth is consumed by the end of life. That is, $\mathbf{f}_{a,1}(t)$ is positive initially but decreases monotonically and becomes negative at some point in the life-cycle. The effect of habit formation is to decrease the growth rate. This acts through the habit price, b_t , which is large for young households but decreases to zero by the end of life. Thus, changes in the demand for risky assets are most different between non-habit and habit forming households early in the life-cycle. Although this may seem to be a result of the fact that effective wealth is most different between young non-habit and habit forming households, note that the difference in the growth rate is not dependent upon the initial habit stock which is the cause of differences in initial effective wealth.

III. Life-Cycle Simulations

To better understand the effects of habit formation, the solution paths for life-cycle consumption and portfolio allocation are simulated. These simulations solve the entire life-cycle problem for a household beginning at age 20, working to age 65 and dying at age 100. The parameters of the model are provided in Table 1. A household begins its life with \$5,000 of financial net wealth and labor earnings of \$30,000 a year which grows by one percent per year. The initial stock of equals the initial flow of labor earnings. The coefficient of relative risk aversion is set to 3.0 which is in the range of plausible values in non-habit forming models. The fixed component of the intertemporal rate of discounting is set to zero but the time varying component from mortality risk at age 20 implies a rate of 0.021. This rate increases over the life-cycle. The habit forming household weighs its current habit stock by 0.875 relative to their current consumption in generating current felicity. The habit accumulates by 25% of the difference between the flow of consumption and the habit stock. The financial parameters of the

model approximate aggregate U.S. data. The real risk free rate of return is set to 3.0% per year while the mean real return to the risky asset is 8.0% per year with a standard deviation of 17 percentage points. For expositional purposes, the price of the risky asset is assumed to earn its expected rate of return over the entire life-cycle of the household.

III.A Optimal Policy Paths and Effective Wealth

Optimal life-cycle consumption is presented in Figure 1. Households without habits experience a doubling of consumption over their working lives after which consumption falls towards the end of life. The degree to which consumption falls following retirement can be modified by slight changes in the mortality rate process. Introducing a weighted habit stock in the felicity function with the parameters in Table 1 has two effects on consumption. First, households decrease their overall consumption thereby generating a small amount of savings out of labor earnings early in life opposed to the large implied negative savings rate of households with no habit. The second effect is that the consumption profile flattens out – a result which follows from the added distaste for downward changes in consumption.

The natural log of consumption is shown in Figure 2. The interpretation of habit forming households having a stronger desire to smooth consumption is clear in this finite horizon model. However, the similarities between habit and non-habit forming households can also be seen in Figure 2 which shows the natural log of the service flow to utility. For non-habit forming households, the utility service flow is simply consumption. For habit forming households, the service flow is the difference between consumption and the weighted habit stock. The slopes of the profiles are essentially identical reflecting the fact that the allocation problem for the habit forming household is simply a transformation of the allocation problem for the non-habit forming household with consumption and wealth being replaced by difference between consumption and the weighted habit stock and effective wealth, respectively. For an infinite horizon household, the slopes would be identical.

The profiles for the demand of risky assets are shown in Figure 3 indicating a stark difference between non-habit and habit forming households. Both habit and non-habit forming households have humped shaped demand profiles with the demand falling to zero at the end of life when all wealth is consumed. However, habit forming households hold as much as 50% less of the risky asset over almost the entire life-cycle. This difference follows directly from the difference in effective wealth, Ω_t , which follows from the habit liability. The life-cycle of effective net wealth, $\Omega_t - h_t$, is shown in Figure 4. The consumption habit liability is largest at the beginning of the life since the household begins with a positive habit stock which needs to be

maintained over the life-cycle. Despite the increasing size of the habit stock with age, the decreasing length of time this needs to be maintained decreases the difference in effective net wealth between habit and on-habit forming households.

III.B The Marginal Propensity to Consume and the Marginal Cost of Consumption

Carroll, Overland, and Weil (2000) and Carroll (2000) have emphasized that habit formation reduces the marginal propensity to consume (*mpc*). The *mpc* out of effective wealth is equivalent to the *mpc* out of net wealth. From (2.19),

$$\frac{\partial c_t}{\partial \Omega_t} = \frac{\partial c_t}{\partial w_t} = A_t (1 + \mathbf{d}b_t)^{-1/g}. \quad (3.1)$$

Clearly, this is less than A_t . It is also the case that A_t is smaller for habit forming households when $g > 1$. The magnitude of the difference in the *mpc* between non-habit and habit forming households can be seen in Figure 5. At age 50, a household without a consumption habit has an *mpc* out of wealth of roughly seven cents to the dollar. In contrast, a household with a consumption habit has an *mpc* of just under two cents to the dollar. As noted above, an extra dollar of wealth is worth less to a household with a consumption habit. This is reflected by less of a response in consumption to a wealth shock.

To see why a dollar of wealth is worth less to a household with a consumption habit, consider the marginal cost of consumption. For households without a consumption habit, current wealth is reduced by one dollar for every dollar of consumption, $\partial w_t / \partial c_t = -1$. However, effective wealth for a household with a consumption habit is reduced by the cost of the extra dollar of consumption *plus* the amount by which the habit stock is increased times the current price of the habit, $\partial \Omega_t / \partial c_t = -(1 + \mathbf{d}b_t)$. This instantaneous effect at each age over the life-cycle is shown in Figure 6. For non-habit forming households, an increase in consumption of one dollar leads to an instantaneous decrease in wealth by one dollar at all ages. In contrast, the magnitude of this effect for a household with a consumption habit is such that effective wealth is reduced by more than four dollars for every dollar of increased consumption among younger households. This magnitude decreases with age as the length of time to maintain the liability – the cost of the habit – decreases. However, it is significantly below the effect on non-habit forming households for most ages. At the end of life, the marginal cost of consumption is identical between non-habit and habit forming households.

III.C Decomposition of Optimal Portfolio Allocation

Differences between net wealth and effective wealth generate portfolio share dynamics over the household's life-cycle. The advice typically offered by financial planners is for a young

household to carry most of their net wealth in risky assets and then gradually switch to riskless assets as they age. The most common reason for this advice is that risk bearing equities are more likely to outperform bonds over longer horizons than shorter horizons. This logic is usually incorrect.⁵ If households can re-balance their portfolios at any time, then there is no difference between the long and short run since all that matters is the length of time between re-balancing. That is, although expected excess returns over riskless assets are increasing over time, so too is the variance which should offset any desires to take advantage of larger expected returns. However, it is the case that households gradually shift their net wealth toward riskless assets as they age since the demand for risky assets is linear in *effective* wealth.

Figure 7 provides a decomposition of the various components that comprise the share of net wealth held in risky assets. Note that this share is given by

$$\frac{\mathbf{a}_t^*}{w_t} = \left(\frac{\mathbf{m} - r}{\mathbf{g}\mathbf{s}^2} \right) \left(1 + \frac{h_t}{w_t} - \frac{b_t k_t}{w_t} \right). \quad (3.2)$$

First, consider the case in which households have no labor income and are not affected by consumption habits, i.e. $h_t = 0$ and $\mathbf{q} = 0$. The standard myopic result holds in this case with the risky asset share independent of net wealth and given by the ratio of the risk premium to the variance corrected for risk aversion. This is the baseline component of the share and is indicated by the dotted line in Figure 7. It is the optimal myopic portfolio allocation result which is identical to the intertemporal optimizing result with only net wealth and no labor earnings or habits. The parameters of the model indicate that roughly 58% of net wealth should be held in the form of risky assets and that this should remain constant over the entire life-cycle.

Now consider the effects of adding a stream of nonstochastic labor income. Younger households will invest heavily in risky assets since h_t will be quite large relative to net wealth. The value of the future stream of income is essentially a large stake in a riskless asset.⁶ As a result, younger households balance their portfolio by demanding risky assets. However, as households age and approach retirement, the present value of future labor income decreases and thus so does the demand for risky assets. This point along with the effects of stochastic labor

⁵ The advice that long-term investors should invest in long-term bonds can be valid under stochastic interest rates. In this case, there are shifts in the investment opportunity set which allow for hedging. Long-term bonds are well suited for conservative long-lived investors to hedge the risk that real interest rates will decline. See Campbell and Viceira (2001) for this argument.

⁶ If there were risk associated with labor earnings that was not entirely idiosyncratic, the share of effective wealth in risky assets would be modified to reflect the hedging against changes in the investment opportunity set associated with the correlation of the stochastic component of labor earnings with market risk. Completely idiosyncratic risk would reduce the value of effective wealth but have no effect on the share of effective wealth. It would, however, affect the risky asset share of net wealth.

earnings were made clear in Bodie, Merton, Samuelson (1992). The effect of labor earnings on the risky asset share can be seen clearly in Figure 7 as the crossed curve which includes the baseline effect. The share starts very large and falls to the baseline level at retirement after which it remains constant.

Finally, consider the case of an initial endowment of net wealth with no labor income and $q > 0$.⁷ While the role of labor income in portfolio allocation ends at retirement, the consumption habit exists throughout the household's life making the demand for risky assets dependent upon wealth and past consumption at all ages. The intuition is that, at any age, the need to maintain a given standard of living is compared to the financial ability to afford that need – the lower the ability, the lower the risks a household can take in their portfolio allocation. As noted by Constantinides (1990), a household's level of risk aversion towards atemporal gambles involving net wealth is heightened by its standard of living, all else equal. Relative risk aversion is given by

$$rra = \frac{-J_{ww}w_t}{J_w} = \mathbf{g} \frac{w_t}{\Omega_t}. \quad (3.3)$$

Thus, the difference between net wealth and effective wealth drives a wedge between the coefficient of risk aversion, \mathbf{g} , and the actual level of relative risk aversion. With habit formation and without human wealth, this wedge acts to increase relative risk aversion. For a young household, net wealth is likely to be small relative to the need to maintain their consumption habit over their entire life-cycle. Even a small initial standard of living can significantly depress the fraction of wealth held in risky assets. As the household ages, the effect of the consumption habit acts through two channels. First, the habit changes depending on the changing consumption patterns of the household. If consumption is rising in the first half of the life-cycle and falling in the latter half – a profile which most realistic parameters generate and indicated in Figure 1 – so too is the household's standard of living. The second effect operates through the decreasing length of time the habit needs to be maintained, i.e. $\dot{b}_t < 0$ with $b_T = 0$. These two effects work in opposite directions. Combined with the life-cycle pattern of net wealth, the net effect of the consumption habit on portfolio allocation, while always having a negative impact on the share of wealth in risky assets, could be rising or falling at different stages of the life-cycle.

The effect of habits on the portfolio share is indicated by the circle curve in Figure 7 which includes the baseline share. Given the positive initial habit stock from Table 1, the habit effect with no labor income implies that young households should invest heavily in the risk free asset to the point of going short in the risky asset. By age 45, households begin investing

positively in the risky asset. The share increases to roughly 20% at retirement after which it falls slowly to zero at the end of life. The combination of the baseline share with the human wealth and habit effect is indicated in Figure 7 by the solid line. Clearly, the human wealth effect dominates for younger households. However, its magnitude is diminished. While households without habits hold close to 150% of their net wealth in risky assets at age 40, households with habits hold only 80%. Although this difference decreases slightly over the life-cycle, it remains roughly on the order of fifty percentage points at most ages.

IV. Empirical Model

IV.A Stochastic Processes for Effective Wealth, Risky Asset Demand and Consumption

In this section, an empirical model is developed to estimate the relative importance of net wealth, human wealth and the habit liability in the demand for risky assets and consumption using aggregate data. The empirical models are based on the optimal policy functions in (2.19). Combining these with the dynamic budget constraint in (2.3), it is shown in the appendix that optimal effective wealth evolves as a geometric random walk with a time varying drift,

$$\ln \Omega_{t+1} = \ln \Omega_t + \mathbf{g}^{-1} [r - \mathbf{r} + (\mathbf{m} - r)^2 / (2\mathbf{s}^2)] - \ln(A_{t+1}/A_t) + \mathbf{x}_{t+1} \quad (4.1)$$

where \mathbf{x} is *i.i.d.* $N(0, (\mathbf{m} - r)^2 / (\mathbf{g}\mathbf{s})^2)$. The stochastic normality of effective wealth is generated by the geometric Brownian motion assumption of asset prices. The time varying component of the drift, $\ln(A_{t+1}/A_t)$, is assumed to be zero. For a single household, this term partially reflects changes in the marginal propensity to consume over the life-cycle. With a representative agent using aggregate data, it reflects changes in the age distribution. However, the aggregate *mpc* will not be sensitive to these changes. To see this, note that the *mpc* changes very little for $t \ll T$.⁸ That is, households with many years remaining to live behave similar to infinite horizon households ($A_{t+1}/A_t \rightarrow 1$ as $T \rightarrow \infty$). Furthermore, the data used below is measured in annual quarters. With this unit of time, changes in the *mpc* between periods are likely to be even smaller.

The empirical model for the demand of risky assets and consumption follow immediately from (2.19) and (4.1),

$$\ln(\mathbf{a}_t) - \mathbf{k}_a - \ln \Omega_{t-1} = \mathbf{x}_t \quad (4.2)$$

and

$$\ln(c_t - \mathbf{q}k_t) - \mathbf{k}_c - \ln \Omega_{t-1} = \mathbf{x}_t, \quad (4.3)$$

⁷ The initial endowment is set to match the value of the model's initial net wealth plus the present discounted value of labor earnings in the case where earnings are nonzero.

⁸ Note that $A_{t+1}/A_t = 1 + (\int_t^{t+1} e^{l_{1t}} \mathbf{I}_{2t} dt) / (\int_{t+1}^T e^{l_{1t}} \mathbf{I}_{2t} dt) \approx 1$ when $t \ll T$.

where the constant terms k_a and k_c are provided in the appendix. These constants are functions of the parameters which include the risk free interest rate, the risk premium, the variance of risky assets, the rate of time discount and the coefficient of risk aversion. Mortality risk is not considered since aggregate data is being used to estimate the model. Also, it is important to note that although the error term, x_t , has a mean of zero, its variance is a function of the risk premium, the variance of risky assets and the coefficient of risk aversion. This complicates the estimation procedure. Note g 's role in the variance of the error term for the demand of risky assets. Higher values of g lower demand. This is reflected in a smaller k_a . However, it also decreases the variance of demand since less wealth is held in that form. As $g \rightarrow \infty$, the demand for risky assets goes to zero along with the variance. For small values of gamma, most of household wealth is held in the form of risky assets and its variance increases. Neglecting this relationship leads to an error in specification. A similar argument can be made for consumption.

The parameter of central interest is the force of habit, q , which is important in determining effective wealth, Ω_{t-1} , as well as the service flow to utility in (4.3). For values of $q > 0$, households behave according the simulations examined in Section III. Although net wealth and human wealth enter effective wealth with unit coefficients, this is relaxed to empirically estimate their importance in determining portfolio allocation and consumption. That is, for the purposes of estimation in this section, $\ln(w_t + h_t - b_t k_t)$ is replaced by $b_0 \ln(w_t + b_1 h_t - b_t k_t)$ where b_0 and b_1 are parameters to be estimated. Note that b_0 is the effective wealth elasticity which should equal one. However, it is likely that b_1 will be less than one, reflecting uncertainty in future labor earnings.

IV.B The Data

In order to estimate either model (4.2) or (4.3), data on household, consumption, labor earnings and net worth is required. Estimation of the risky asset demand function also requires detailed wealth data to define risky versus non-risky assets. Quarterly aggregate time series data is used for these proposes. Seasonally adjusted personal consumption expenditures and labor earnings data come from the National Income and Product Accounts. Personal consumption expenditures are also decomposed into non-durable and durable consumption. Non-durable consumption includes non-durable expenditures and services. Labor earnings include wages and salaries, other labor income, proprietors' income and net transfers payments. Household wealth data comes from the Federal Reserve's Flow of Funds Z.1 historical tables (b100). Risky assets are defined to include corporate and non-corporate equities plus mutual funds.

All data is converted to 1999 dollars using the consumer price index of all urban consumers from the Bureau of Labor Statistics (CPI-U for 1999 is equal to 167.2 with base 1982-1984). Also, the data is normalized by annual Census estimates of the number of households in the United States to control for changing demographics over the past half century. Data covering the years 1952I through 2000IV is used in all analyses.

Many of the parameters in the model are identified weakly from the nonlinearities in the solution. Rather than estimate these parameters, only \mathbf{b}_0 , \mathbf{b}_1 , \mathbf{q} and \mathbf{g} are estimated. It is assumed that the rest of the parameters take on the values in Table 1 with the exception of the rate of time preference which is set to 0.08 since there is no mortality risk. It is also assumed that the representative agent is 40 years old with an age of retirement equal to 65 and a life span of 90 years.

Human wealth is computed by first estimating the mean growth rate of labor earnings over the sample period. This is roughly 1.2% per year. Human wealth is then computed as $h_t = y_t[1 - e^{-(r-g)25}]/(r-g)$ where $r=0.03$ and $g=0.012$. The habit stock, k_t , is based on the personal expenditures series. Unless specified otherwise, the non-durable consumption series is used. It is assumed that the starting value of the habit stock in 1952I is equal to the value of consumption in that quarter since this is the steady state value. All values after that are computed iteratively on a quarterly basis using the difference equation:

$$k_t = k_{t-1}e^{-((1+d)^{0.25}-1)} + (1 - e^{-((1+d)^{0.25}-1)})c_{t-1},$$

with $\mathbf{d} = 0.25$.

IV.C Demand for Risky Assets: Estimation Results

Both models (4.2) and (4.3) are estimated by maximum likelihood. The contribution of a single observation to the likelihood function of the demand for risky assets is given by

$$\ln L_t = \ln(\mathbf{f}(\ln(\mathbf{a}_t) - \mathbf{k}_a - \mathbf{b}_0 \ln(w_{t-1} + \mathbf{b}_1 h_{t-1} - b k_{t-1}))/\mathbf{s}_x) - \ln(\mathbf{s}_x) \quad (4.4)$$

where $\mathbf{f}(z)$ is the probability density function of a standard normal random variable, $b = \mathbf{q}(1 - e^{-(r+d(1-q)50)})/(r + \mathbf{d}(1-q))$ with $r=0.03$ and $\mathbf{d}=0.25$, and $\mathbf{s}_x = (\mathbf{m} - r)/(\mathbf{g}\mathbf{s})$ with $(\mathbf{m} - r) = 0.05$ and $\mathbf{s} = 0.17$. The likelihood function is maximized with respect to \mathbf{b}_0 , \mathbf{b}_1 , \mathbf{q} and \mathbf{g} . An estimate of \mathbf{s}_x can be computed given an estimate of \mathbf{g} . The standard error of \mathbf{s}_x follows from the estimate of \mathbf{g} and its standard error using a first order Taylor approximation. The likelihood function is not globally concave. Although the results are not presented, various initial

conditions are experimented with in order to check the robustness of the results presented in this section.

Table 2 presents the maximum likelihood results for the demand of risky assets. The first column estimates the model assuming $\mathbf{b}_1 = 0$ and $\mathbf{q} = 0$. This is the intertemporal portfolio allocation model of Merton (1969, 1971). Not surprisingly, net wealth plays a significant role in the demand for risky assets. The net wealth elasticity of risky asset demand, \mathbf{b}_0 , is statistically less than one with a value of 0.921 and a standard error of 0.004. Despite the fact that the manifestation of the equity premium puzzle is that too little wealth is held in the form of risky assets, this does not show up in the coefficient of risk aversion, estimated to be 1.839 with a standard error of 0.092. This could be because the distribution of risky asset ownership in the population is highly skewed. That is, the demand for risky assets per household based on aggregate data does not adequately reflect the risky asset ownership of the average household. As a result, the assumption of the representative agent may be inadequate.

A common criticism of behavioral models that involve “wealth” is the failure to include human wealth. The second column of Table 2 adds human wealth to net wealth. The effect of net wealth is still statistically less than one but with a value of 0.948 and a standard error of 0.009. Surprisingly, the effect of human wealth is trivial and slightly negative. This could be explained if there were an omitted variable that was highly correlated with human wealth and negatively correlated with the demand for risky assets. Clearly, the habit stock, which is a weighted sum of lagged consumption could be such a variable. The estimate of the coefficient of risk aversion changes little with the addition of human wealth.

Column 3 of Table 2 reports the results of estimating the risky asset demand model including human wealth and the habit liability. The coefficient on net wealth is 0.990 with a standard error of 0.006. This is not significantly different from the theoretical value of one. As suspected, the coefficient on human wealth is positive and very significant but with a value much less than one. The value of 0.180 could be reflecting the decrease in value of future labor earnings from uncertainty. The point estimate of the force of habit parameter, \mathbf{q} , is 0.739, with a standard error of 0.023. The implied price of one unit of the consumption habit is \$7.69 which, in turn, implies a marginal cost of consumption of \$2.92. That is the price of an additional unit of consumption is increased by \$1.92 because of the effect it will have on the habit stock. The estimate of \mathbf{g} is 2.48. However, this is not the actual level of relative risk aversion as noted above in Section III. This is examined further in Section IV.E below.

The predictive performance of Models *i*, *ii*, and *iii* of Table 2 is shown in Figure 8. Actual risky asset demand is presented as the solid bold line. The demand for corporate equities increased over the 1950's and 1960's before falling precipitously during the 1970's, reaching its lowest point in the early 1980's. Over the period of 1984I to 2000I the series increased by more than 300% before falling by 13% in the year 2000. The solid line in Figure 8 shows the predicted value using the estimated Model *i* which only includes net wealth. The model under-predicts risky asset demand through the 1950's and 1960's and over-predicts during the 1970's up to 1998 when it roughly approximates the true series but still under-predicts. Model *ii* performs similar to Model *i*, reflecting the estimated trivial effect of human wealth. The inclusion of the habit liability in effective wealth performs remarkably well. The model still under-predicts demand in the 1950's and 1960's but by much less than Model *i* or *ii*. Most remarkable is that the model closely tracks the fall in risky asset demand through the 1970's. There is an over-prediction in the 1980's but this is eliminated in the 1990's when the model is almost coincident with the true series.

The habit model makes a clear prediction about the behavior of the demand for risky assets. Namely, for a given level of wealth, increases in consumption will lead to an increase in the habit stock which, in turn, should make households more risk averse since their habit has become more costly. This results in a fall in the demand for risky assets. The 1960's saw a large increase in wealth. However, the increase in consumption over this same period was larger in the sense that the liability induced by the increased habit stock led to a fall in effective wealth. The fall in total wealth in the 1970's only exacerbated the fall in effective wealth. The consequence was a large drop in the demand for risky assets. Without the additional effect of the increased habit liability, this fall would not have been as pronounced, as indicated by the predicted demand for risky assets excluding the habit liability (Models *i* and *ii* of Table 2). It was only in the 1990's that wealth increased enough to make household's sufficiently able to "afford" their consumption habits and, despite increasing consumption, significantly increase their demand for risky assets.

IV.D Demand for Consumption: Estimation Results

Now consider the consumption model of (4.3). The contribution of a single observation to the likelihood function of consumption is given by

$$\ln L_t = \ln \left(\mathbf{f}(\ln(c_t - \mathbf{q}k_t) - \mathbf{k}_c - \mathbf{b}_0 \ln(w_{t-1} + \mathbf{b}_1 h_{t-1} - b k_{t-1})) / \mathbf{s}_x \right) - \ln(\mathbf{s}_x). \quad (4.5)$$

The likelihood function is maximized with respect to $\mathbf{b}_0, \mathbf{b}_1, \mathbf{q}$ and \mathbf{g} . Consumption is initially defined as total personal expenditures, i.e. durable plus non-durable. The results are reported in Table 3. As with Table 2, the first column presents the estimates of the assuming $\mathbf{b}_1 = 0$ and

$q = 0$. The net wealth elasticity of consumption is larger than one with a value of 1.107 and a standard error of 0.001. Again, the coefficient of risk aversion, g , is not estimated to be excessively large and within the range of plausible values. Unlike the demand for risky assets, the inclusion of human wealth greatly improves the likelihood function. The wealth elasticity of consumption is closer to one with a value of 1.038 and a standard error of 0.004. The effect of human wealth on consumption is larger than with the demand for risky assets. A dollar increase in human wealth increases effective wealth by forty-nine cents. The coefficient of risk aversion is quite large with a value of 12.172 and a standard error of 0.618. This may seem consistent with the literature that estimates unreasonably large values for relative risk aversion. However, since effective wealth includes human wealth, the wedge between actual relative risk aversion and the coefficient of risk aversion, g , implies a much smaller level of risk aversion as examined below in Section IV.E.

The habit liability is included in effective wealth in Model *iii* of Table 3. The force of habit, q , is estimated as to be -0.163 indicating a durability of consumption. The point estimate is much less significant than the estimate using the demand for risky assets but significant nevertheless with a standard error of 0.080. The finding of consumption durability in aggregate consumption data is consistent with Dunn and Singleton (1986) but inconsistent with Ferson and Constantinides (1991) who find a dominating habit persistence. The estimation procedure used here is different from that of earlier research however in that, rather than estimating the sign of a coefficient on lagged consumption in the Euler equation, the closed form solution is estimated with the restriction that the variance of the error term is endogenous to the coefficient of risk aversion. As with Model *ii*, the estimate of the coefficient of risk aversion is large and significant. Note that since $\hat{q} < 0$, human wealth and the habit liability (or in this case, the durability “asset”) both act to decrease the actual level of risk aversion.

The personal expenditures series is composed of both durable and non-durable expenditures. Clearly, expenditures on durable goods should bias towards q being negative and thus indicating durability of consumption in household decision making rather than habit formation. The model examined in Section II assumes that consumption goods are not durable. In Table 4, personal expenditures are decomposed into non-durable and durable consumption. The estimated value of q is insignificant in the Model *iii* version of non-durable consumption indicating neither habit persistence nor durability. In contrast, the same model estimated using the durable consumption indicates a strong degree of durability with q estimated to -2.957 with a standard error of 0.407.

Predicted log consumption using Models *i*, *ii*, and *iii* are shown in Figures 9a and 9b along with the actual values. Figure 9a presents the results for non-durable consumption while Figure 9b presents the durable consumption results. Non-durable real personal expenditures per household has trended upward over the past fifty years ending the century over twice as large as it was in 1952. This trend was disrupted only in the in 1970's when consumption growth was stagnate. All three models predict this trend relatively well. Using only net wealth, Model *i* over-predicts the fall in consumption in the 1970's and over-predicts in the latter half of the 1990's. Model's *ii* and *iii* perform better with both predicted series being almost coincident with the actual consumption expenditure series. Actual durable real personal expenditures per household has also trended upward but with much more volatility. No predicted series performs remarkably well. Despite the significance of the consumption durability in Model *iii*, the estimated model over-predicted expenditures through the 1960's and greatly over-predicts the fall in the 1970's and early 1980's.

IV.E Implied Time Series of Relative Risk Aversion

The implied level of actual relative risk aversion toward temporal gambles can be computed using (3.3) and the estimates from Model *iii* of Tables 2 and 4. The estimated force of habit in the risky asset demand equation along with the reduced value of human wealth ($\hat{b}_1 = 0.180$) imply that net wealth is greater than effective wealth at all times. This increases the degree of relative risk aversion from the estimate of $\hat{g} = 2.480$. In contrast, the insignificance of the habit liability for non-durable consumption and estimated durability of durable consumption in the consumption equation imply net wealth is less than effective wealth at all times. This reduces the degree of relative risk aversion from the estimates of 10.158 and 9.875 for non-durable and durable consumption, respectively. The time series estimates of the actual level of relative risk aversion are presented in Figure 10. Although relative risk aversion is very low based on the estimated durable consumption function, the estimated non-durable consumption function as well as estimated risky asset demand function both indicate plausible values over all years. More interesting however is the degree of relative variability produced from the risky asset demand function. This is an attractive feature since it can help explain movements in the asset market.

For periods in which net wealth is not much larger than effective wealth – for instance, a period of low net wealth or a period following high consumption, both experienced in the 1970's – households will be more risk averse and thus less likely to purchase risky assets. This is exactly what is seen in Figure 10 for the risky asset demand derivation of risk aversion which increased

from 3.66 in 1972^{III} to 5.74 in 1981^{II}. In contrast, after a period of low consumption in which the standard of living is driven downward thereby decreasing the habit liability and making wealth more in line with effective wealth, households will be more willing to take on risk. This is indicative of the 1980's which followed a decade of stagnate consumption growth. With rising net wealth and a depressed habit liability, the level of relative risk aversion dropped to 4.28 in 1987^{II}. Although consumption has grown much during the 1990's, the ability to afford the resulting increasing habit liability from rising wealth has led to a level of relative risk aversion of 4.05 in 2000^{III}. The shaded areas indicate the peak-to-trough business cycle dates as set by the National Bureau of Economic Research. Note that increases in the relative risk aversion series, as defined by the estimated risky asset demand model, lead all recession periods.

The relative risk aversion series generated by the estimated non-durable consumption function is relatively flat over the past fifty years. Furthermore, the series has a slight trend downward through the 1970's after which a slight trend upward is indicated. This is inconsistent with the actual behavior of households who have increased their holdings of risky assets over the past two decades.

V. Conclusion

Time non-separable utility in the form of habit formation has the attractive feature of being able to explain a number of existing puzzles in macroeconomics and finance. In this paper, habit formation is recast in a way that notes its forward looking nature in a life-cycle context of consumption and portfolio choice. A household's habit liability is defined as the cost of maintaining its current standard of living over the remaining life-cycle. The behavior of habit forming households is shown to closely match that of non-habit forming households where net wealth is reduced by the habit liability. The difference between net wealth and the habit liability is defined as effective wealth. Human wealth in the form of future labor earnings is added to effective wealth as well. Habit forming households have a higher degree of patience and a smoother life-cycle consumption profile. The demand for risky assets is significantly reduced for households with a habit liability. Life-cycle simulations suggest that risky asset demand for a habit forming household is roughly 60% that of the same household without a habit liability over most ages. As a share of net wealth, the difference in the demand for risky assets between a household with and without a habit liability ranges from 40 to 200 percentage points depending on the stage of the life-cycle. Essentially, households with a higher standard of living for a given level of wealth cannot afford to be as risky as households with a lower standard of living. This is directly a result of the difference in effective wealth and reflected in risky asset demand differences.

Previous research has attempted to find indications of habit formation using the consumption Euler equation. The results have been mixed. While Ferson and Constantinides (1991) find a significant dominating habit persistence in monthly, quarterly and annual data, Heaton (1993) finds only weak evidence of habits. However, Heaton (1995) finds stronger evidence of habits when allowing for local substitutability but habit formation at lower frequencies. These authors use seasonally adjusted aggregate consumption expenditures on nondurables and services. Alternatively, Dynan (2000) uses micro panel data to estimate the consumption Euler equation for households with the service flow to utility generated by the weighted first difference in consumption along with preference shifters. The results suggest households are not effected by habits.

Unlike this previous research, closed form solutions for both the consumption and risky asset demand function are estimated in this paper. Seasonally adjusted, quarterly aggregate data on household balance sheets, non-durable consumption expenditures and labor earnings are used for estimation. Results suggest that habit formation is highly significant in the determination of the demand for risky assets. The estimated force of habit implies a marginal cost of consumption roughly three times the cost of standard time separable models. This would explain the excessively small marginal propensity to consume estimated in the literature. In contrast, estimates based on the consumption function suggest no role for habits using non-durable expenditures and a strong role for durability using durable expenditures. An estimate of the time series of relative risk aversion using the estimated risky asset demand model suggests significant variability consistent with asset market behavior.

Appendix

A. Solution Verification to the Hamilton-Jacobi-Bellman Partial Differential Equation

Using the proposed solution for the value function (2.17) along with the implied optimal policy functions (2.7), the components of the HJB equation (2.8) are

$$\begin{aligned}
(i) \quad & u(s_t^*, t) = \frac{a_t}{1-g} A_t \Omega_t^{1-g} (1 + \mathbf{d}b_t)^{(g-1)/g} \\
(ii) \quad & J_t = -(\mathbf{r} - \mathbf{h}ue^u) \frac{a_t}{1-g} \Omega_t^{1-g} - \frac{g a_t}{1-g} A_t^{-1} \Omega_t^{1-g} \dot{A}_t + a_t \Omega_t^{-g} \dot{h}_t - a_t \Omega_t^{-g} \dot{b}_t k_t \\
(iii) \quad & J_w \mathbf{a}_t^* (\mathbf{m} - r) = a_t \Omega_t^{1-g} \frac{(\mathbf{m} - r)^2}{g \mathbf{s}^2} \\
(iv) \quad & J_w r w_t = a_t \Omega_t^{-g} r w_t \\
(v) \quad & J_w y_t = a_t \Omega_t^{-g} y_t \\
(vi) \quad & -J_w s_t^* = -a_t A_t \Omega_t^{1-g} (1 + \mathbf{d}b_t)^{-1/g} \\
(vii) \quad & -J_w \mathbf{q} k_t = -a_t \Omega_t^{-g} \mathbf{q} k_t \\
(viii) \quad & J_y g y_t = a_t \Omega_t^{-g} g h_t \\
(ix) \quad & J_k \mathbf{d} s_t^* = -a_t A_t \Omega_t^{1-g} (1 + \mathbf{d}b_t)^{-1/g} \mathbf{d} b_t \\
(x) \quad & -J_k (\mathbf{d}_k - \mathbf{d}_c \mathbf{q}) k_t = a_t \mathbf{d} (1 - \mathbf{q}) b_t \Omega_t^{-g} k_t \\
(xi) \quad & \frac{1}{2} J_{ww} \mathbf{a}_t^{*2} \mathbf{s}_2 = -\frac{1}{2} a_t \frac{(\mathbf{m} - r)^2}{g \mathbf{s}^2} \Omega_t^{1-g}
\end{aligned}$$

where $a_t \equiv e^{-rt} A_t^{1-g} (1-g)^{-1}$. Noting that $\dot{h}_t = (r-g)h_t - y_t$, consider the penultimate term in (ii) along with equations (v) and (viii). These imply

$$a_t \Omega_t^{-g} \dot{h}_t + a_t \Omega_t^{-g} y_t + a_t \Omega_t^{-g} g h_t = a_t \Omega_t^{-g} r h_t. \quad (\text{A.1})$$

Now consider the last term in (ii) along with equations (vii) and (x), noting that $\dot{b}_t = b_t (r + \mathbf{d}(1-\mathbf{q})) - \mathbf{q}$. These imply

$$-a_t \Omega_t^{-g} \dot{b}_t k_t - a_t \Omega_t^{-g} \mathbf{q} k_t + a_t \mathbf{d} (1 - \mathbf{q}) b_t \Omega_t^{-g} k_t = -a_t \Omega_t^{-g} r b_t k_t. \quad (\text{A.2})$$

The sum of (iv), (A.1) and (A.2) yields

$$\begin{aligned}
a_t \Omega_t^{-g} r w_t + a_t \Omega_t^{-g} r h_t - a_t \Omega_t^{-g} r b_t k_t &= a_t \Omega_t^{-g} r (w_t + h_t - b_t k_t) \\
&= a_t r \Omega_t^{1-g}.
\end{aligned} \quad (\text{A.3})$$

Adding together the components (i) through (xi), substituting in (A.3), and multiplying by $(1-\mathbf{g})\mathbf{g}^{-1}a_t^{-1}A_t\Omega_t^{-(1-\mathbf{g})}$, the HJB equation in (2.8) is reduced to a second order differential equation in time,

$$\dot{A}_t = \left[\left(\frac{1-\mathbf{g}}{\mathbf{g}} \right) \left(r - \frac{\mathbf{r}}{1-\mathbf{g}} + \frac{1}{2} \frac{(\mathbf{m}-r)^2}{\mathbf{g}\mathbf{s}^2} \right) - \mathbf{h}u e^w \right] A_t + \left[(1+\mathbf{d}b_t)^{(\mathbf{g}-1)/\mathbf{g}} \right] A_t^2. \quad (\text{A.4})$$

The terminal condition is provided by the assumption that all wealth is consumed by the end of life. Since consumption is a flow in continuous time while wealth is a stock, this condition implies $\lim_{t \rightarrow T} c_t^*/(w_t + h_t) = \infty$. Noting that the standard of living is a stock that will remain finite and that both $h_T = 0$ and $b_T = 0$, this reduces to

$$\lim_{t \rightarrow T} s_t^*/(\Omega_t(1+\mathbf{d}b_t)^{-1/\mathbf{g}}) = \lim_{t \rightarrow T} A_t = \infty. \quad (\text{A.5})$$

A transformation of $z_t \equiv A_t^{-1}$ yields a first order differential equation in z_t with terminal condition $\lim_{t \rightarrow T} z_t = 0$. The solution is given by the inverse of (2.18).

B. Linear policy functions if and only if HARA felicity

The “if” part is clear from (2.19). To see the “only if” part, assume $s_t^* = m\Omega_t + n$ and $\mathbf{a}_t^* = p\Omega_t + q$. Since consumption is a linear function of the service flow to utility, it too is a linear function of effective wealth. Noting from (2.13) that $J_w = J_\Omega$, $J_{ww} = J_{\Omega\Omega}$, $J_k = -J_\Omega b_t$ and $J_{kw} = -J_{\Omega\Omega} b_t$, the first order condition for the service flow to utility in (2.7) implies that $u'(s_t^*) = e^{r_t} J_\Omega(1+\mathbf{d}b_t)$ and $u''(s_t^*) \partial s_t^* / \partial \Omega_t = e^{r_t} J_{\Omega\Omega}(1+\mathbf{d}b_t)$. The assumed solution for s_t^* then implies

$$\frac{-mu''(s_t^*)}{u'(s_t^*)} = \frac{-J_{\Omega\Omega}}{J_\Omega}. \quad (\text{A.6})$$

The first order condition for \mathbf{a}_t^* in (2.7) along with the assumed solution imply

$$\left(\frac{-J_{\Omega\Omega}}{J_\Omega} \right) = \left(\frac{\mathbf{s}^2 p}{\mathbf{m}-r} \Omega_t + \frac{\mathbf{s}^2 q}{\mathbf{m}-r} \right)^{-1}. \quad (\text{A.7})$$

Combining (A.6) and (A.7) yields

$$\frac{-u''(s_t^*)}{u'(s_t^*)} = (m's_t^* + n')^{-1} \quad (\text{A.8})$$

where $m' = \mathbf{s}^2 p(\mathbf{m}-r)^{-1}$ and $n' = \mathbf{s}^2 (mq - np)(\mathbf{m}-r)^{-1}$. By definition, (A.8) implies that $u(s)$ is a member of the HARA family.

C. Optimal growth rate of the service flow to utility and portfolio allocation

From the dynamic budget constraint (2.3) along with the optimal policy functions (2.19), the diffusion process for optimal effective wealth is given by

$$\begin{aligned} d\Omega_t &= d(w_t + h_t - b_t k_t) \\ &= dw_t^* + (rh_t - y_t)dt - \dot{b}_t k_t dt - b_t d(s_t^* - (1-q)k_t)dt \\ &= \Omega_t \left(r + \frac{(m-r)^2}{g\mathbf{s}^2} - A_t (1 + \mathbf{d}b_t)^{(g-1)/g} \right) dt + \Omega_t \frac{(m-r)}{g\mathbf{s}} dz_t. \end{aligned} \quad (\text{A.9})$$

Taking the total time differential of the optimal service flow to utility yields

$$ds_t^* = \frac{s_t^*}{A_t} \dot{A}_t + \frac{s_t^*}{\Omega_t} d\Omega_t - \frac{s_t^*}{g(1 + \mathbf{d}b_t)} \dot{b}_t. \quad (\text{A.10})$$

Dividing by s_t^* , substituting in (A.9) and (A.4) yields the growth rate in (2.20). The growth rate for the demand of risky assets in (2.22) follows directly from taking the total time differential of the optimal policy function and substituting in (A.9).

D. Discrete time stochastic process for effective wealth

Ito's Lemma implies that $d\Omega = (\Omega_t + \Omega_{zz}/2)dt + \Omega_z dz$ where explicit time subscripts are dropped and subscripts indicate partial derivatives. Thus, from (A.9) it must be that $\Omega_z = \Omega(m-r)/(g\mathbf{s})$ which implies that $\Omega = \exp(z(m-r)/(g\mathbf{s}) + f(t))$ for some function of time, $f(t)$. Using this partial solution for Ω and noting that $\Omega_t + \Omega_{zz}/2$ must equal the first term in (A.9), the solution for $f(t)$ follows from

$$f'(t) = r + \frac{(m-r)^2}{g\mathbf{s}^2} - A_t (1 + \mathbf{d}b_t)^{(g-1)/g} - \frac{(m-r)^2}{2g^2\mathbf{s}^2}. \quad (\text{A.11})$$

Solving this for $f(t+1)$ given $f(t)$ yields the discrete time solution for optimal effective wealth:

$$\Omega_{t+1} = \Omega_t \exp \left[\left(r + \frac{(m-r)^2}{g\mathbf{s}^2} \left(1 - \frac{1}{2g} \right) \right) (t+1) - \int_t^{t+1} A_p (1 + \mathbf{d}b_p)^{(g-1)/g} dp + \frac{(m-r)}{g\mathbf{s}} z_{t+1} \right] \quad (\text{A.12})$$

where z is *i.i.d.* $N(0,1)$. This can be simplified slightly by noting from (A.4) that

$$A_t (1 + \mathbf{d}b_t)^{(g-1)/g} = \frac{\partial \ln A_t}{\partial t} - \left(\frac{1-g}{g} \right) \left(r - \frac{r}{1-g} + \frac{1}{2} \frac{(m-r)^2}{g\mathbf{s}^2} \right) \quad (\text{A.13})$$

since $\dot{A}_t/A_t = \partial \ln A_t / \partial t$ and there is no mortality risk for the purposes of Section IV. Taking the integral of (A.13) and substituting the result into (A.12) yields the solution for optimal effective wealth as indicated in (4.1).

E. Constants for empirical model

The discrete time version of the optimal policy functions are given as

$$\begin{aligned}\ln(\mathbf{a}_{t+1}) &= \ln\left(\frac{\mathbf{m}-r}{\mathbf{g}\mathbf{s}^2}\right) + \ln\Omega_{t+1} \\ \ln(c_{t+1} - \mathbf{q}k_t) &= \ln\left(A_{t+1}(1 + \mathbf{d}b_{t+1})^{(g-1)/g}\right) + \ln\Omega_{t+1}.\end{aligned}\tag{A.14}$$

substituting (4.1) into these yields (4.2) and (4.3) with constants given by

$$\begin{aligned}\mathbf{k}_a &= \ln\left(\frac{\mathbf{m}-r}{\mathbf{g}\mathbf{s}^2}\right) + \mathbf{g}^{-1}\left(r - \mathbf{r} + \frac{(\mathbf{m}-r)^2}{2\mathbf{s}^2}\right) \\ \mathbf{k}_c &= \ln\left(-\frac{(1-\mathbf{g})}{\mathbf{g}}\left(r - \frac{\mathbf{r}}{1-\mathbf{g}} + \frac{(\mathbf{m}-r)^2}{2\mathbf{s}^2}\right)\right) - \mathbf{g}^{-1}\left(r - \mathbf{r} + \frac{(\mathbf{m}-r)^2}{2\mathbf{s}^2}\right)\end{aligned}\tag{A.15}$$

where the first term for \mathbf{k}_c comes from the first term in (A.14) which is replaced by (A.13), noting the assumption that $\partial \ln A_{t+1} / \partial t = 0$.

References

- Abel, Andrew B., "Operative Gift and Bequest Motives ." *American Economic Review*, December 1987, 77 (5), 1037-1047.
- Bodie, Zvi, Merton, Robert C., Samuelson, William F., "Labor Supply Flexibility and Portfolio Choice in a Life-Cycle Model." *Journal of Economic Dynamics and Control*, 1992, 16, 427-449.
- Boyd, John H. III. "Symmetries, Dynamic Equilibria, and the Value Function," in *Conservation Laws and Symmetry: Applications to Economics and Finance*, R. Sato, R. V. Ramachandran (editors). Boston, MA: Kluwer Academic Publishers, 1990, 225-259.
- Campbell, John Y., Cochrane, John H., "By Force of Habit: A Consumption-Based Explanation of Aggregate Stock Market Behavior." *Journal of Political Economy*, 1999, 107 (2), 205-251.
- _____, "Explaining the Poor Performance of Consumption-Based Asset Pricing Models." *Journal of Finance*, 2000, 55 (6), 2863-2878.
- Campbell, John Y., Deaton, Angus S., "Why is Consumption So Smooth?" *Review of Economic Studies*, 1989, 56 (3), 357-373.
- Campbell, John Y., Viceira, Luis M., "Who Should Buy Long Term Bonds?" *American Economic Review*, 2001, 91 (1), 99-127.
- Carroll, Christopher D., "'Risky Habits' and the Marginal Propensity to Consume Out of Permanent Income or, How Much Would a Permanent Tax Cut Boost Japanese Consumption?" *NBER Working Paper Series*, 2000, W7839.
- Carroll, Christopher D., Overland, Jody, Weil, David N., "Saving and Growth with Habit Formation." *American Economic Review*, 2000, 90 (3), 341-355.
- Carroll, Christopher D., Weil, David N., "Saving and Growth: A Reinterpretation." *Carnegie-Rochester Conference Series*, 1994, 40, 133-192.
- Constantinides, George M., "Habit Formation: A Resolution of the Equity Premium Puzzle." *The Journal of Political Economy*, 1990, 98 (3), 519-543.
- Dunn, Kenneth B., Singleton, Kenneth J., "Modeling Term Structure of Interest Rates under Non-Separable Utility and Durability of Goods." *Journal of Financial Economics*, 1986, 17 (1), 27-55.
- Dynan, Karen E., "Habit Formation in Consumer Preferences: Evidence from Panel Data." *American Economic Review*, 2000, 90 (3), 391-406.
- Ferson, Wayne E., Constantinides, George M., "Habit Persistence and Durability in Aggregate Consumption." *Journal of Financial Economics*, 1991, 29 (2), 199-240.
- Hansen, Lars Peter, Jagannathan, Ravi, "Restrictions on Intertemporal Marginal Rates of Substitution Implied by Asset Returns." *Journal of Political Economy*, 1991, 99, 225-262.
- Heaton, John, "The Interaction Between Time-Nonseparable Preferences and Time Aggregation." *Econometrica*, 1993, 61 (2), 353-385.
- _____, "An Empirical Investigation of Asset Pricing with Temporally Dependent Preference Specifications." *Econometrica*, 1995, 63 (3), 681-717.
- Mehra, Rajnish, Prescott, Edward C., "The Equity Premium: A Puzzle." *Journal of Monetary Economics*, 1985, 15, 145-161.
- Merton, Robert C., "Lifetime Portfolio Selection under Uncertainty: The Continuous-Time Case." *Review of Economics and Statistics*, 1969, 51 (3), 247-257.
- _____, "Optimum Consumption and Portfolio Rules in a Continuous-Time Model." *Journal of Economic Theory*, 1971, 3, 373-413.
- Sundaresan, Suresh M., "Intertemporally Dependent Preferences and the Volatility of Consumption and Wealth." *Review of Financial Studies*, 1989, 2 (2), 73-89.

Table 1: Simulation Parameters and Initial Conditions

	No Habit	Habit
<i>Parameters</i>		
Length of Life (T)	100	100
Age of Retirement (R)	65	65
g	0.010	0.010
γ	3.000	3.000
ρ	0.000	0.000
ε	0.007	0.007
ν	0.070	0.070
θ	0.000	0.875
δ	0.000	0.250
r	0.030	0.030
μ	0.080	0.080
σ	0.170	0.170
<i>Initial Conditions</i>		
Age	20	20
Labor Income	30	30
Financial Wealth	5	5
Habit Stock	0	30

Table 2: Estimated Parameters of Risky Asset Demand Model, 1952I to 2000IV

	<i>i</i>	<i>ii</i>	<i>iii</i>
β_0	0.921 (0.004)	0.948 (0.009)	0.990 (0.006)
β_1		-0.069 (0.018)	0.180 (0.030)
θ			0.739 (0.023)
γ	1.839 (0.092)	1.861 (0.092)	2.480 (0.122)
σ_ξ	0.160 (0.000)	0.158 (0.000)	0.119 (0.000)
Log-Likelihood	83.6	87.3	144.2

Table 3: Estimated Parameters of Consumption Model, 1952I to 2000IV

	<i>i</i>	<i>ii</i>	<i>iii</i>
β_0	1.107 (0.001)	1.038 (0.004)	1.043 (0.005)
β_1		0.491 (0.036)	0.528 (0.043)
θ			-0.163 (0.080)
γ	4.160 (0.206)	12.172 (0.618)	12.327 (0.625)
σ_ξ	0.071 (0.000)	0.024 (0.000)	0.024 (0.000)
Log-Likelihood	244.5	448.2	450.9

Table 4: Estimated Parameters of Consumption Model by Consumption Type, 1952I to 2000IV

	Non-Durable Consumption			Durable Consumption		
	<i>i</i>	<i>ii</i>	<i>iii</i>	<i>i</i>	<i>ii</i>	<i>iii</i>
β_0	1.097 (0.001)	1.038 (0.004)	1.040 (0.005)	0.932 (0.002)	0.830 (0.014)	0.876 (0.009)
β_1		0.376 (0.031)	0.383 (0.035)		1.070 (0.320)	3.569 (0.624)
θ			-0.063 (0.098)			-2.957 (0.407)
γ	4.395 (0.218)	10.152 (0.516)	10.158 (0.517)	2.902 (0.145)	4.233 (0.215)	9.875 (0.503)
σ_ξ	0.067 (0.000)	0.029 (0.000)	0.029 (0.000)	0.101 (0.000)	0.069 (0.000)	0.030 (0.000)
Log-Likelihood	254.8	412.1	412.3	172.3	242.6	406.6

Figure 1: Consumption

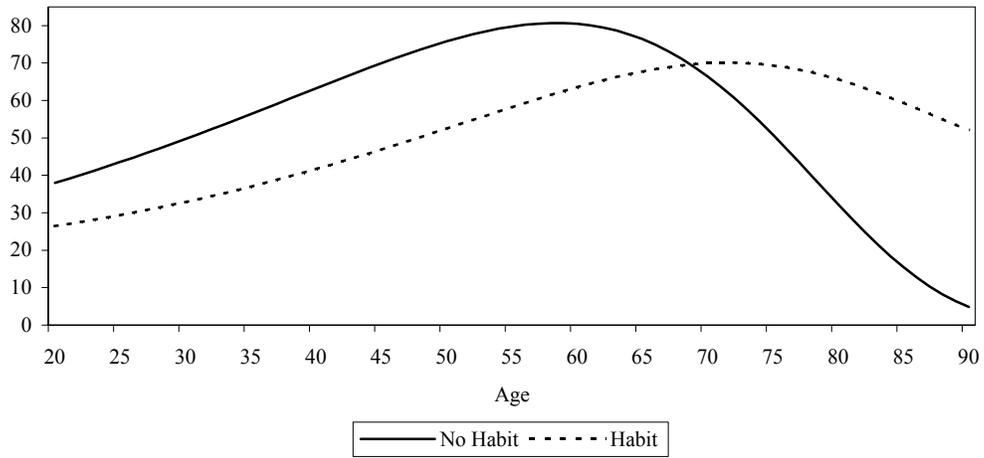


Figure 2: Log Consumption and Service Flow

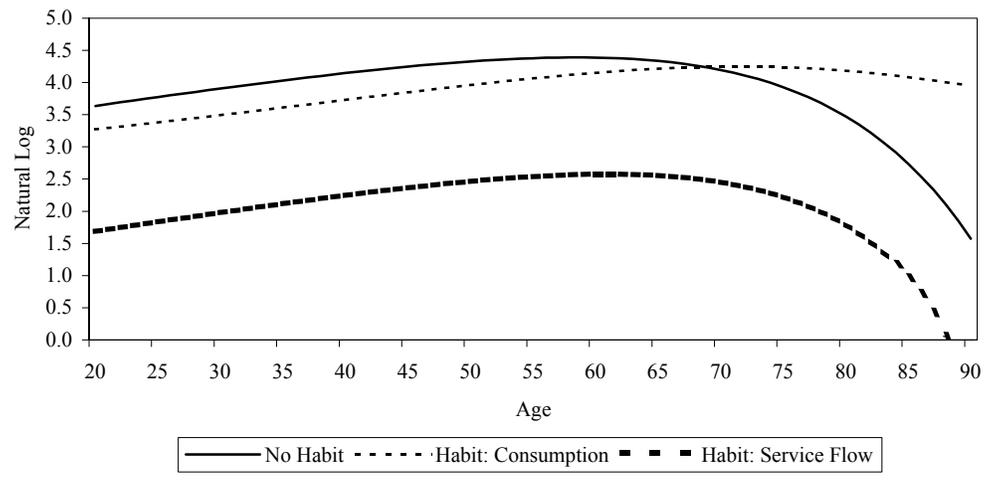


Figure 3: Risky Asset Demand

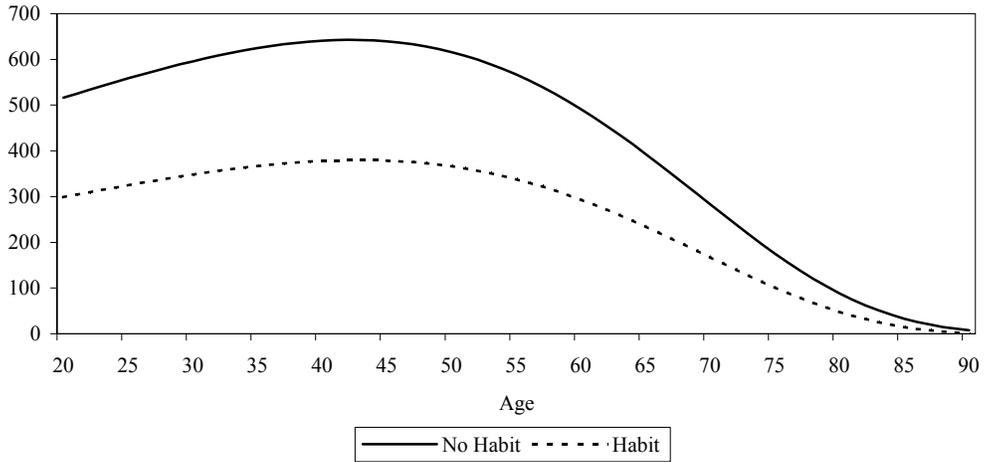


Figure 4: Effective Wealth Less Human Wealth

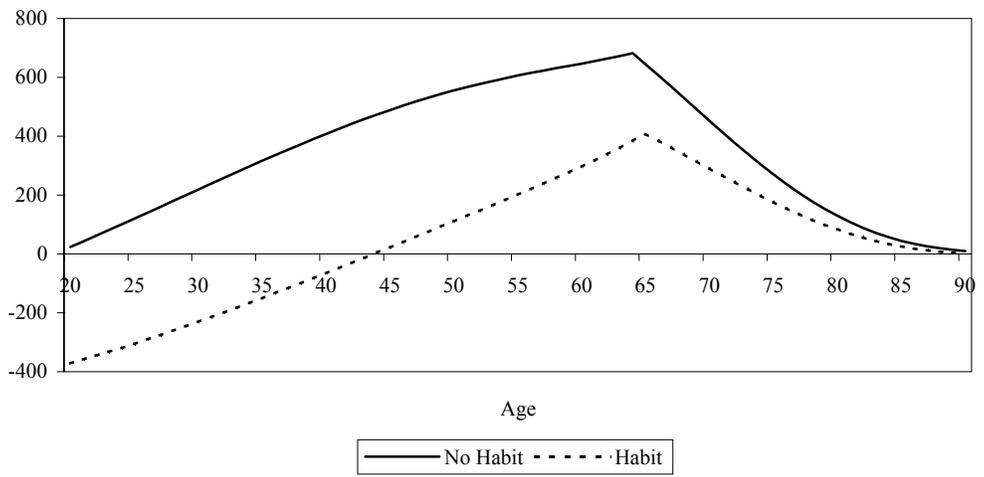


Figure 5: Marginal Propensity to Consume Out of Wealth

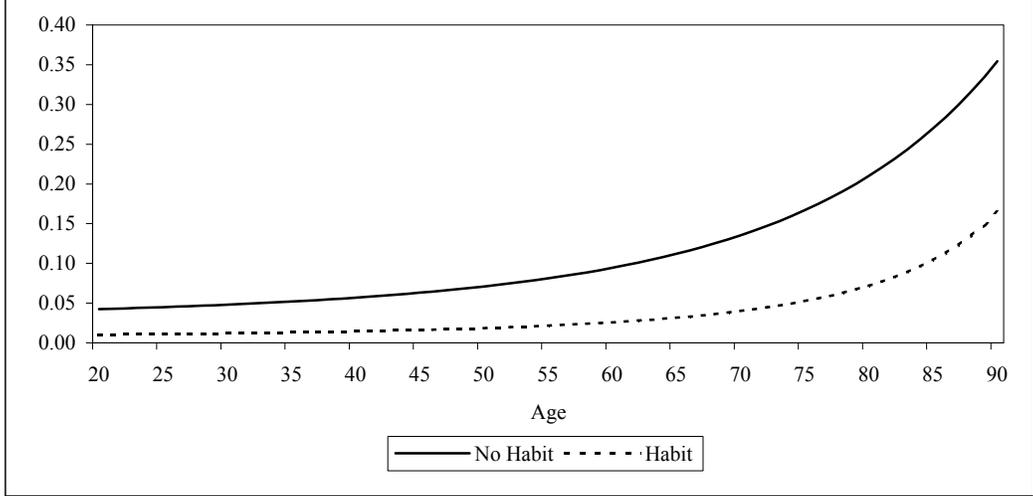


Figure 6: Marginal Cost of Consumption

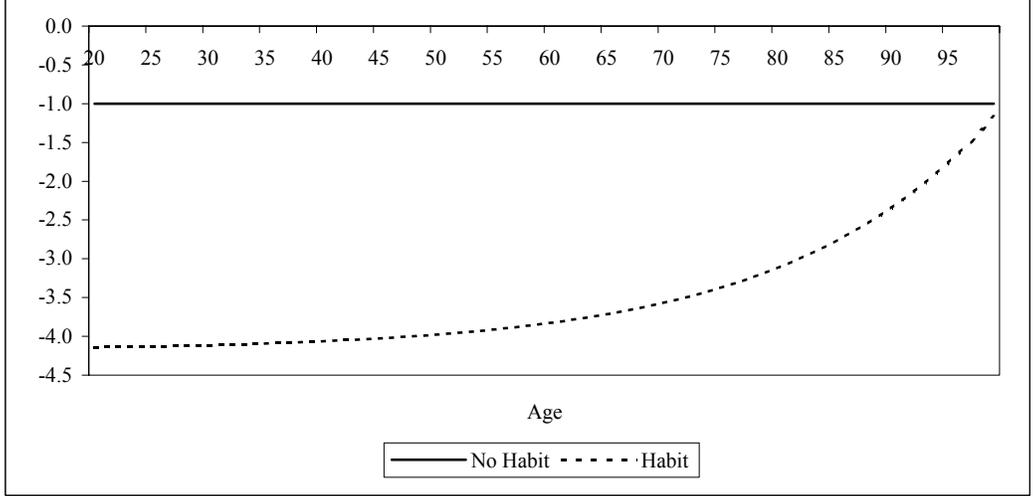


Figure 7: Risky Asset Share of Financial Wealth

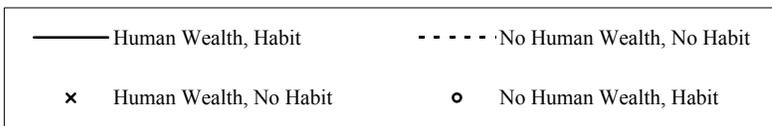
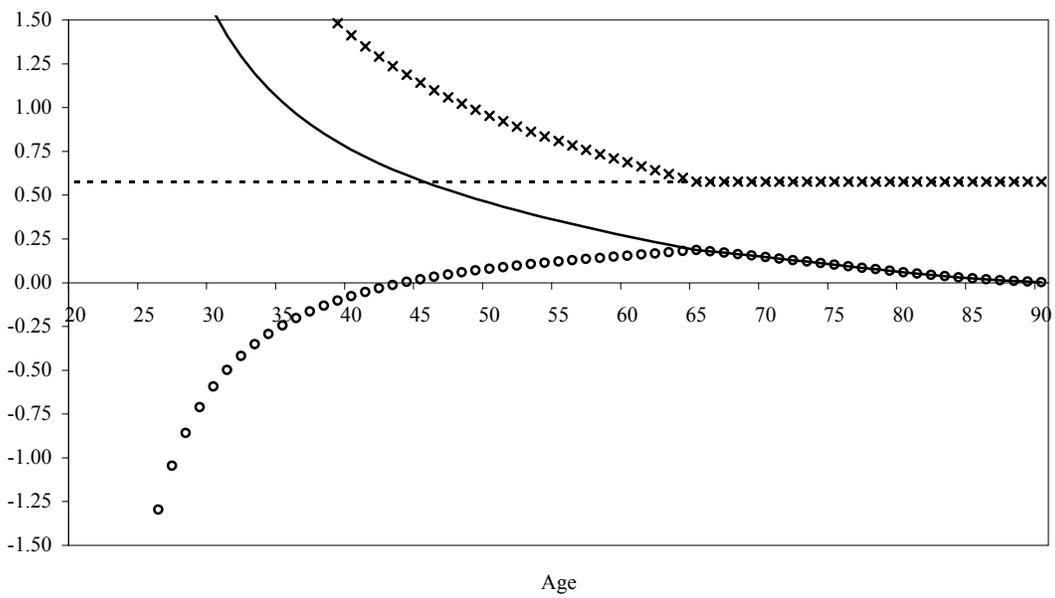
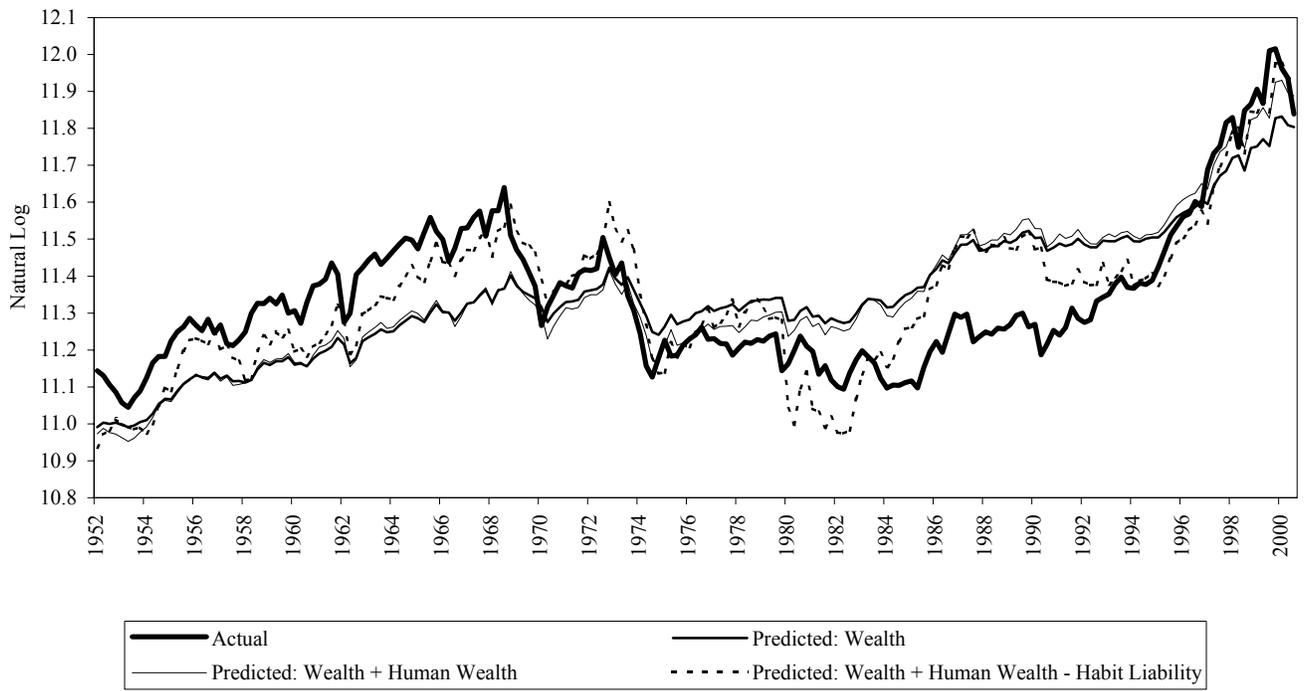


Figure 8:
Real Demand for Risky Assets Per Household with Non-Durable Consumption Habit Liability, 1952I to 2000IV



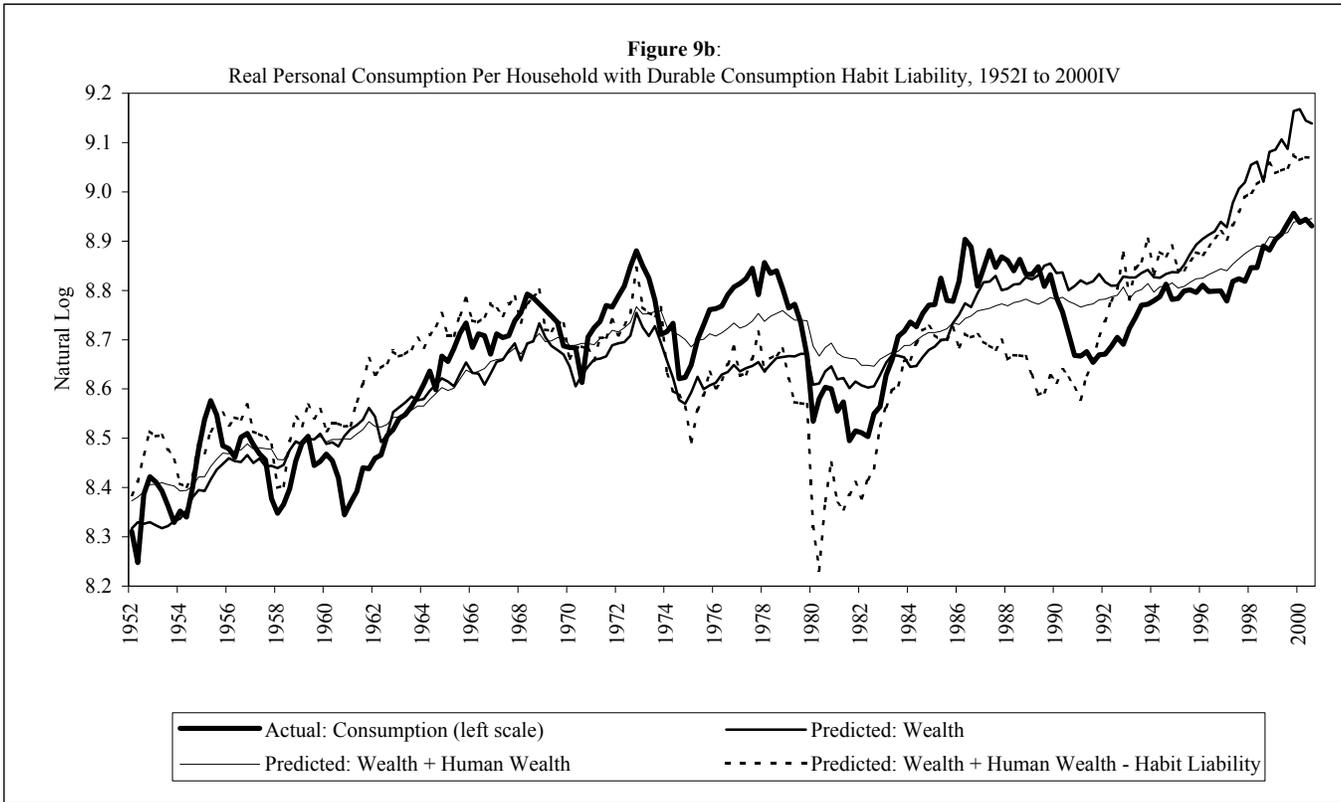
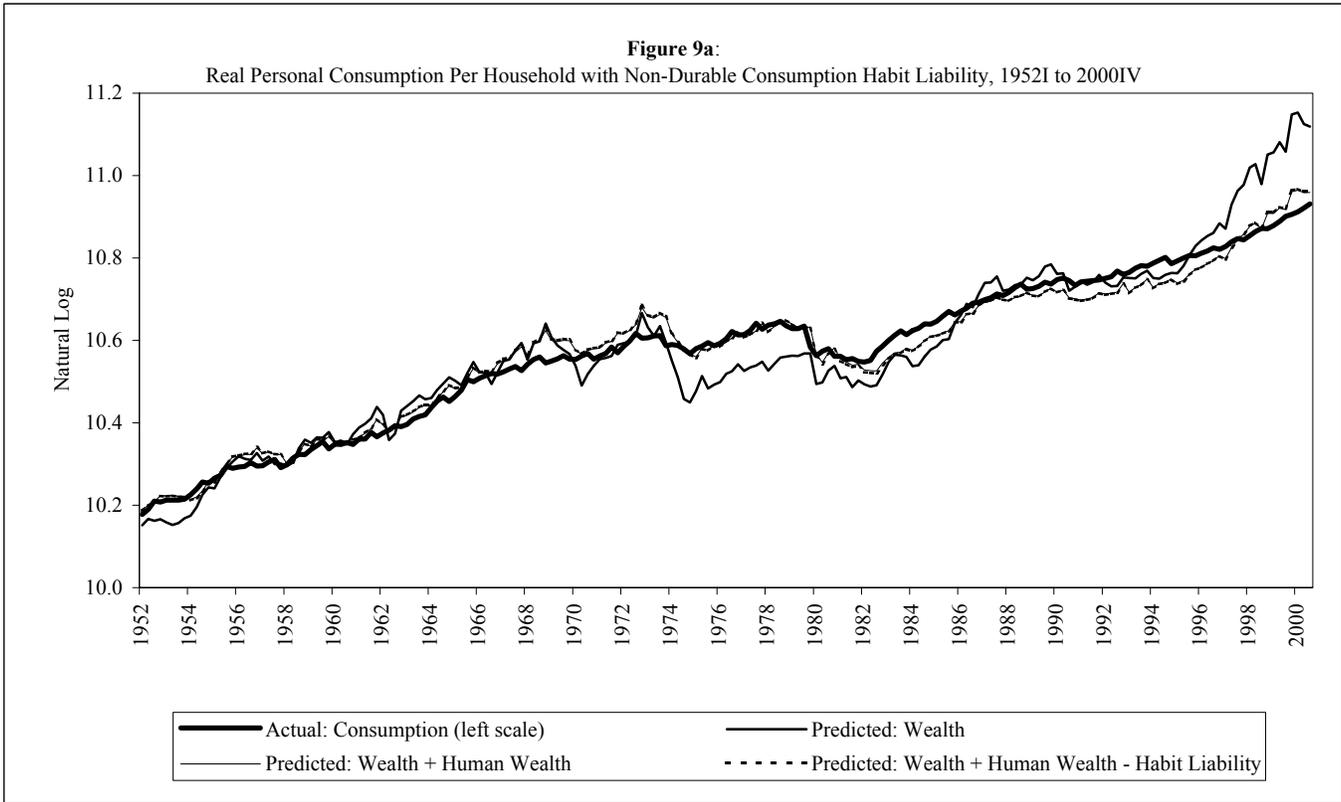


Figure 10:
Relative Risk Aversion, 1952I to 2000IV

