Some (more) Nuclear Structure

Lecture 2

Low-energy Collective Modes and Electromagnetic Decays in Nuclei

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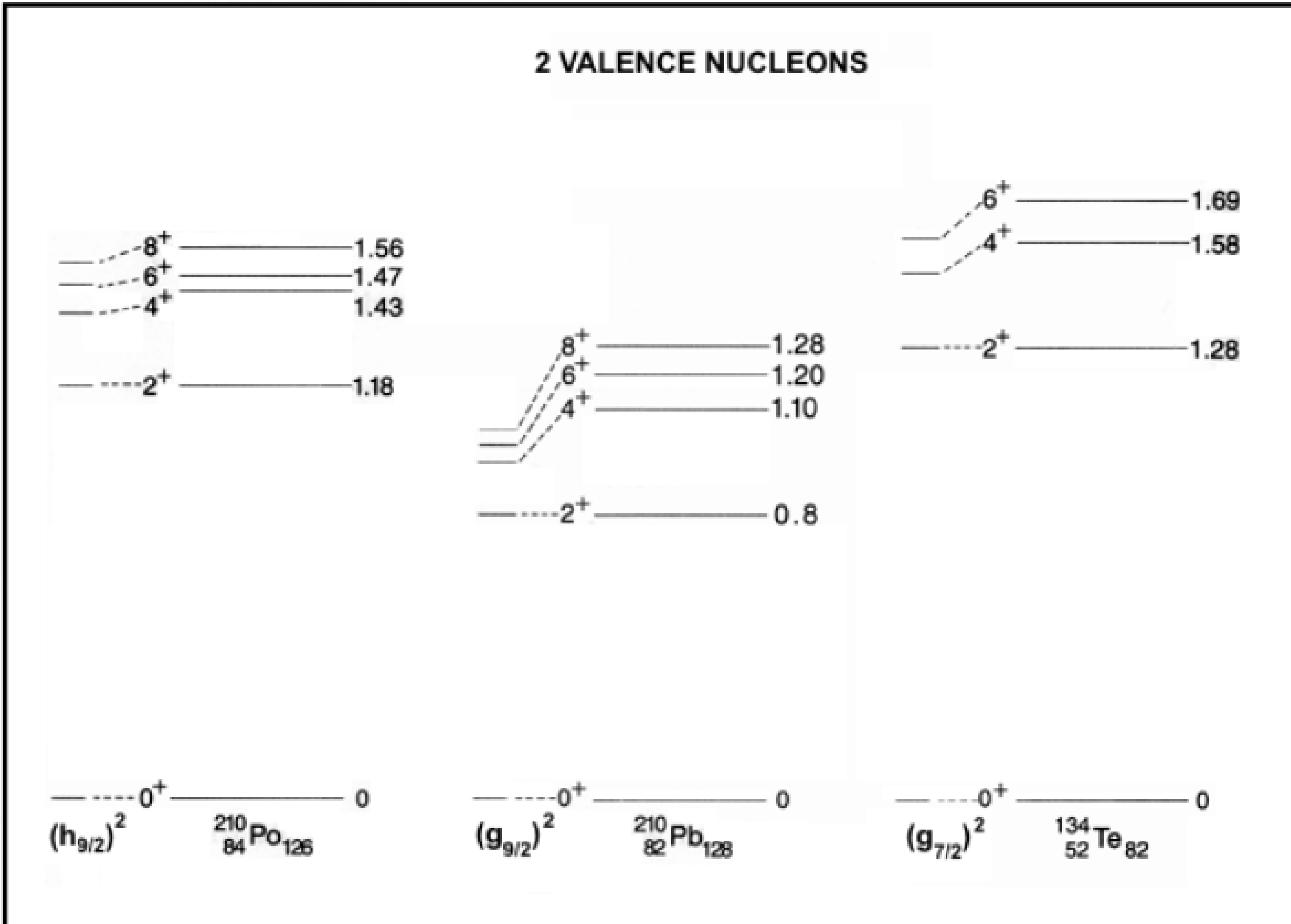
Outline of Lectures 1& 2

- 1) Overview of nuclear structure 'limits'
  - Some experimental observables, evidence for shell structure
  - Independent particle (shell) model
  - Single particle excitations and 2 particle interactions.

- 2) Low Energy Collective Modes and EM Decays in Nuclei.
  - Low-energy quadrupole vibrations in nuclei
  - Rotations in even-even nuclei
  - Vibrator-rotor transitions, E-GOS curves

What about 2 nucleons outside a closed shell ?



Residual Interactions?

- We need to include any addition changes to the energy which arise from the interactions between valence nucleons.
- This is in addition the mean-field (average) potential which the valence proton/neutron feels.
- Hamiltonian now becomes $H = H_0 + H_{residual}$
- 2-nucleon system can be thought of as an inert, doubly magic core plus 2 interacting nucleons.
- Residual interactions between these two 'valence' nucleons will determine the energy sequence of the allowed spins / parities.

What spins can you make?

- If two particles are in identical orbits (j^2), then what spins are allowed?

Two possible cases:

- Same particle, e.g., 2 protons or 2 neutrons = even-even nuclei like ^{42}Ca, 2 neutrons in $f_{7/2} = (\nu f_{7/2})^2$

  We can couple the two neutrons to make states with spin/parity $J^{\pi}=0^{+}$, 2^{+}, 4^{+} and 6^{+}

  These all have T=1 in isospin formalism, intrinsic spins are anti-aligned with respect to each other.

- Proton-neutron configurations (odd-odd)

  e.g., ^{42}Sc, 1 proton and 1 neutron in $f_{7/2}$

  We can couple these two make states with spin / parity 0^{+}, 1^{+}, 2^{+}, 3^{+}, 4^{+}, 5^{+}, 6^{+} and 7^{+}.

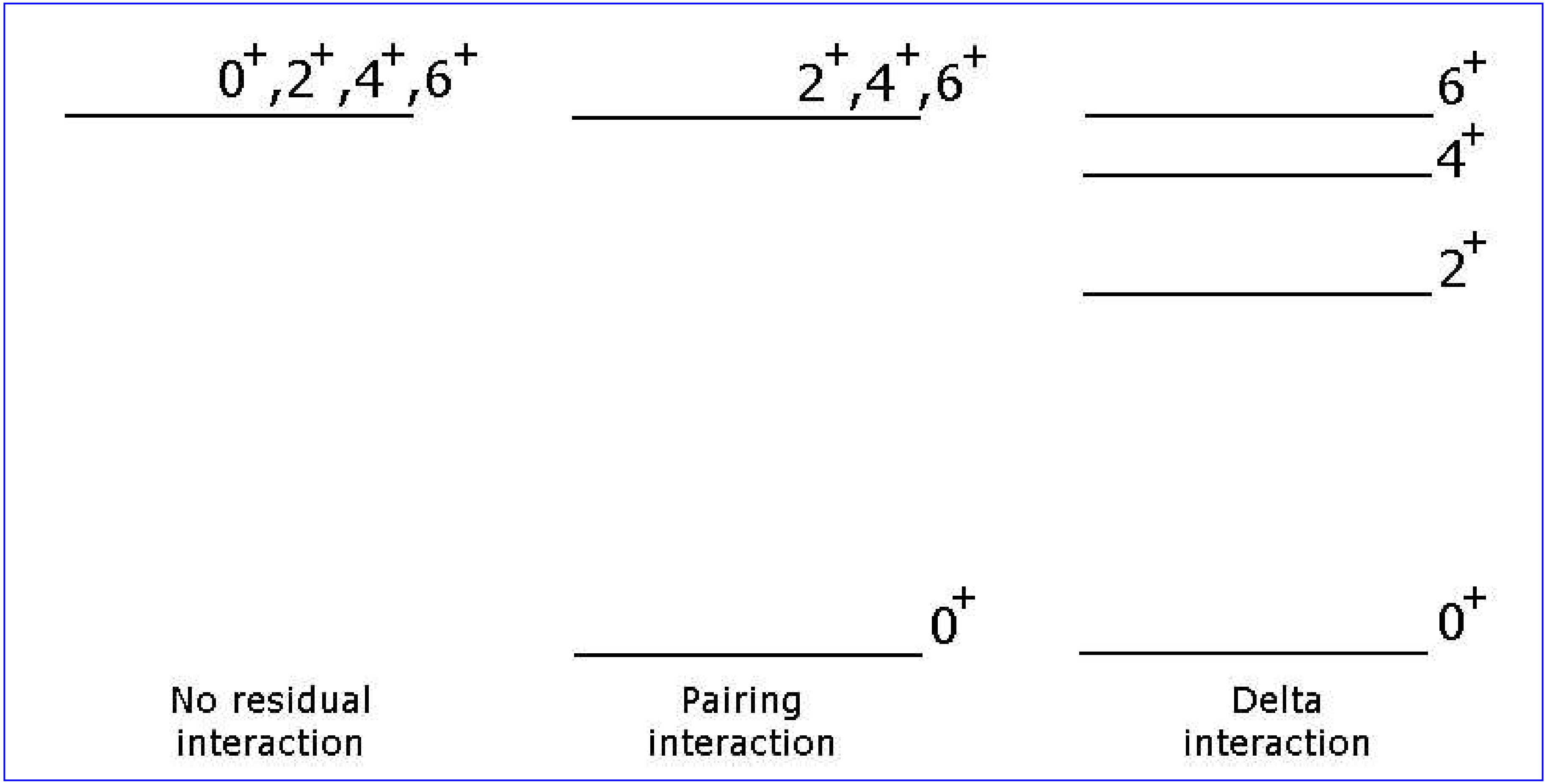
Even spins have T=1 (S=0, intrinsic spins anti-aligned);

Odd spins have T=0 (S=1, intrinsic spins aligned) m – scheme showing which J_{tot} values are allowed for $(f_{7/2})^2$ coupling of two identical particles (2 protons or 2 neutrons).

$j_1 = 7/2$ m_1	$j_2 = 7/2$ m_2	M	J
7/2	5/2	6	
7/2	3/2	5	
7/2	1/2	4	
7/2	−1/2	3	6
7/2	−3/2	2	
7/2	−5/2	1	
7/2	−7/2	0	
5/2	3/2	4	
5/2	1/2	3	
5/2	−1/2	2	4
5/2	−3/2	1	
5/2	−5/2	0	
3/2	1/2	2	
3/2	−1/2	1	2
3/2	−3/2	0	
1/2	−1/2	0	0

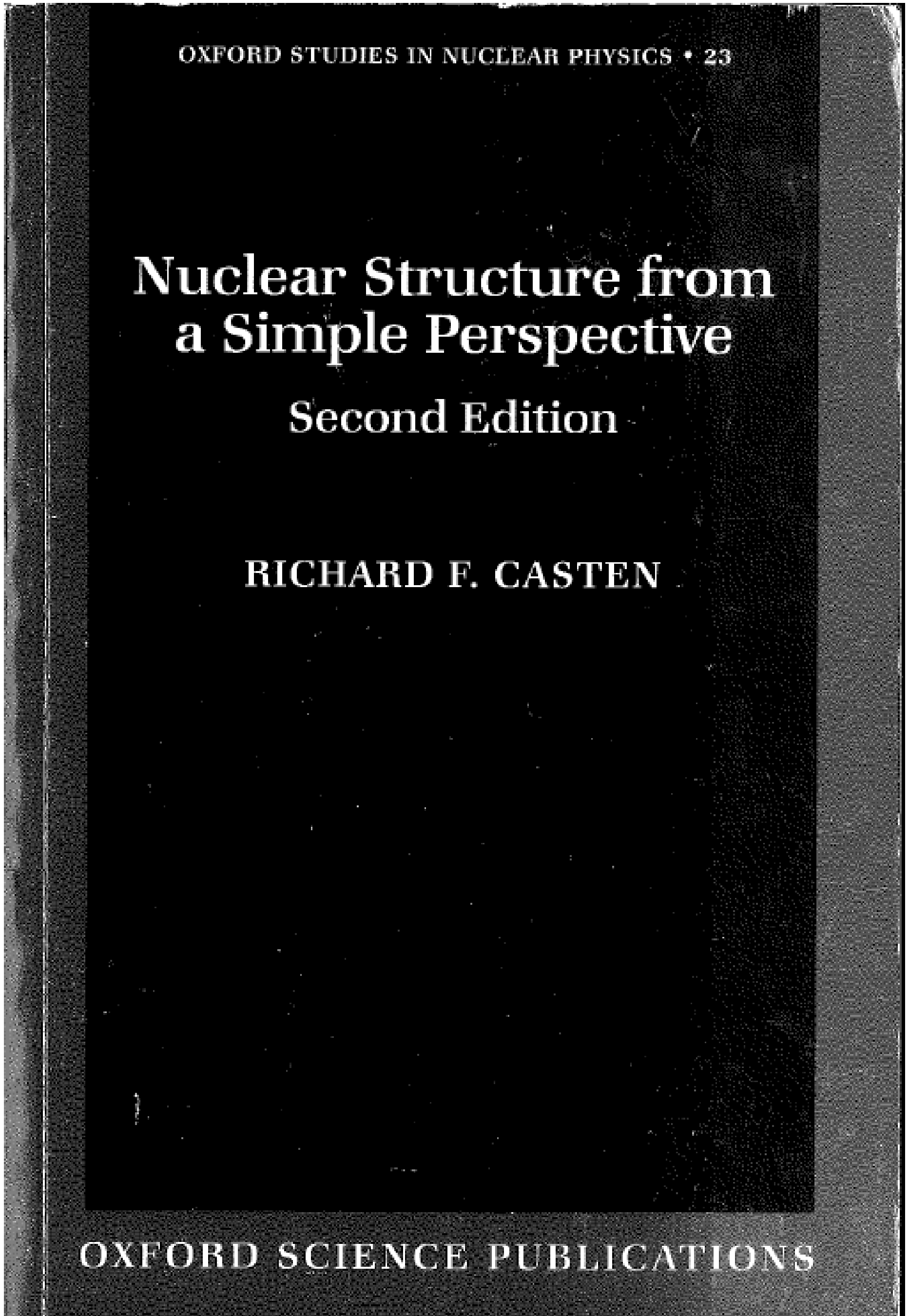
* Only positive total M values are shown. The table is symmetric for $M < 0$.

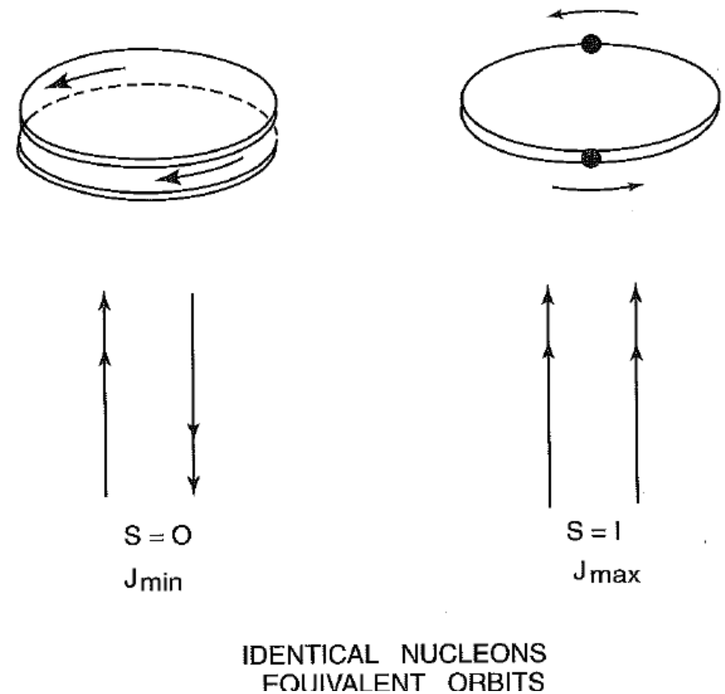
Note, that only even spin states are allowed.



Schematic for $(f_{7/2})^2$ configuration.
4 degenerate states if there are no residual interactions.

Residual interactions between two valence nucleons give additional binding, lowering the (mass) energy of the state.





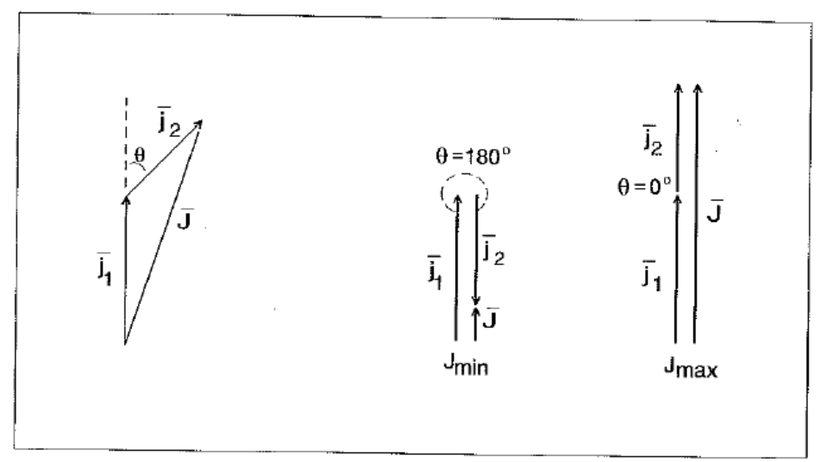
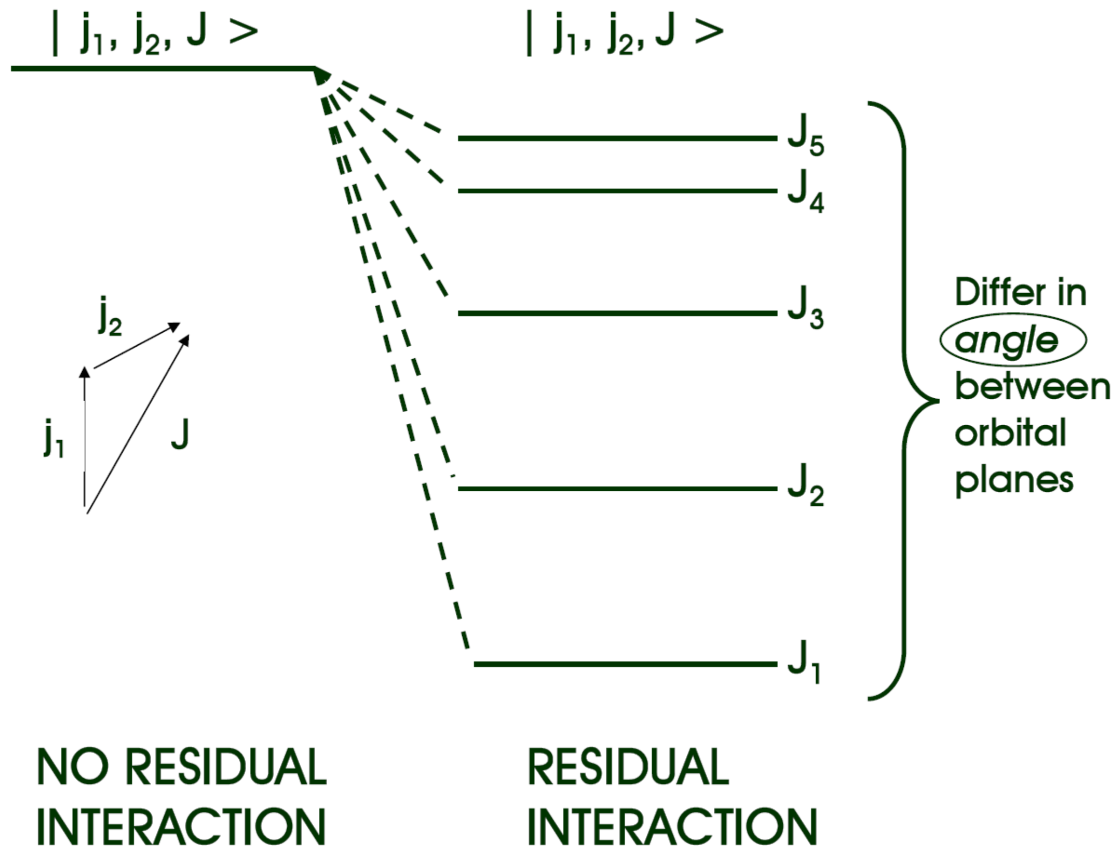


FIG. 4.8. Definition and schematic illustration of some of the ideas used in the geometrical analysis of short-range residual interactions.

Residual Interactions—Diagonal Effects

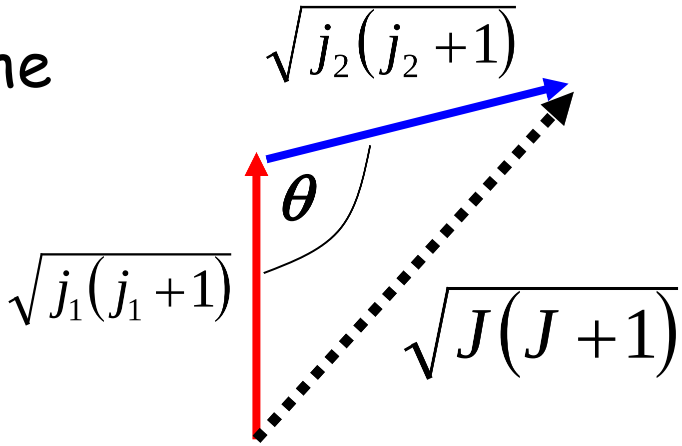
Consider 2 particles, in orbits j_1, j_2 coupled to spin J_i, and interacting with a residual interaction, V_{12}.

2 Identical Nucleons



Geometric Interpretation of the δ Residual Interaction for a j^2 Configuration Coupled to Spin J

Use the cosine rule and recall that the magnitude of the spin vector of spin $j = [\, j\,(j+1)\,]^{-1/2}$



$$J^2 = j_1^2 + j_2^2 - 2\, j_1 j_2 \cos(\theta)$$

therefore

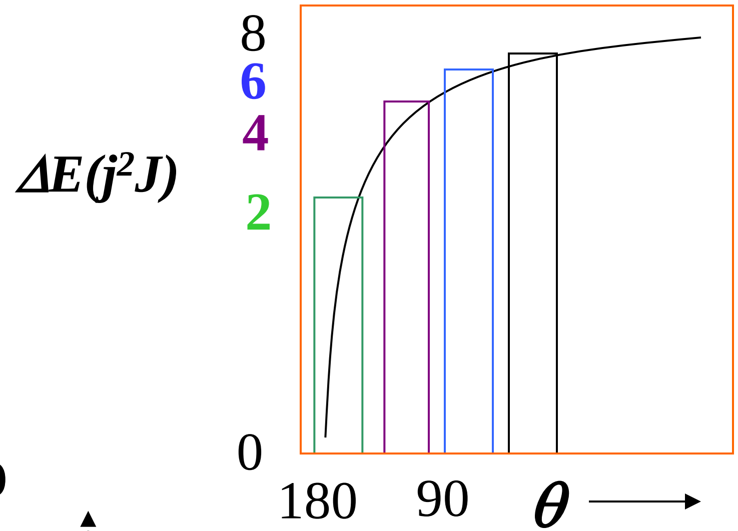
$$J(J+1) = j_1(j_1+1) + j_2(j_2+1) - \sqrt{j_1(j_1+1)}\sqrt{j_2(j_2+1)}\cos(\theta)$$

$$\therefore \text{for } j_1 = j_2 = j \quad \cos^{-1}\left[\frac{J(J+1) - 2j(j+1)}{j(j+1)}\right]$$

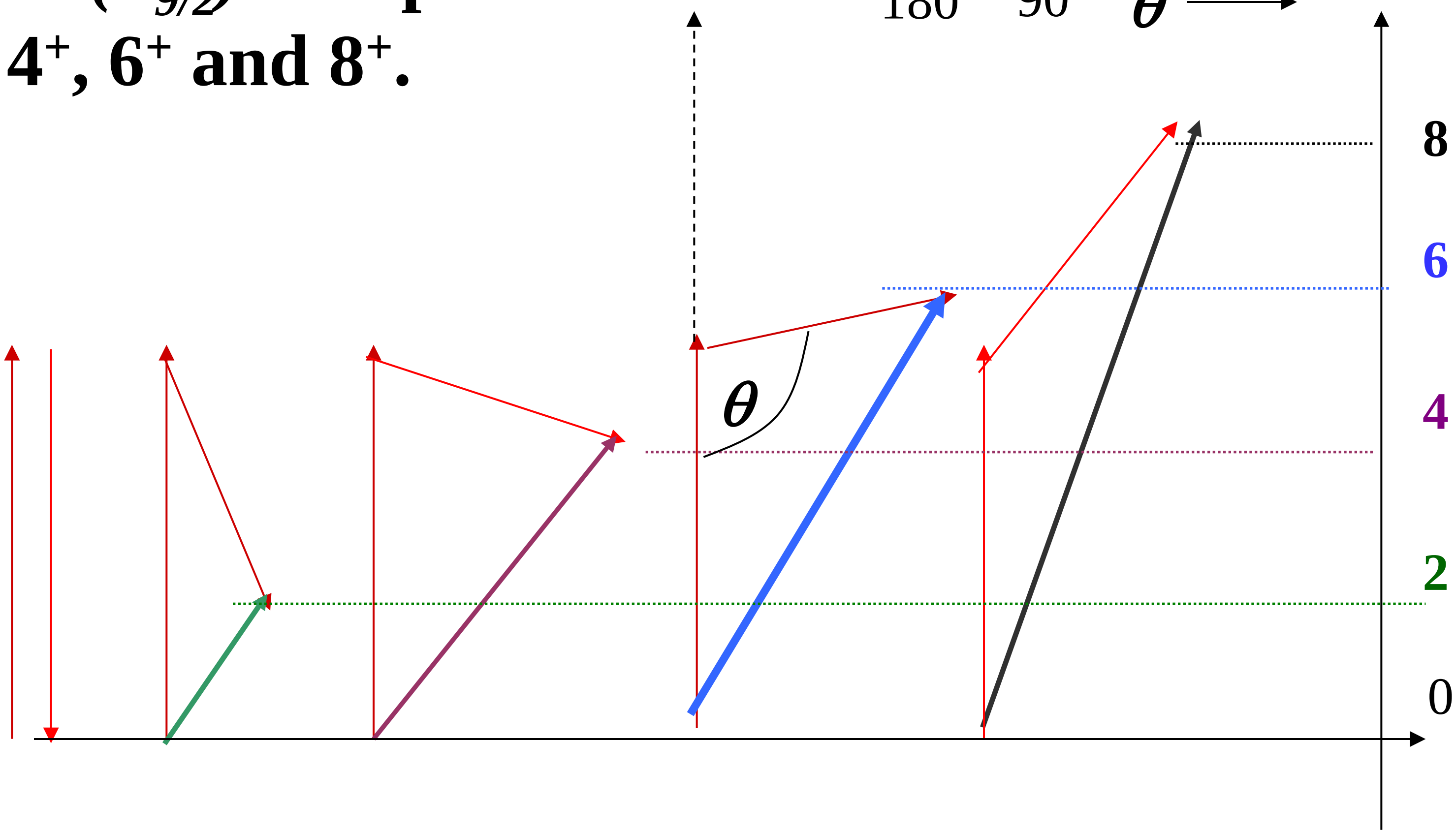
δ–interaction gives nice simple geometric rationale for **Seniority Isomers** from

$\Delta E \sim -V_o F_r \tan(\theta/2)$

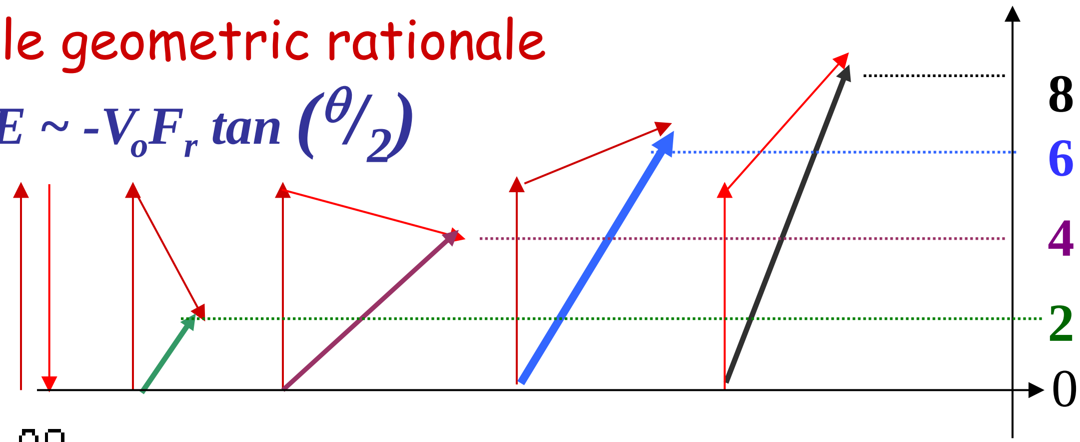
for $T=1$, even J

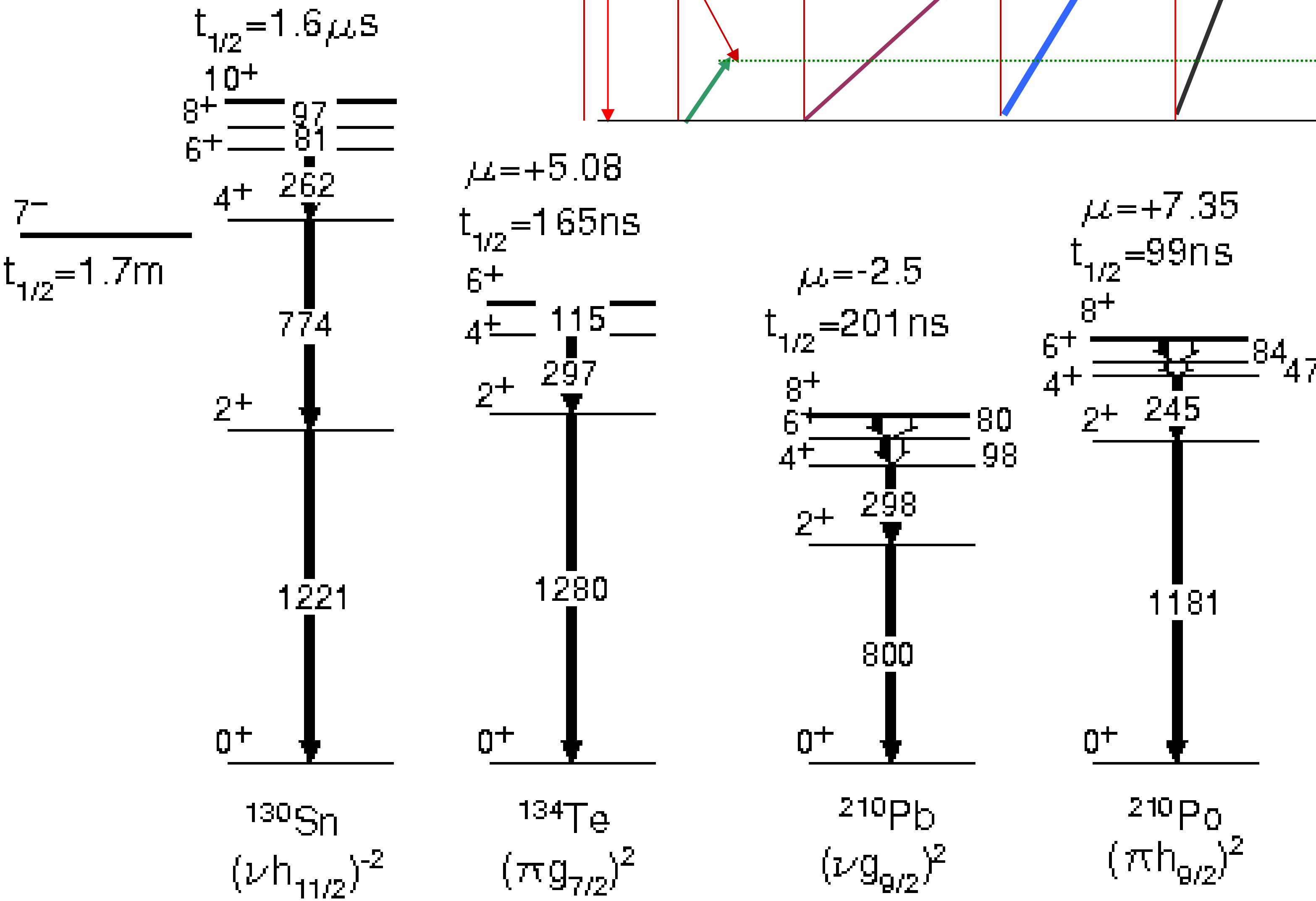


e.g. $J^{\pi} = (h_{9/2})^2$ **coupled to 0^+, 2^+, 4^+, 6^+ and 8^+.**



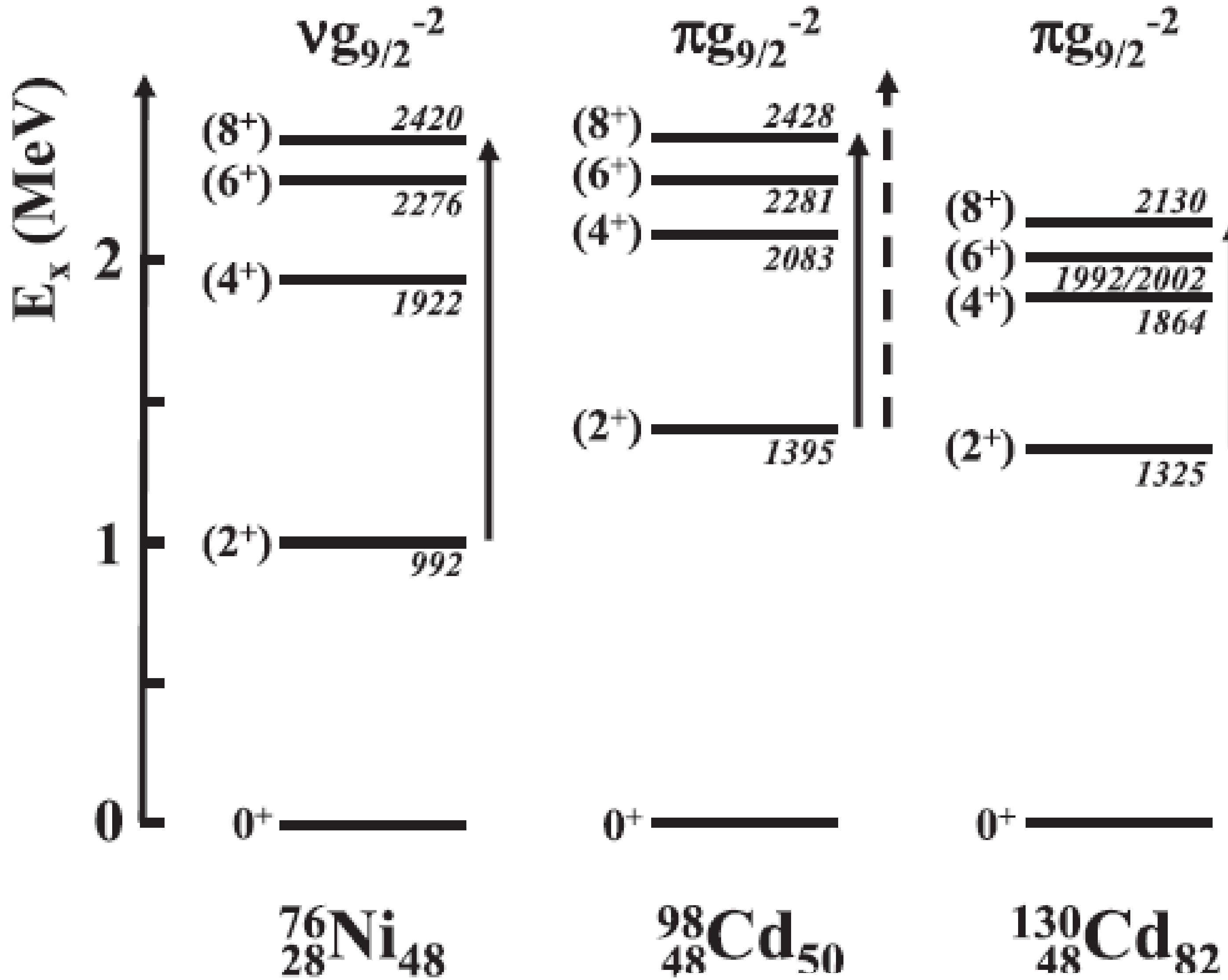
δ – interaction gives nice simple geometric rationale for **Seniority Isomers** from $\Delta E \sim -V_o F_r \tan(\theta/2)$ for $T=1$, even J





See e.g., Nuclear structure from a simple perspective, R.F. Casten Chap 4.)

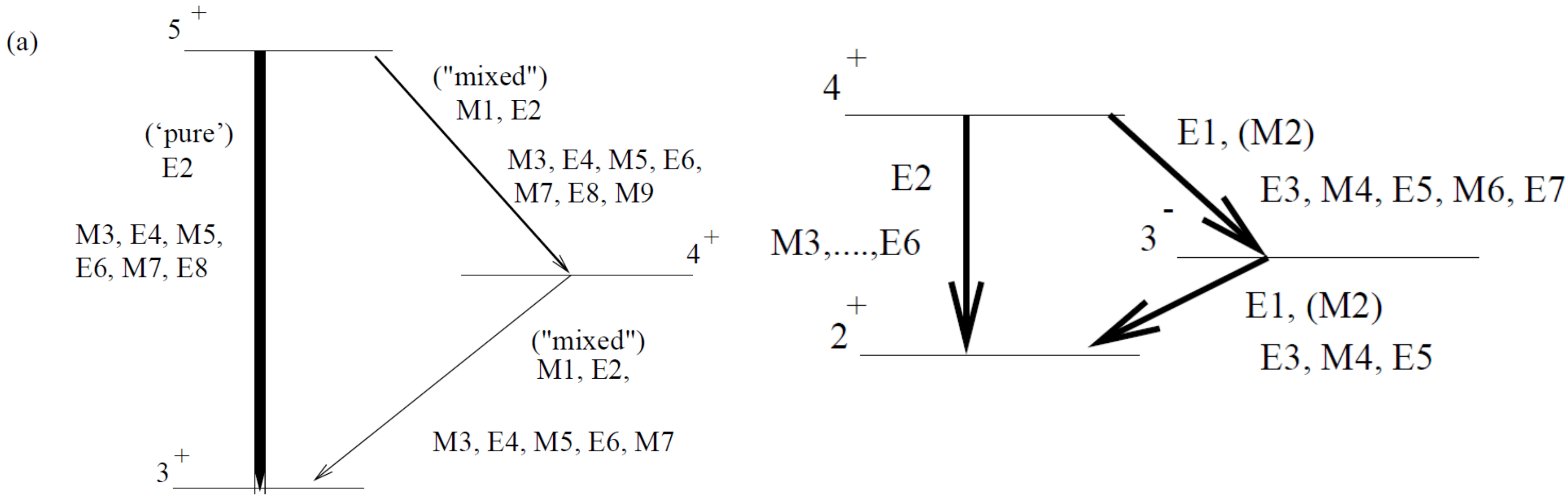
Note, 2 neutron or 2 proton **holes** in doubly magic nuclei show spectra like 2 proton or neutron **particles**.



A. Jungclaus et al., PRL 99, 132501 (2007)

Basic EM Selection Rules?

$$|I_i + I_f| \geq L \geq |I_i - I_f|$$



The transition probability for a state decaying from state J_i to state J_f, separated by energy E_γ, by a transition of multipole order L is given by [1, 7]

$$T_{fi}(\lambda L) = \frac{8\pi(L+1)}{\hbar L\left((2L+1)!!\right)^2}\left(\frac{E_\gamma}{\hbar c}\right)^{2L+1} B(\lambda L : J_i \rightarrow J_f) \qquad (1.1.2)$$

where $B(\lambda L : J_i \rightarrow J_f)$ is called the *reduced matrix element.*

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where $B(\lambda L : J_i \to J_f)$ is called the *reduced matrix element.*

Measuring the lifetime (decay probability) of a nuclear state thus gives a value for the $B(\lambda L : J_i \to J_f)$.

For lifetimes, τ in units of seconds where the transition *probability* per unit second, $T = \frac{1}{\tau}$, (E_γ in MeV),

$$T(E1) = 1.587 \times 10^{15} E_\gamma^3 B(E1) \qquad (5.0.4)$$

$$T(E2) = 1.223 \times 10^{9} E_\gamma^5 B(E2) \qquad (5.0.5)$$

$$T(E3) = 5.698 \times 10^{2} E_\gamma^7 B(E3) \qquad (5.0.6)$$

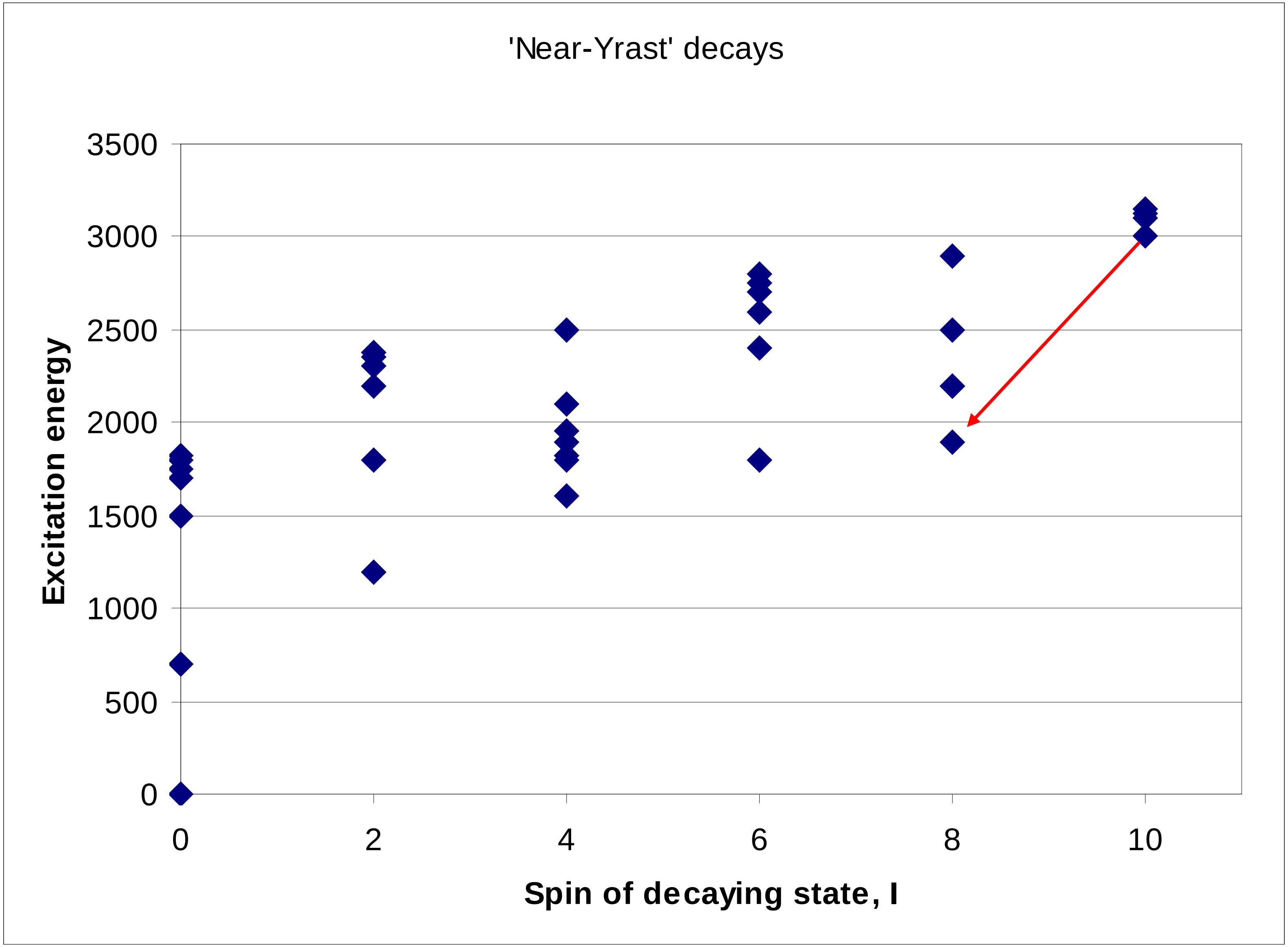
$$T(M1) = 1.779 \times 10^{13} E_\gamma^3 B(M1) \qquad (5.0.7)$$

$$T(M2) = 1.371 \times 10^{7} E_\gamma^5 B(M2) \qquad (5.0.8)$$

$$T(M3) = 6.387 \times E_\gamma^7 B(M3) \qquad (5.0.9)$$

The units of $B(E\lambda)$ are $e^2\mathrm{fm}^{2\lambda}$ and the units of $B(M\lambda)$ are $(e\hbar 2Mc)^2\,(\mathrm{fm})^{2\lambda-2}$.

The EM transition rate depends on $E\gamma^{2\lambda+1}$,, the highest energy transitions for the lowest λ are (generally) favoured.
This results in the preferential population of yrast and near-yrast states.



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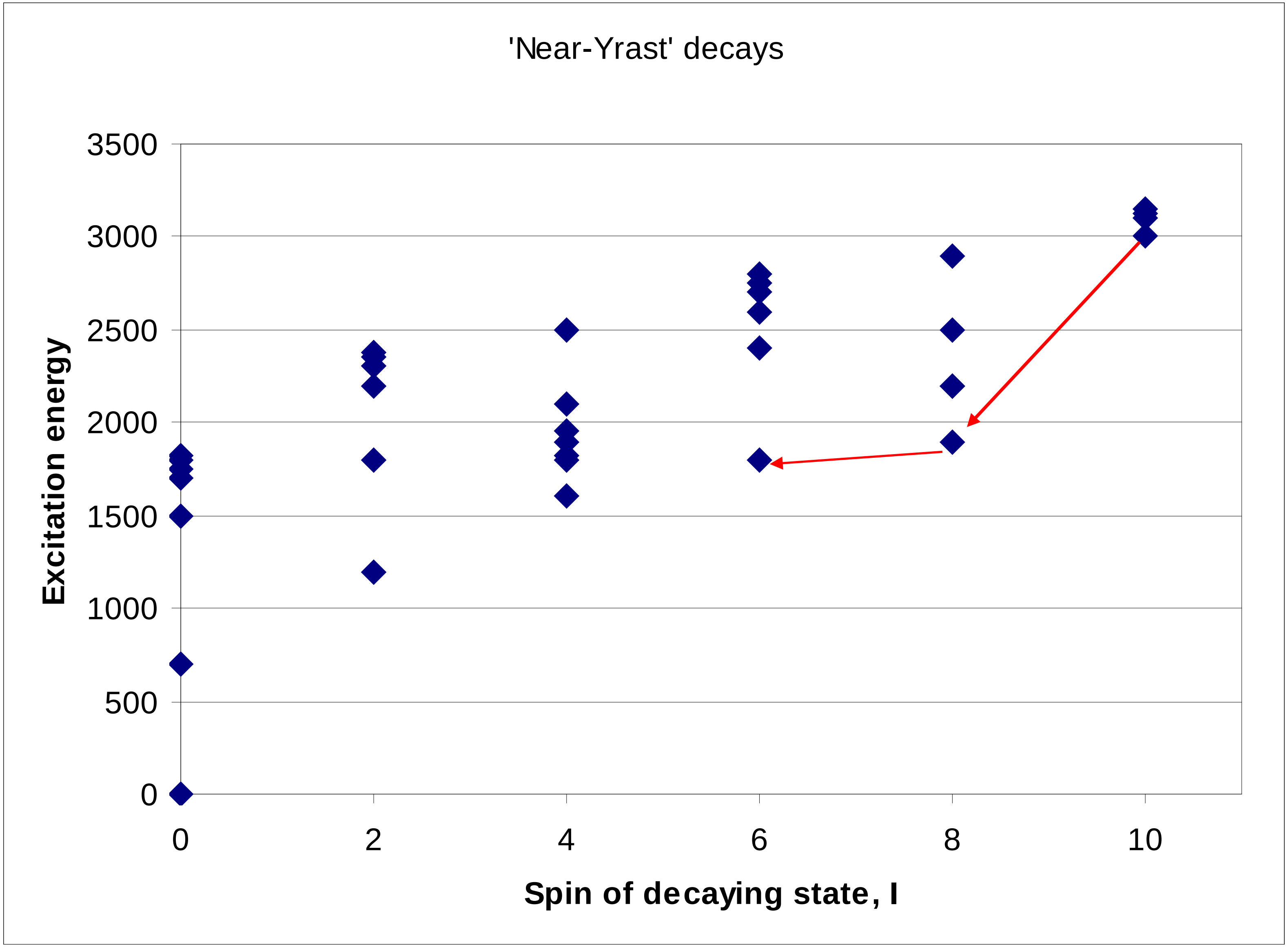

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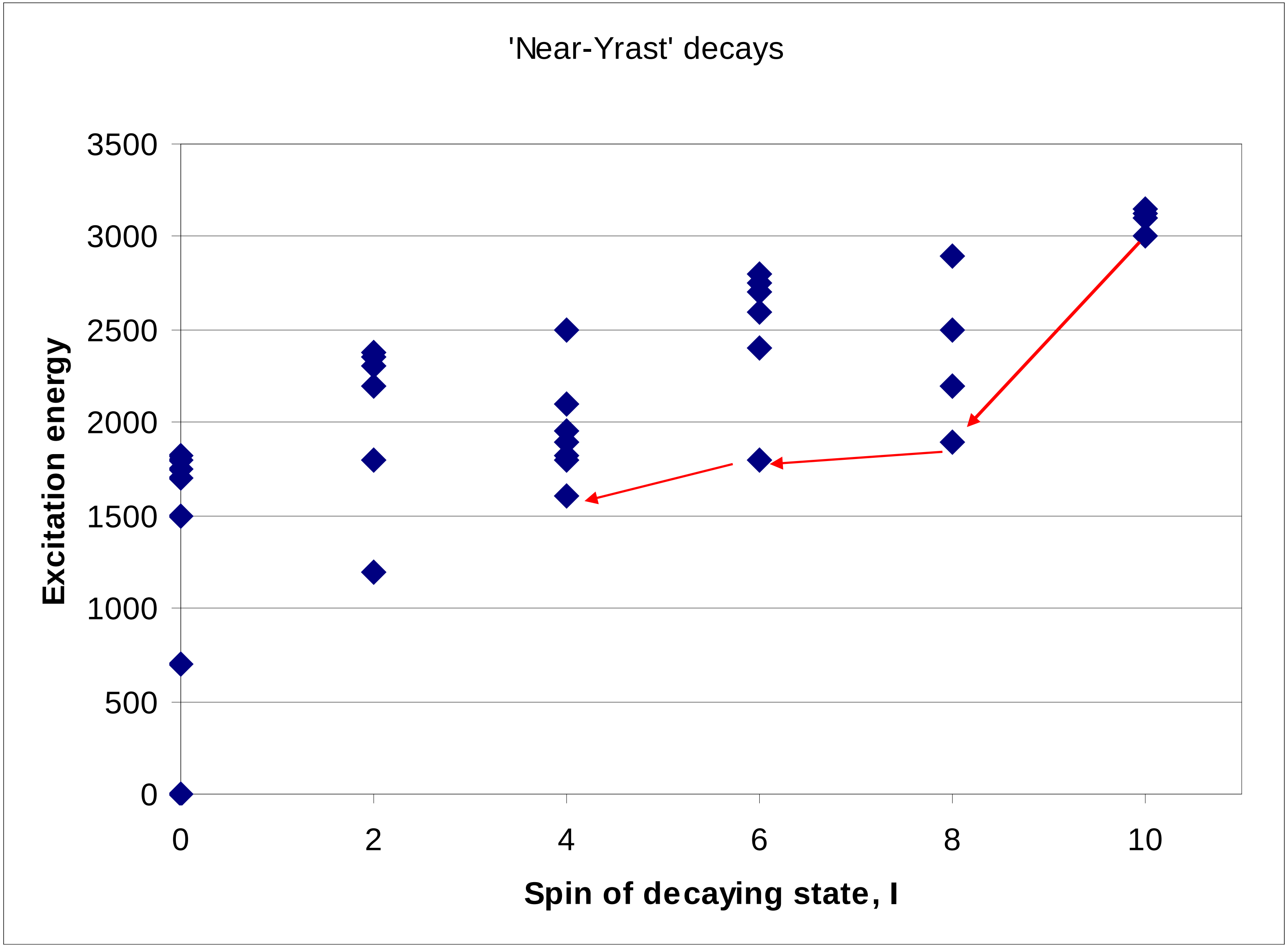

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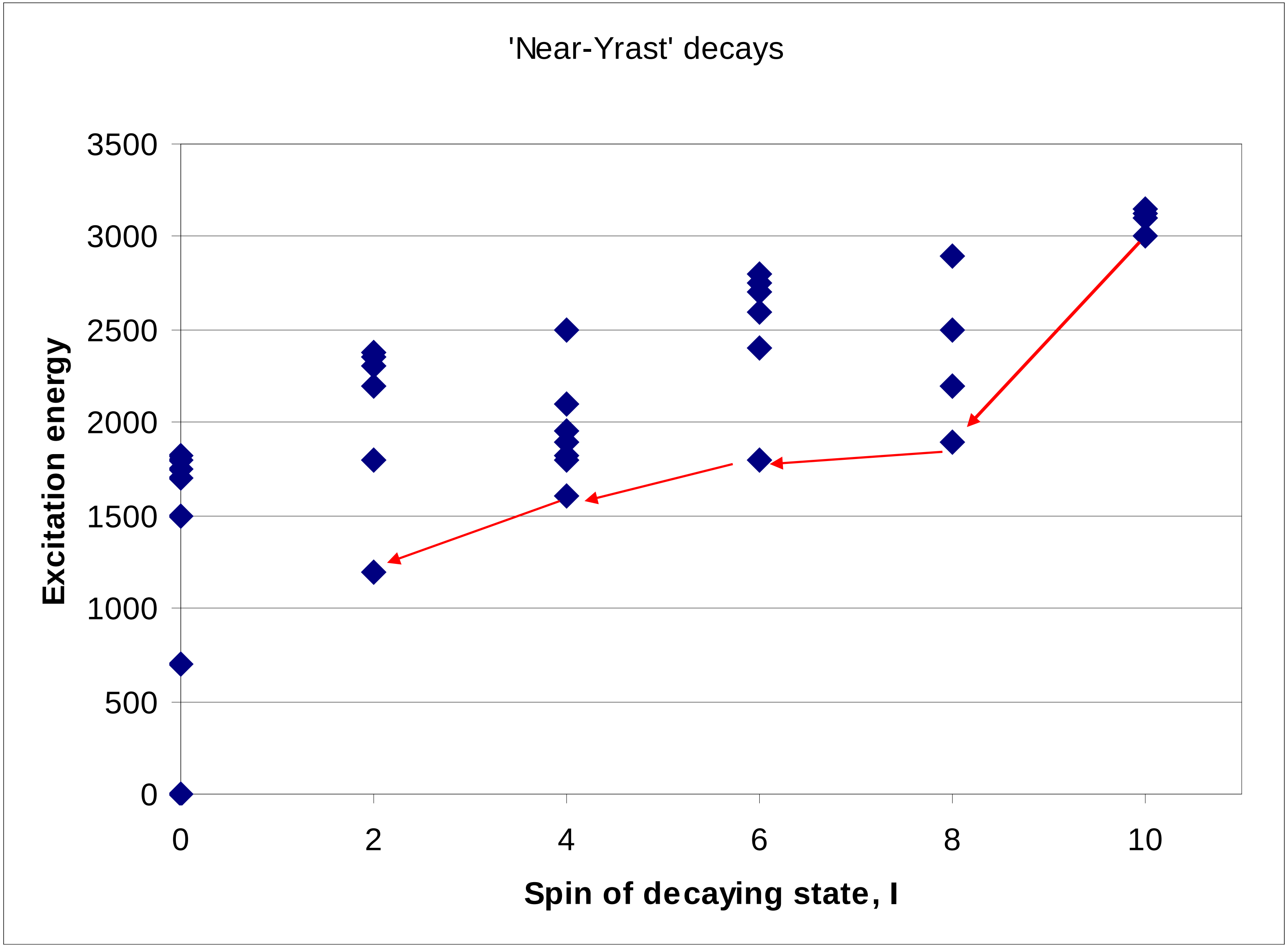

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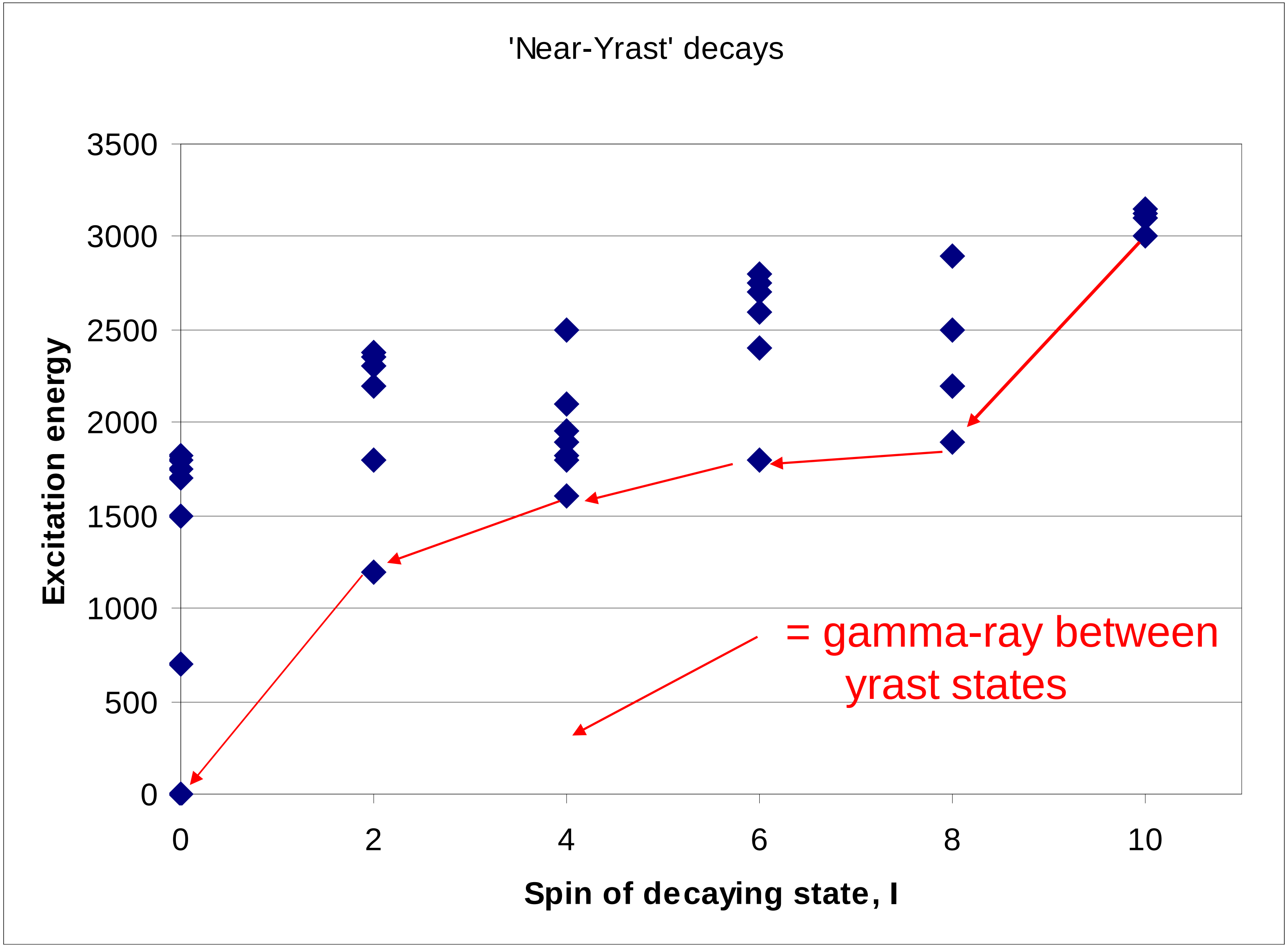
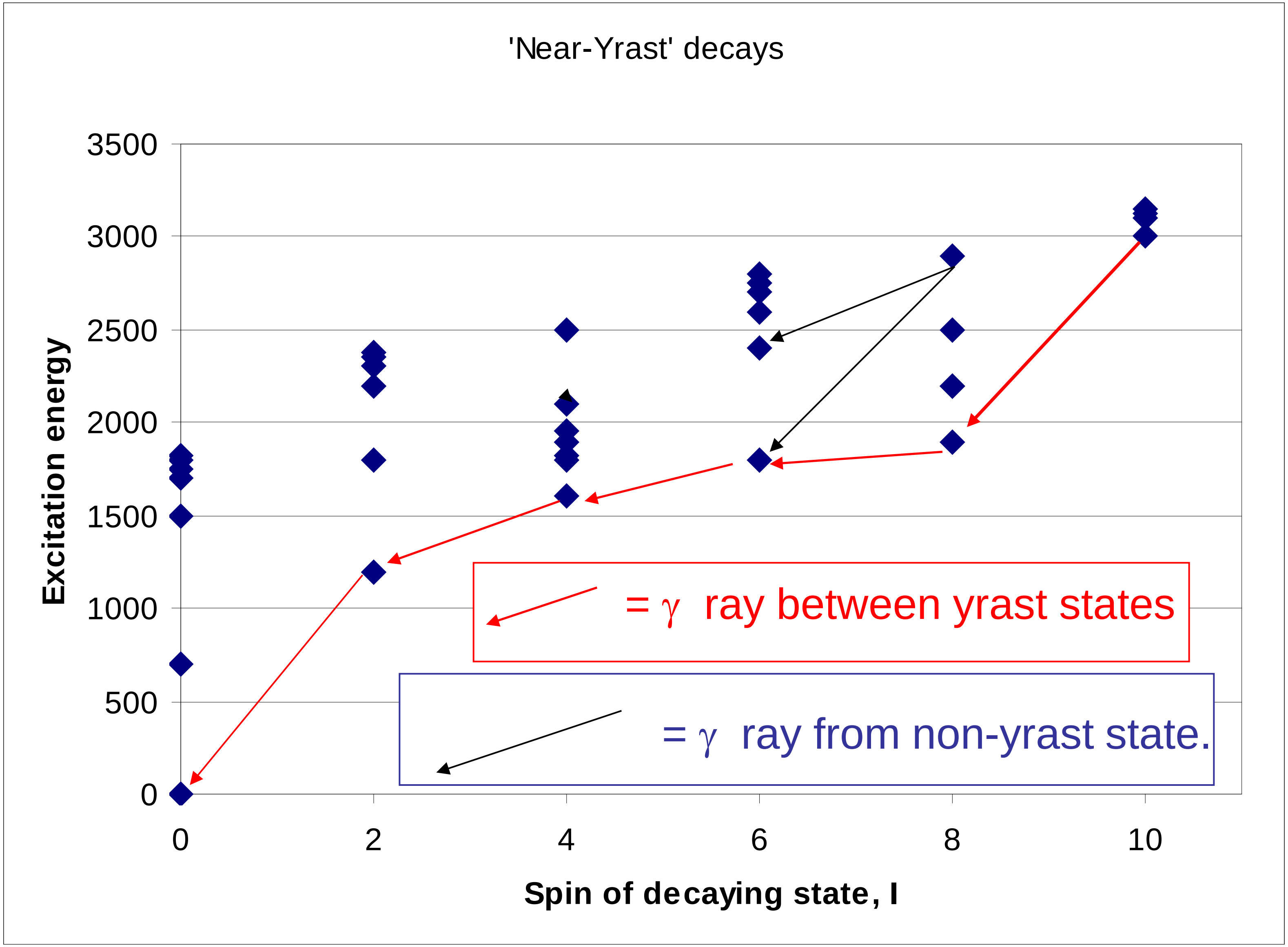


The EM transition rate depends on $E\gamma^{2\lambda+1}$, (for E2 decays E_γ^5)
Thus, the highest energy transitions for the lowest λ are usually favoured.
Non-yrast states decay to yrast ones (unless very different ϕ , K-isomers)



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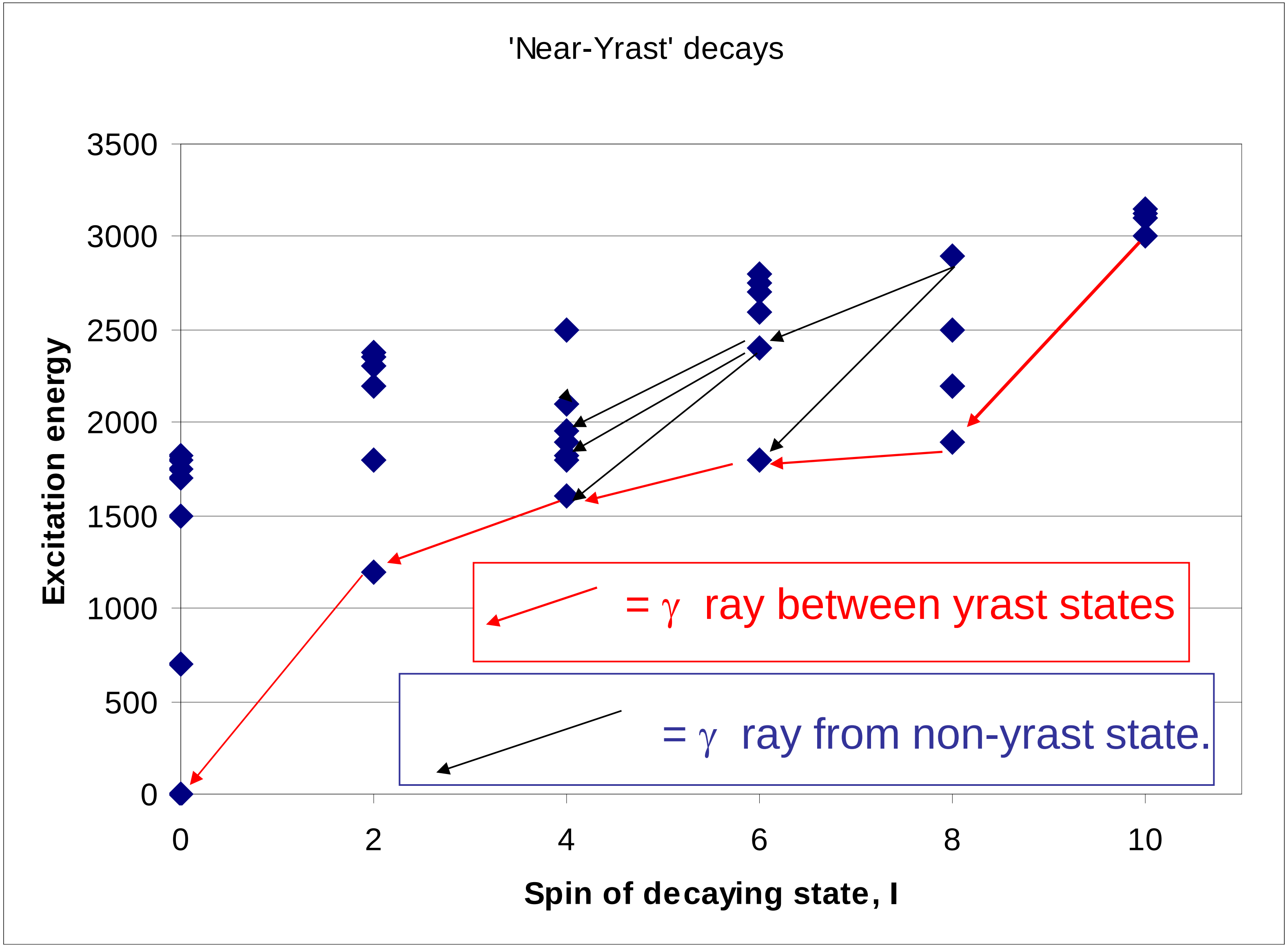

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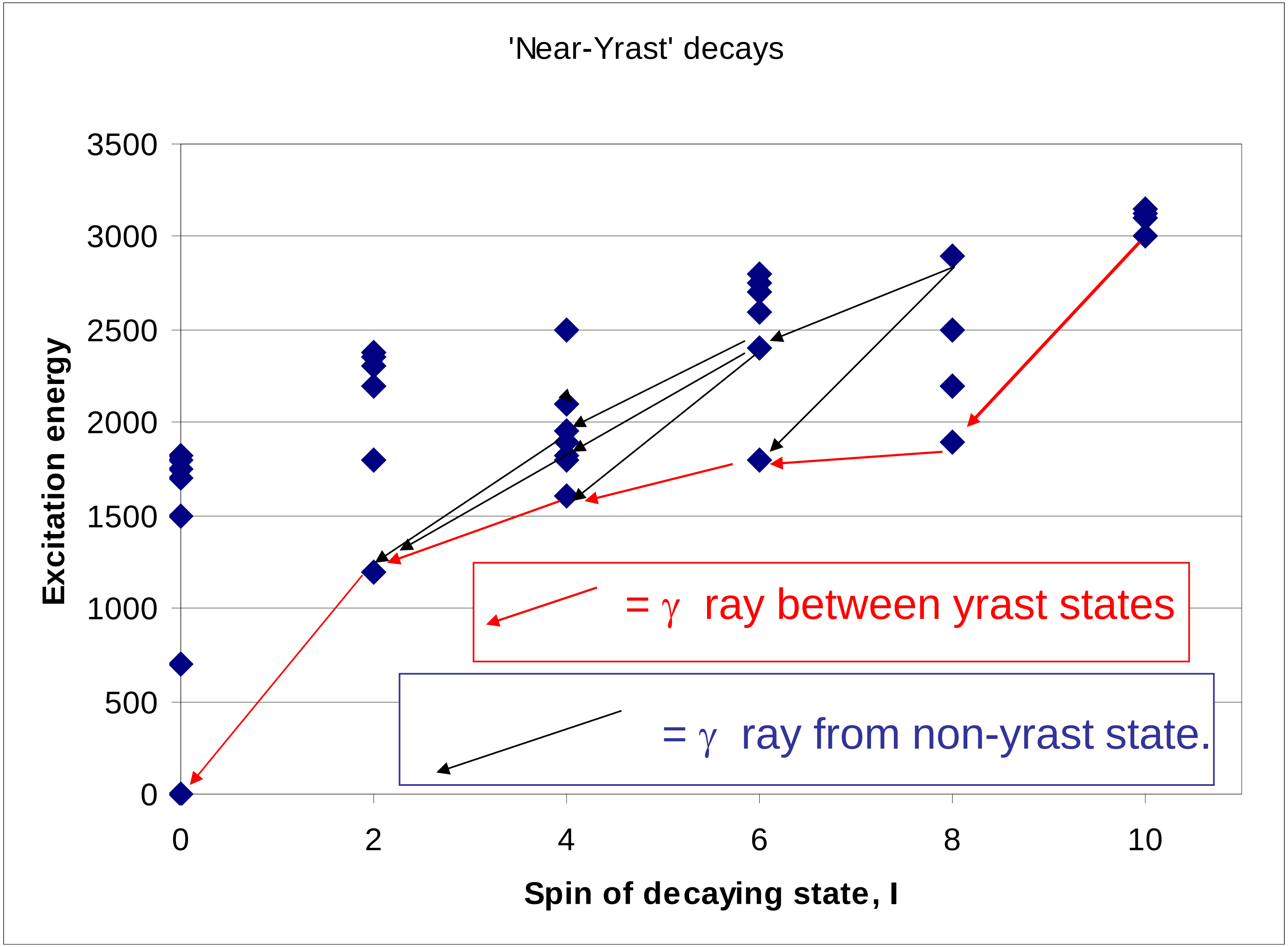

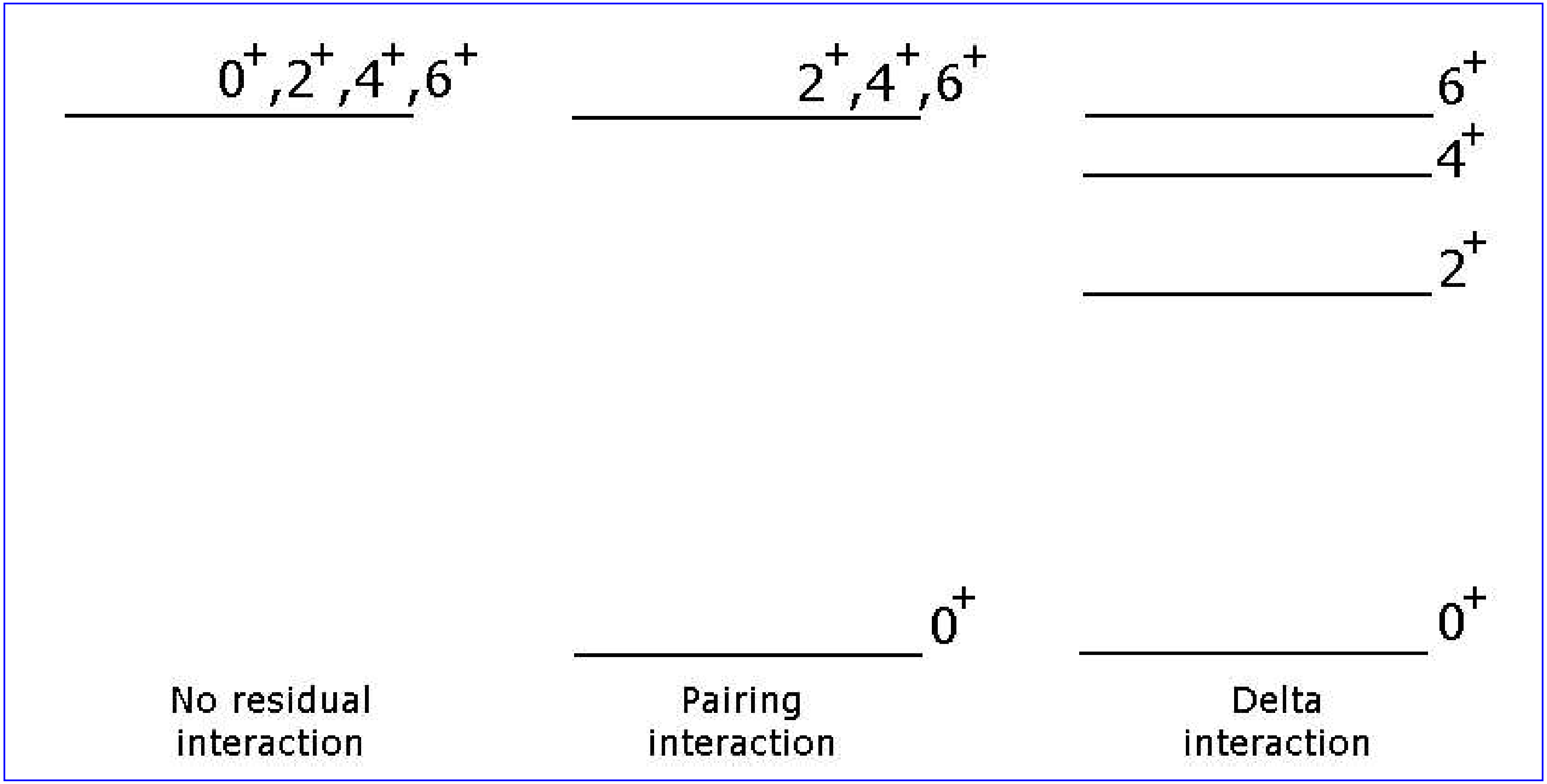

Schematic for $(f_{7/2})^2$ configuration.
4 degenerate states if there are no residual interactions.

Residual interactions between two valence nucleons give additional binding, lowering the (mass) energy of the state.

The effective interaction between nucleons deduced from nuclear spectra*

Reviews of Modern Physics, Vol. 48, No. 2, Part I, April 1976
191

John P. Schiffer

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B. General features in the experimental matrix elements

Several comparisons of the experimental matrix elements seem worthwhile before embarking on detailed analyses. To gain a qualitative impression of spectra we adopt plots of matrix elements as a function of θ_{12}, the classical angle of orientation between the two angular momentum vectors j_1 and j_2 coupled to J which gives a measure of the overlap of their orbital wavefunctions. θ_{12} is defined by

$$\cos\theta_{12} = \frac{J(J+1) - j_1(j_1+1) - j_2(j_2+1)}{2\sqrt{j_1 j_2 (j_1+1)(j_2+1)}} \ . \tag{II.2}$$

$$\Delta E(j^2 J) \approx \frac{-V_0 F_R}{\pi} \tan\frac{\theta}{2} \quad (T = 1,\ J \text{ even})$$

$$\Delta E\left(j_p j_n J\right)_{j_p = j_n} = \frac{-V_0 F_R}{\pi}\left(\cot\frac{\theta}{2}\right)\left[1 + \frac{1}{\cos^2\left(\frac{\theta}{2}\right)}\right] \quad (T = 0,\ J \text{ odd})$$

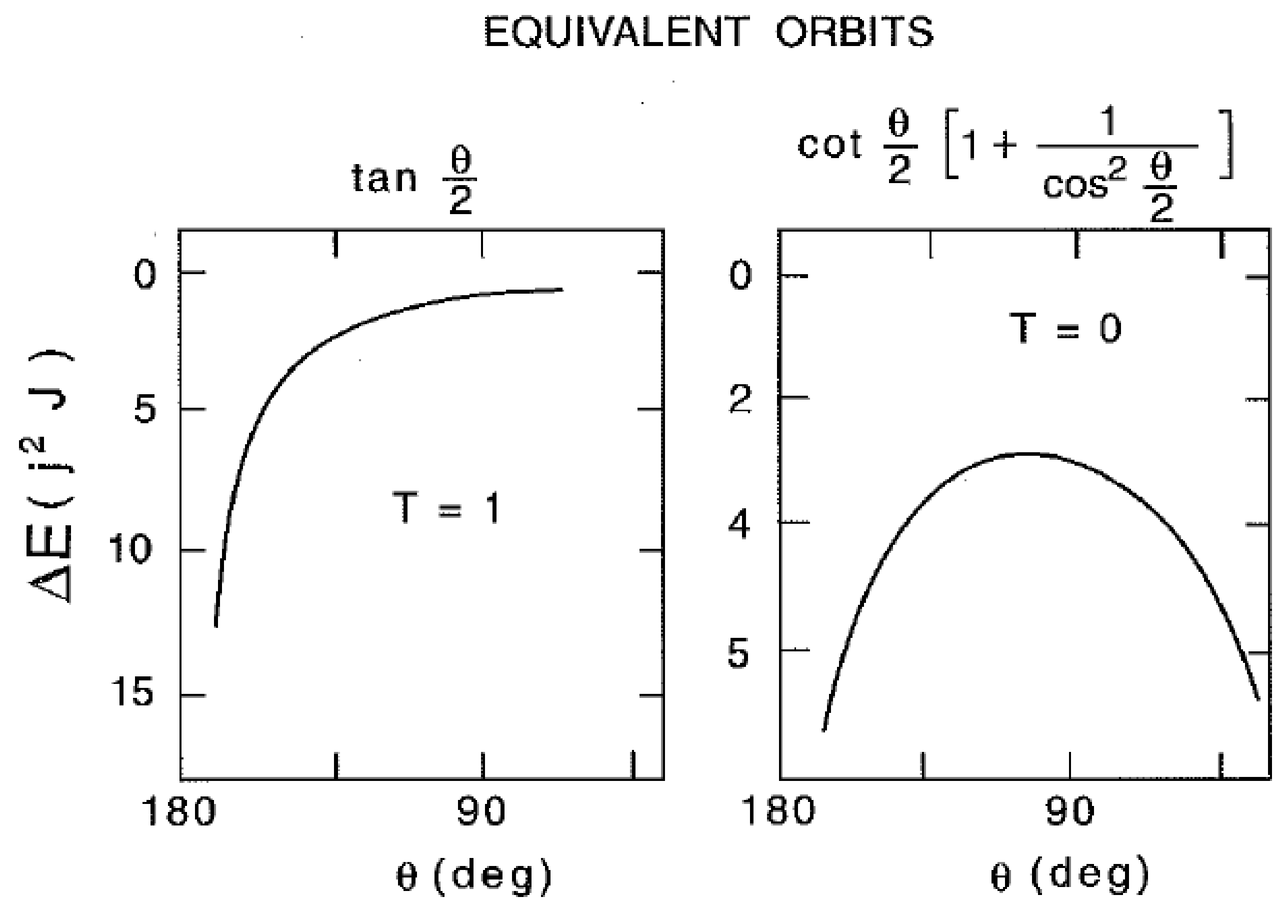

FIG. 4.9. Angular dependence of the δ-function residual interaction strength (lower values correspond to more attractive residual interactions) for two particles in equivalent orbits. (Left) The $T = 1$ (J even) states. (Right) The $T = 0$ (J odd) states. The analytic expressions are indicated above their respective plots.

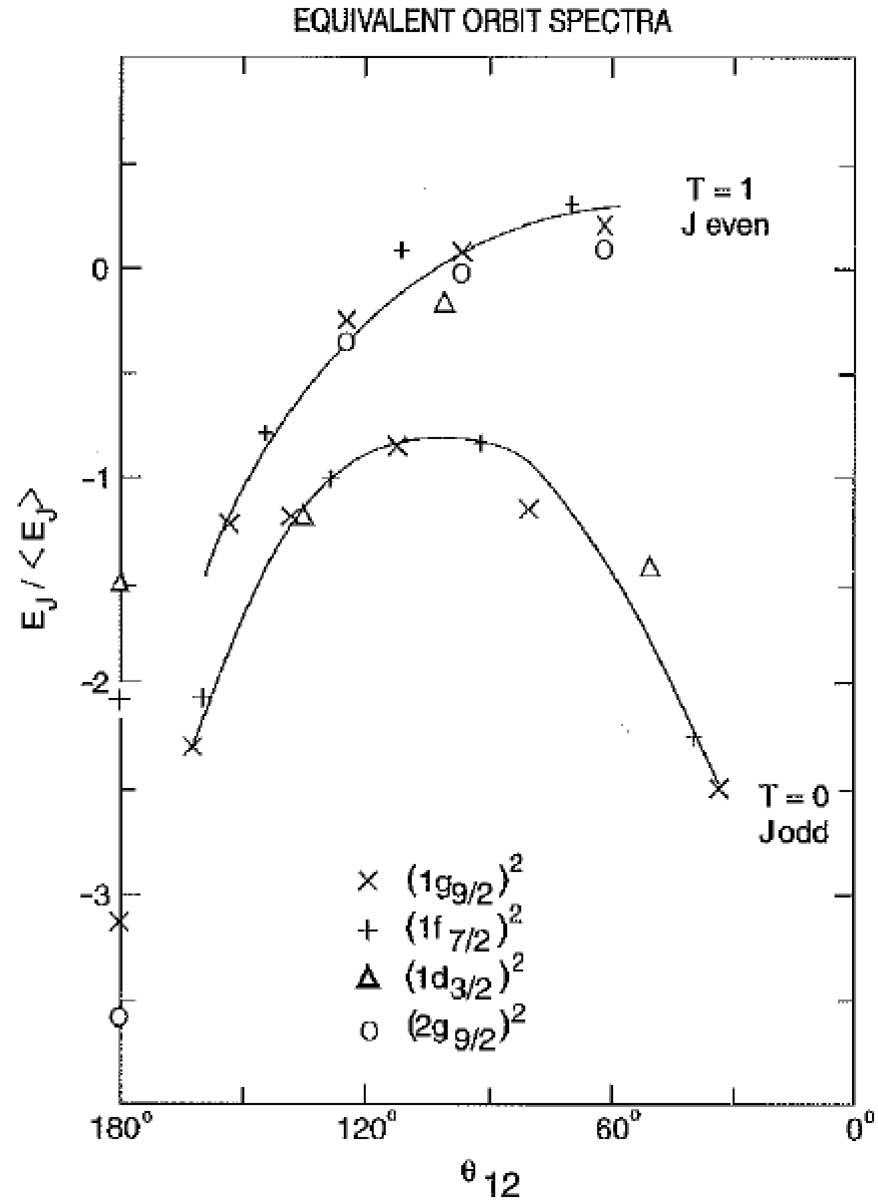

FIG. 4.10. Empirical proton–neutron multiplets for two particle equivalent orbit configurations for comparison with the behavior shown in Fig. 4.9. The curves are drawn through the data (Schiffer, 1971).

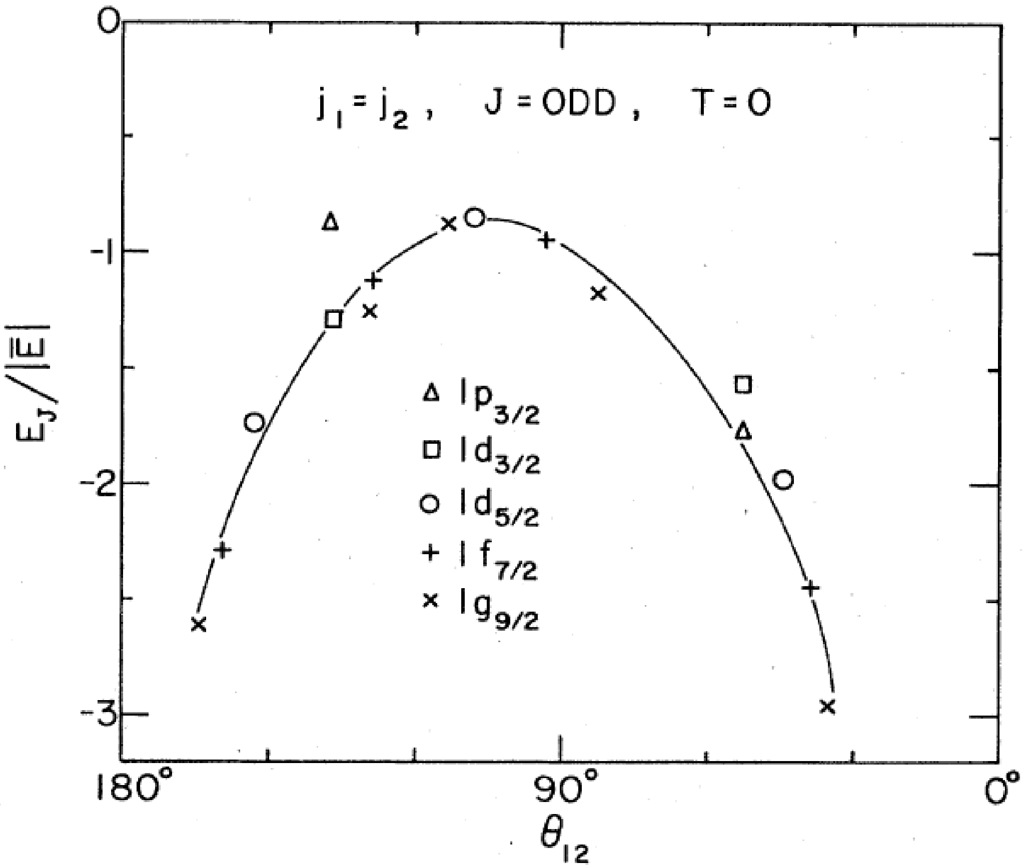

FIG. 2. Comparison of data from various multiplets with $j_1 = j_2$ and $T=0$. The values of the matrix elements are divided by $\bar{E} \equiv \sum_J [J] E_J / \sum_J [J]$ to display the similarities in the J dependence (or θ dependence) of the various multiplets.

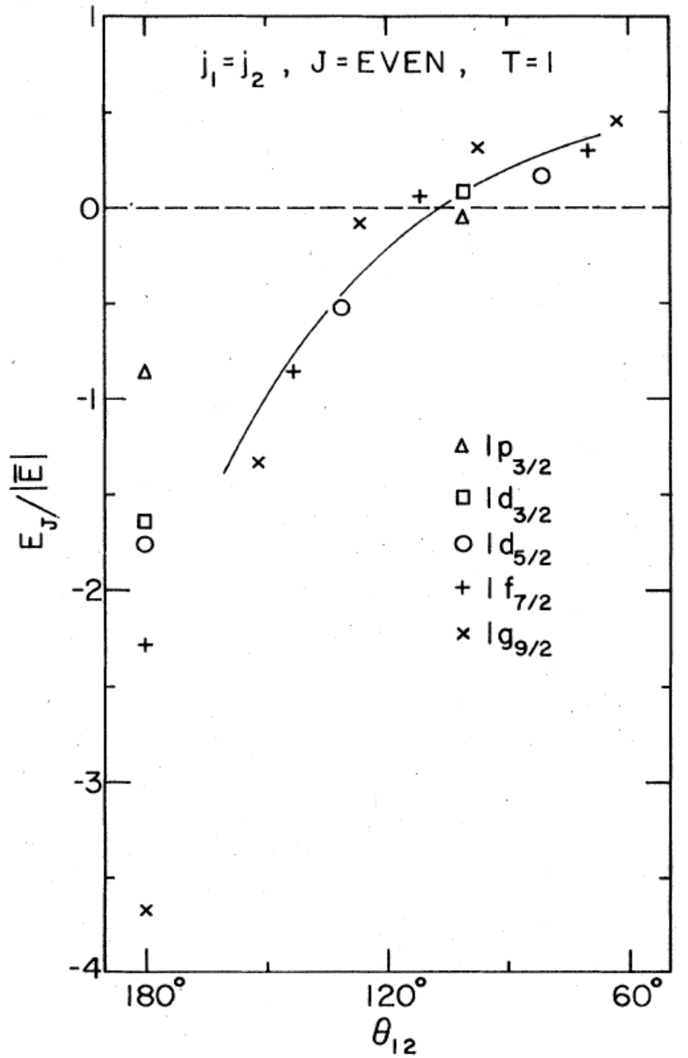

FIG. 3. Comparison of data from various multiplets with $j_1 = j_2$ and $T=1$. The values of the matrix elements are divided by $\bar{E} \equiv \sum_J [J] E_J / \sum_J [J]$ to display the similarities in the J dependence (or θ dependence) of the various multiplets.

J. P. Schiffer and W. W. True: Effective interaction between nucleons
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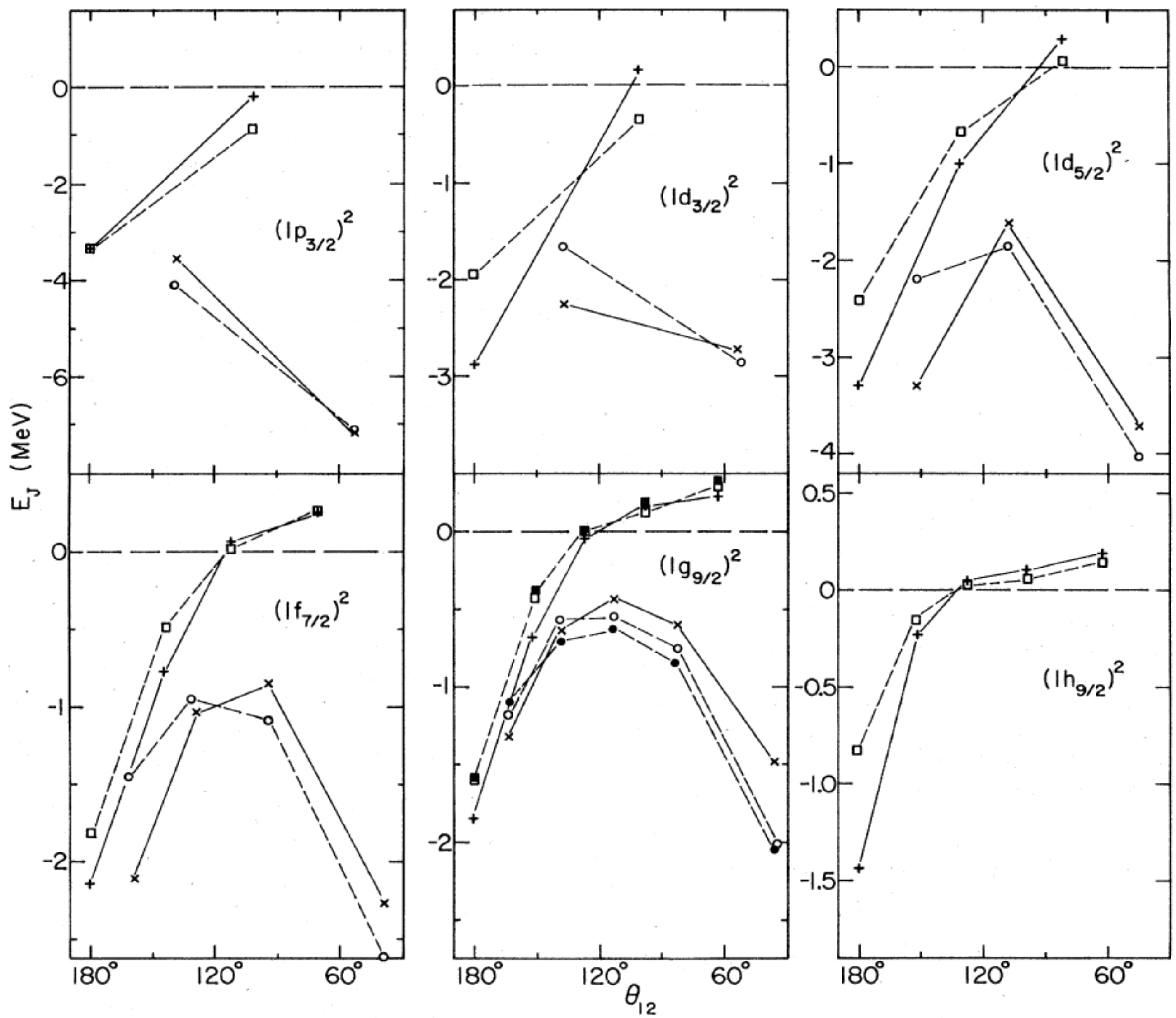


FIG. 4. Plots of the various $j_1=j_2$ multiplets. The data (crosses) are compared to calculations (circles for $T=0$ and squares for $T=1$) with the 12-parameter interaction of Table XVII ($r_2=2.0$ fm). The solid points for the $(1g_{9/2})^2$ multiplet represent calculations with the purely central five-parameter interaction of Table XIII.

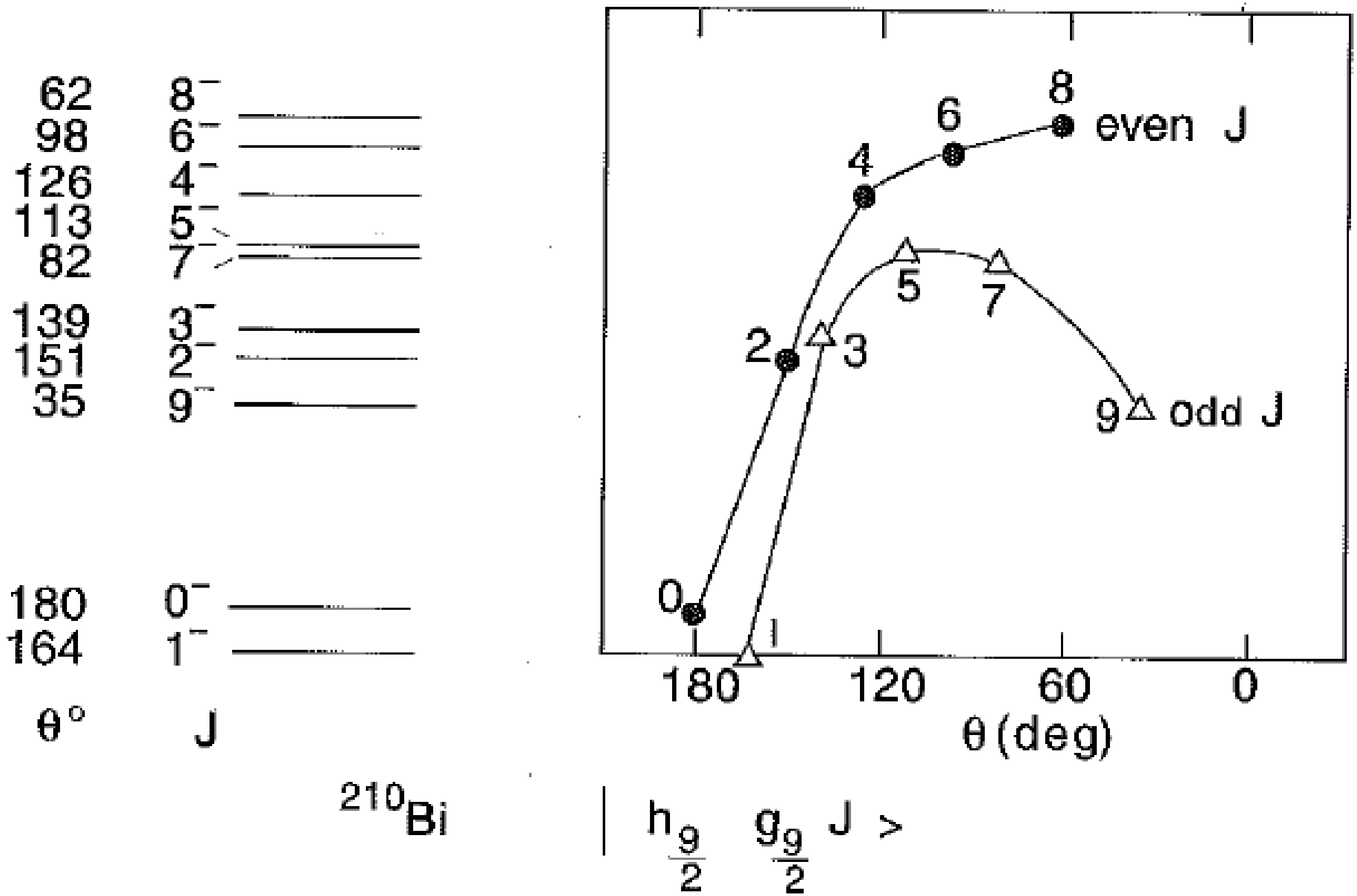
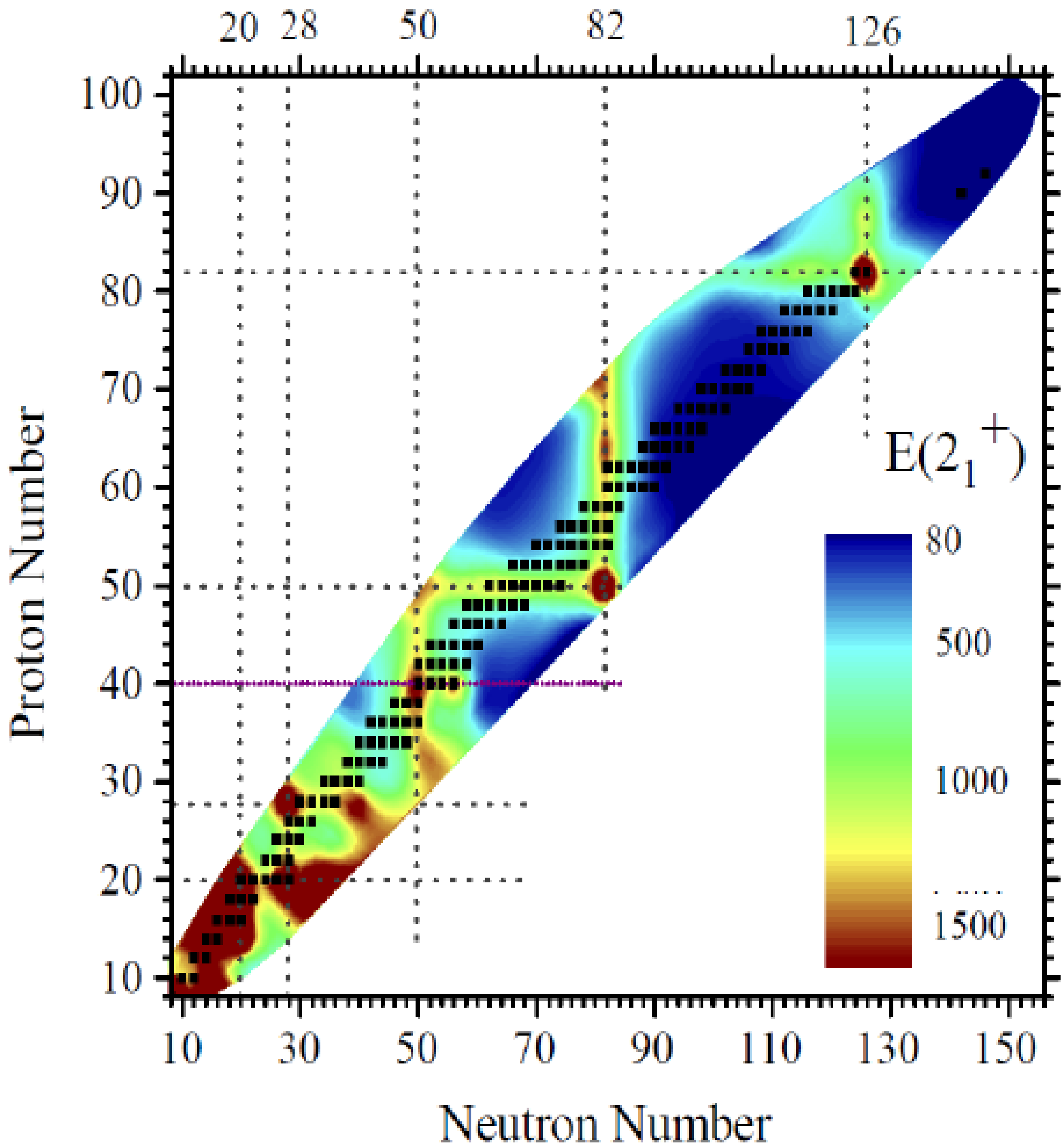
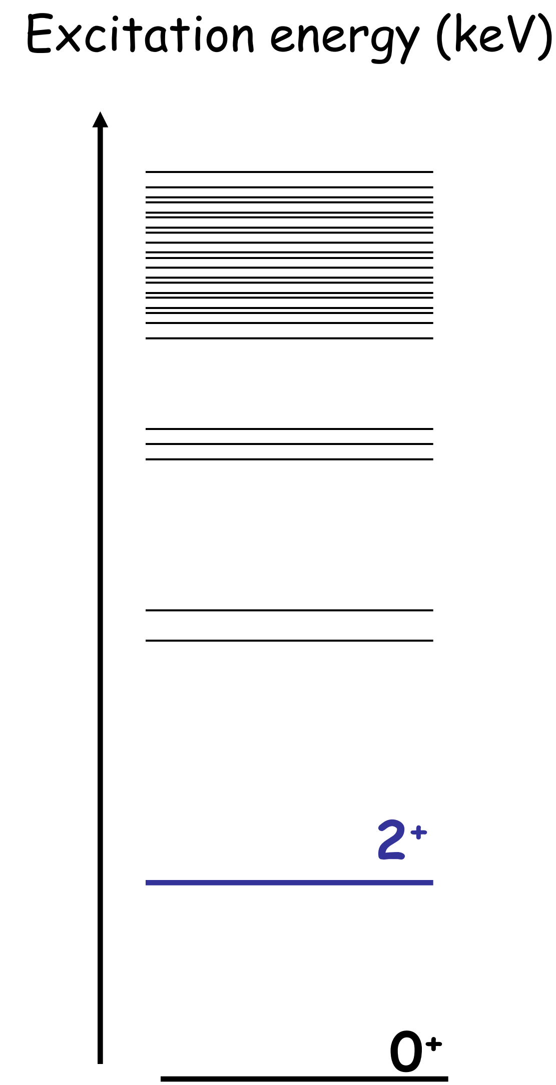


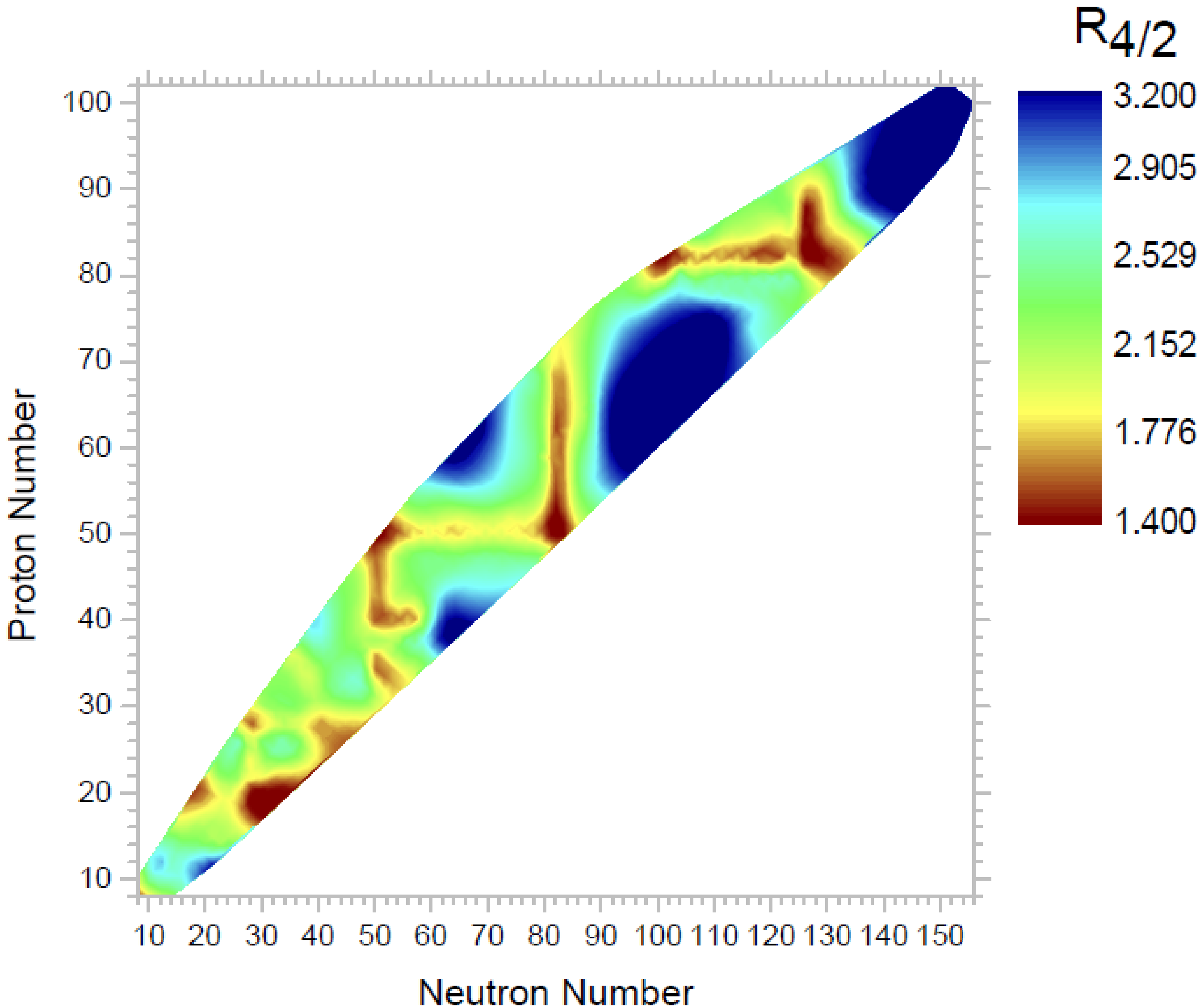
FIG. 4.12. A geometrical analysis of the $|h_{9/2}g_{9/2}J\rangle$ p–n multiplet in ^{210}Bi. The empirical levels are shown on the left along with the semiclassical angle between the orbits of the two nucleons. The right side shows that the levels split into two families, according to J even or J odd. The solid lines are drawn to connect the points.

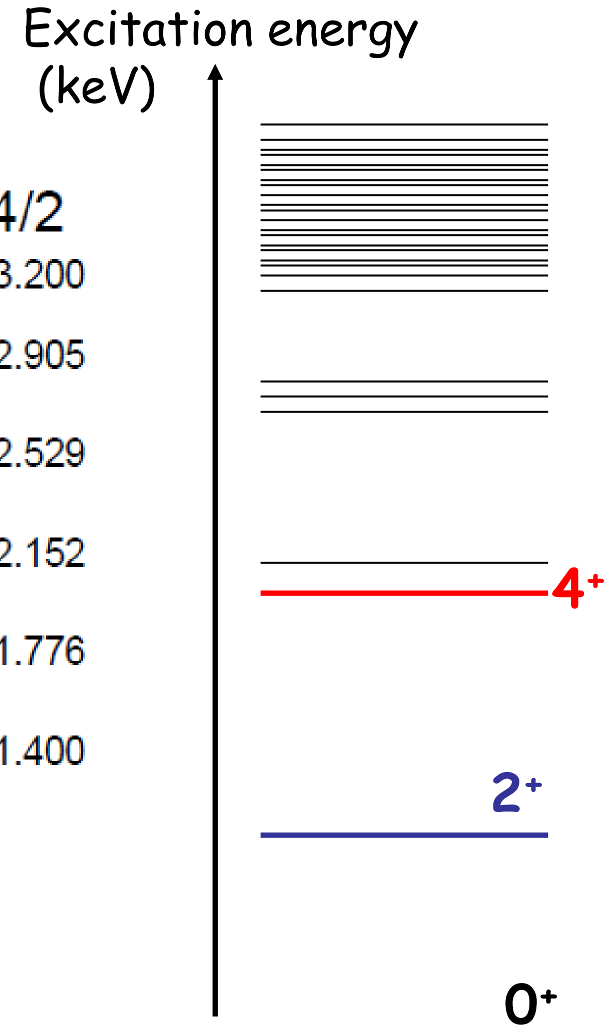




Ground state
Configuration.
Spin/parity $I^{\pi}=0^{+}$;
E_x = 0 keV

4+/2+ energy ratio: mirrors 2^+ systematics.





Ground state Configuration.
Spin/parity $I^{\pi}=0^+$;
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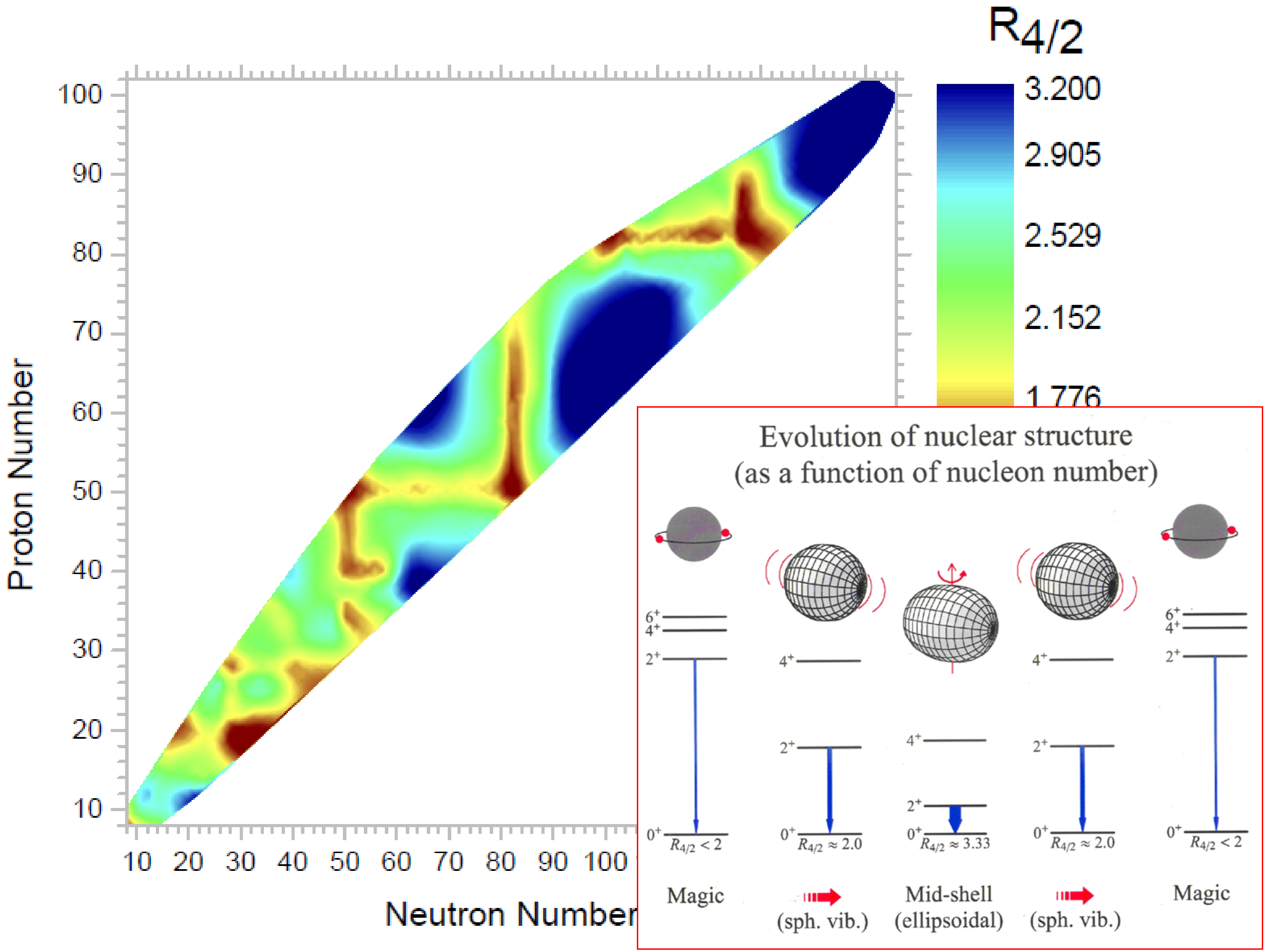

R4/2
3.200
2.905
2.529
2.152
1.776
Proton Number
100
90
80
70
60
50
40
30
20
10
Neutron Number
10
20
30
40
50
60
70
80
90
100
Evolution of nuclear structure
(as a function of nucleon number)
6+
4+
2+
0+
$R_{4/2} < 2$
Magic
4+
2+
0+
$R_{4/2} \approx 2.0$
(sph. vib.)
4+
2+
0+
$R_{4/2} \approx 3.33$
Mid-shell
(ellipsoidal)
4+
2+
0+
$R_{4/2} \approx 2.0$
(sph. vib.)
6+
4+
2+
0+
$R_{4/2} < 2$
Magic

Evolution of nuclear structure
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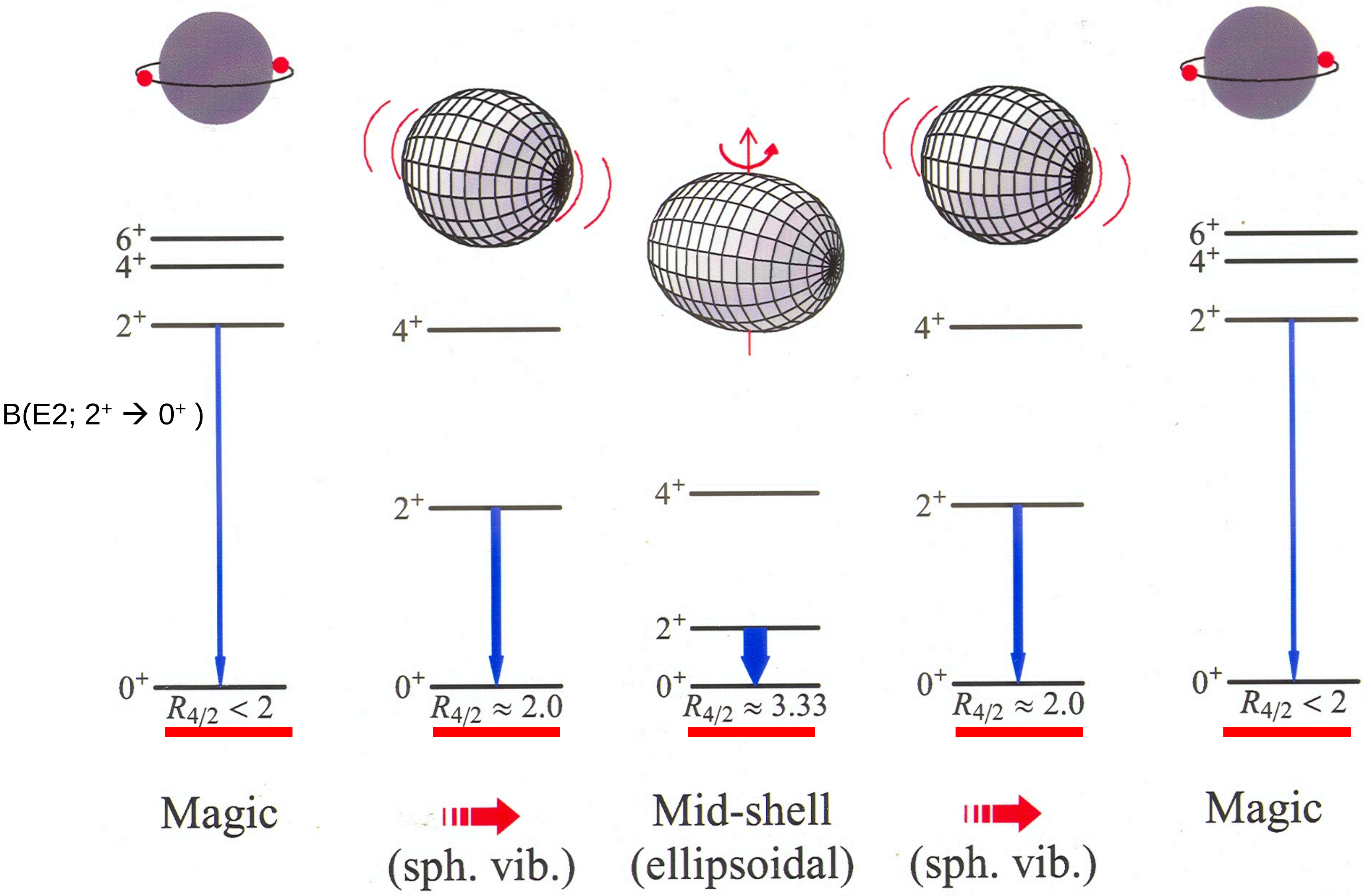
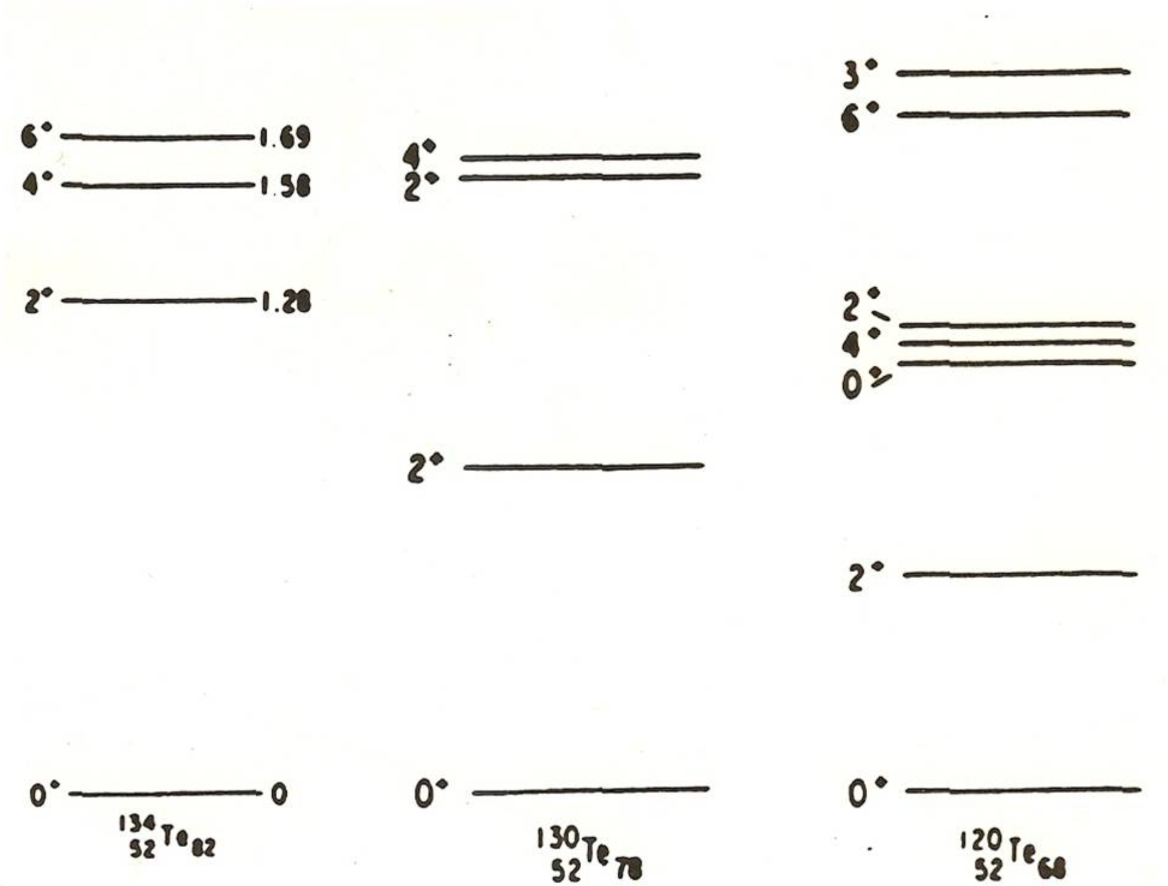


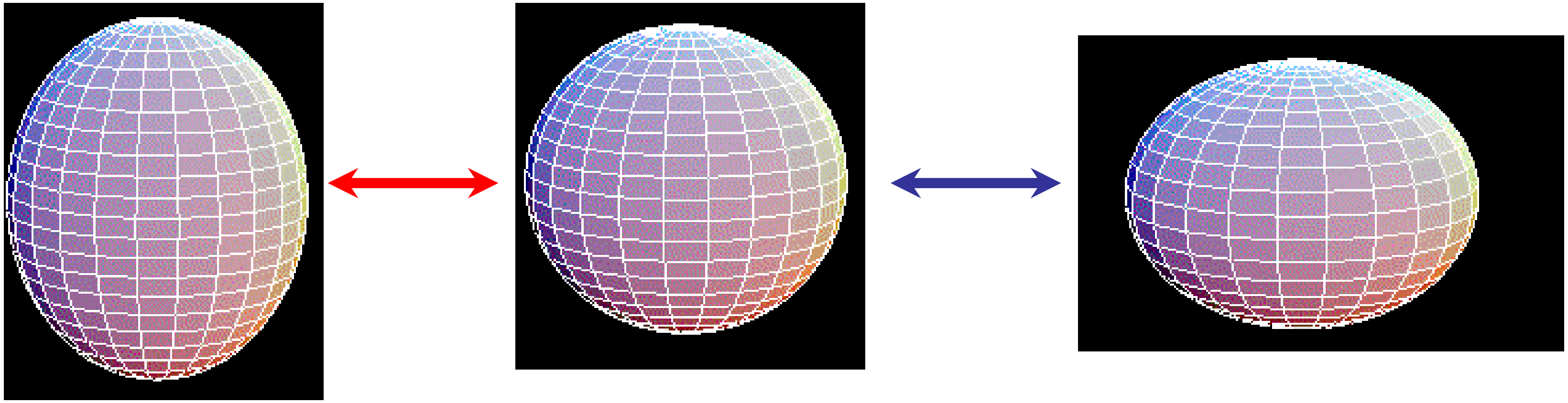
What about <u>both</u> valence neutrons and protons? In cases of a few valence nucleons there is a lowering of energies, development of multiplets.

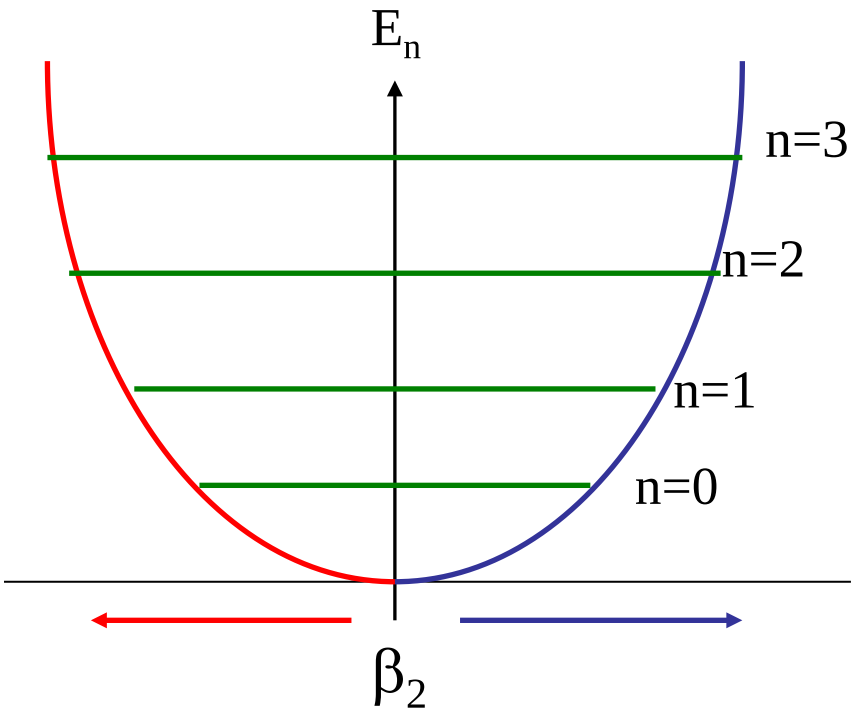
$R_{4/2} \rightarrow \sim 2\text{-}2.4$

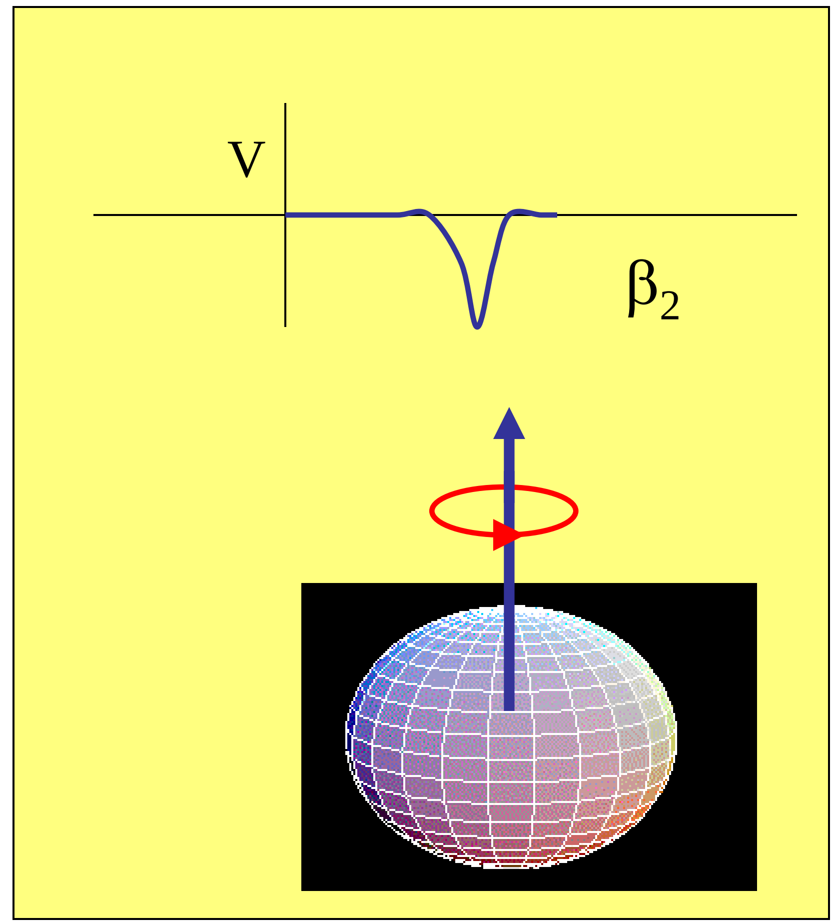


Quadrupole Vibrations in Nuclei ?

- Low-energy quadrupole vibrations in nuclei ?
  - Evidence?
  - Signatures?
  - Coupling schemes ?







http://npl.kyy.nitech.ac.jp/~arita/vib

Low energy vibrations

$\lambda = 1$ is equivalent to motion of center of mass

Lowest physical mode is $\lambda = 2$

$\boxed{\text{Quadrupole}}$

$$\Downarrow$$

$$Y_{2\mu}$$

2^+ ________

0^+ ________

We can use the m-scheme to see what states we can make when we couple together 2 quadrupole phonon excitations of order J=2ħ

.

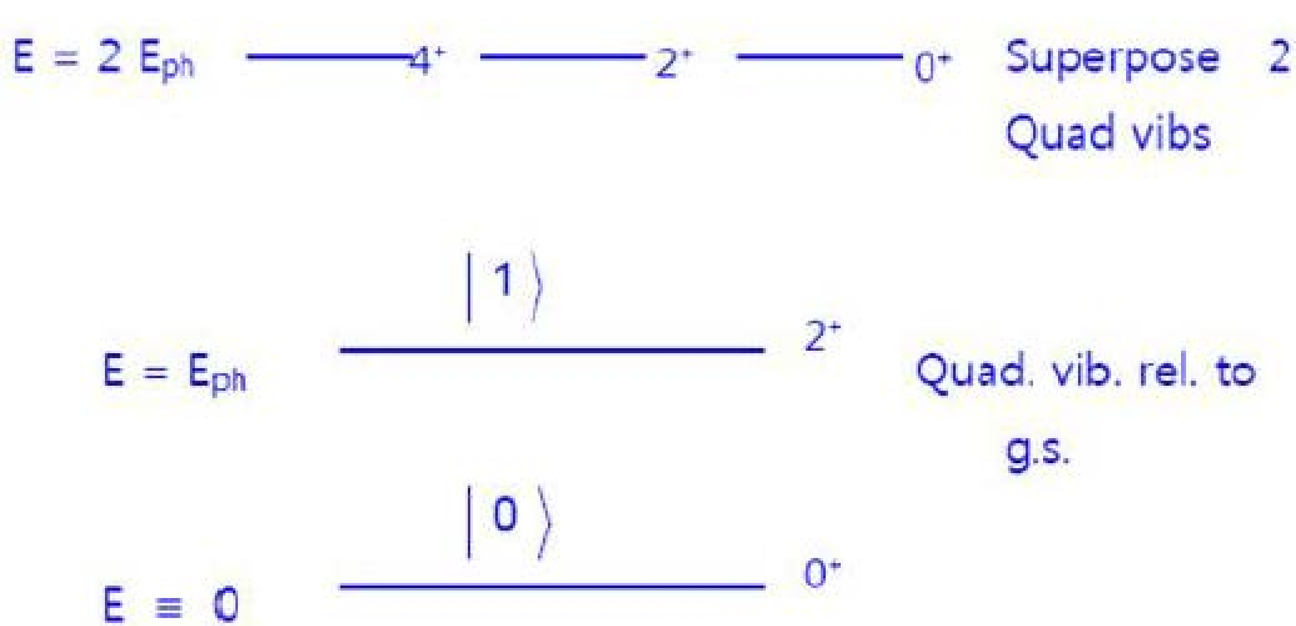
(Note phonons are bosons, so we can couple identical 'particles' together).

$J_1 = 2$ m_1	$J_2 = 2$ m_2	$M = \sum m_i$	J
2	2	4	
2	1	3	
2	0	2	4
2	−1	1	
2	−2	0	
1	1	2	
1	0	1	2
1	−1	0	
0	0	0	0

*Only positive total M values are shown: the table is symmetric for $M < 0$. The full set of allowable m_i values giving $M \geq 0$ is obtained by the conditions $m_1 \geq 0$, $m_2 \leq m_1$.

$J_1 = 2$ m_1	$J_2 = 2$ m_2	$M = \sum m_i$	J
2	2	4	
2	1	3	
2	0	2	4
2	−1	1	
2	−2	0	
1	1	2	
1	0	1	2
1	−1	0	
0	0	0	0

*Only positive total M values are shown: the table is symmetric for $M < 0$. The full set of allowable m_i values giving $M \geq 0$ is obtained by the conditions $m_1 \geq 0$, $m_2 \leq m_1$.

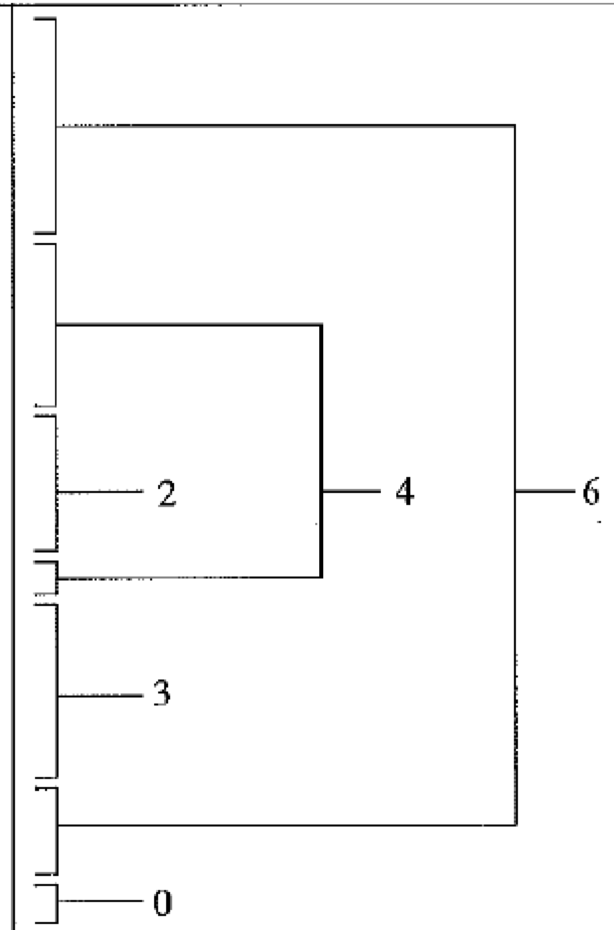


From, *Nuclear Structure From a Simple Perspective*, by R.F. Casten, Oxford University Press.

Table 6.2 *m scheme for three-quadrupole phonon states**

$J_1 = 2$ m_1	$J_2 = 2$ m_2	$J_3 = 2$ m_3	M
2	2	2	6
2	2	1	5
2	2	0	4
2	2	−1	3
2	2	−2	2
2	1	1	4
2	1	0	3
2	1	−1	2
2	1	−2	1
2	0	0	2
2	0	−1	1
2	0	−2	0
2	−1	−1	0
1	1	1	3
1	1	0	2
1	1	−1	1
1	1	−2	0
1	0	0	1
1	0	−1	0
0	0	0	0

J



$J = 6, 4, 3, 2, 0$

*Only positive total M values are shown; the table is symmetric for $M < 0$. The full set of allowable m_i values giving $M \geq 0$ is obtained by the conditions $m_1 \geq 0$, $m_3 \leq m_2 \leq m_1$.

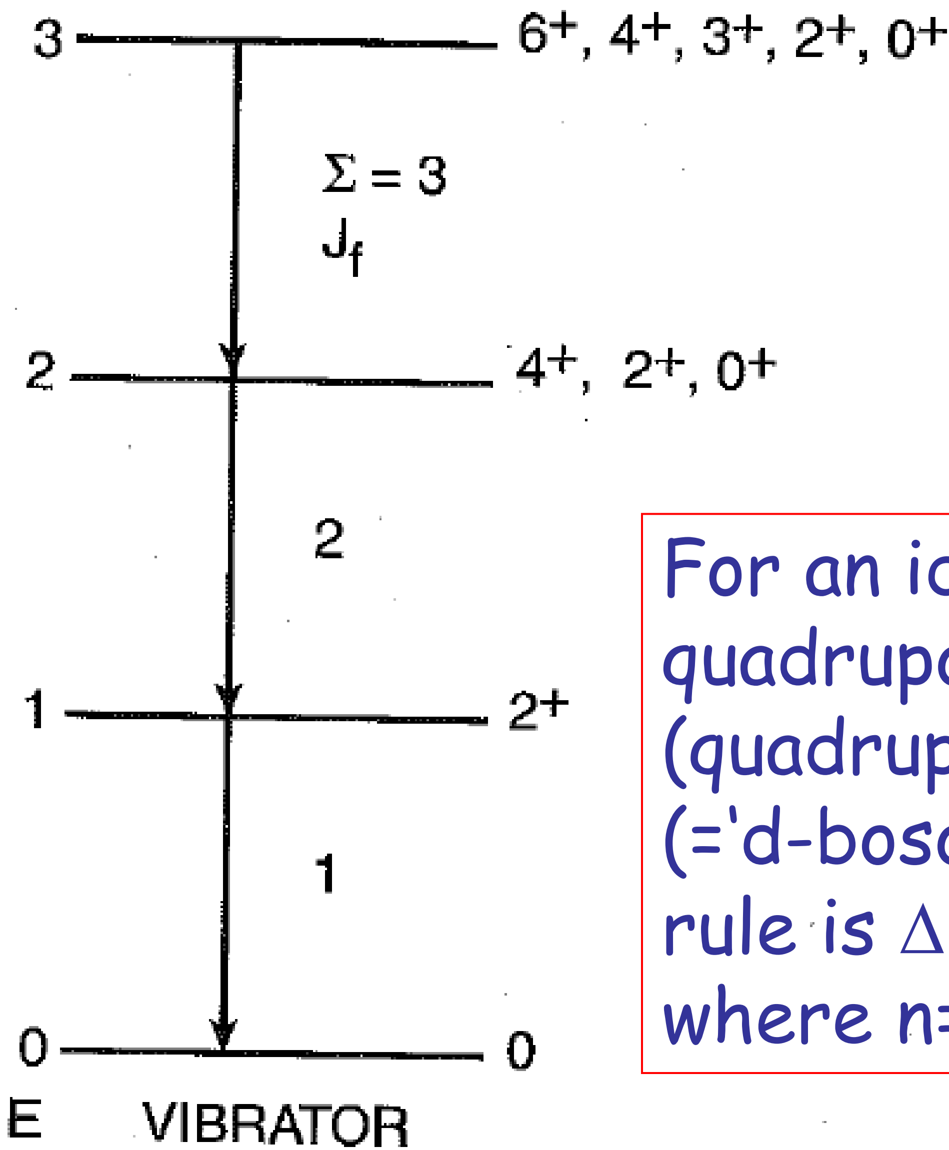


FIG. 6.2. Low-lying levels of the harmonic vibrator phonon model.

For an idealised quantum quadrupole vibrator, the (quadrupole) phonon (='d-boson') selection rule is $\Delta n=1$, where n=phonon number.

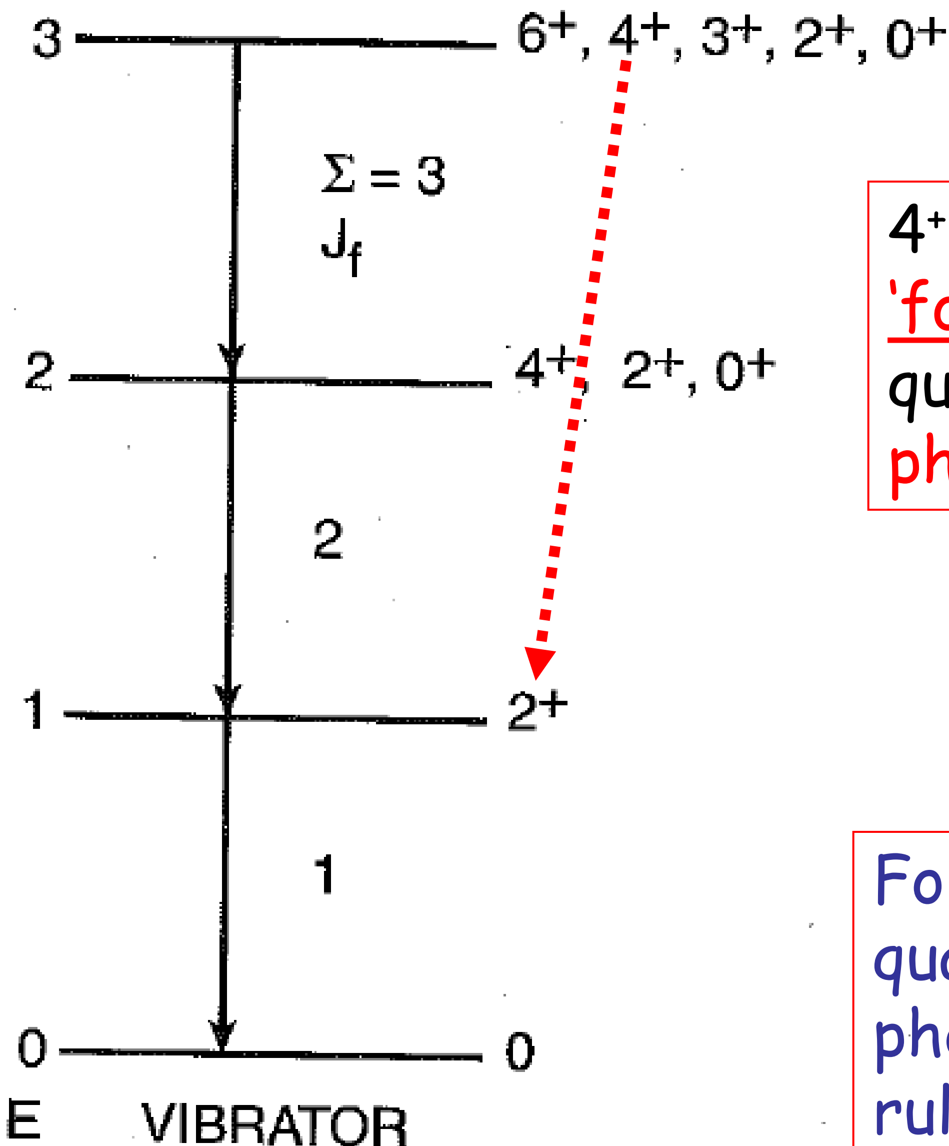


FIG. 6.2. Low-lying levels of the harmonic vibrator phonon model.

$4^+ \rightarrow 2^+$ E2 from n=3 →n=1 is 'forbidden' in an idealised quadrupole vibrator by phonon selection rule.

For an idealised quantum quadrupole vibrator, the phonon (='d-boson') selection rule is $\Delta n=1$

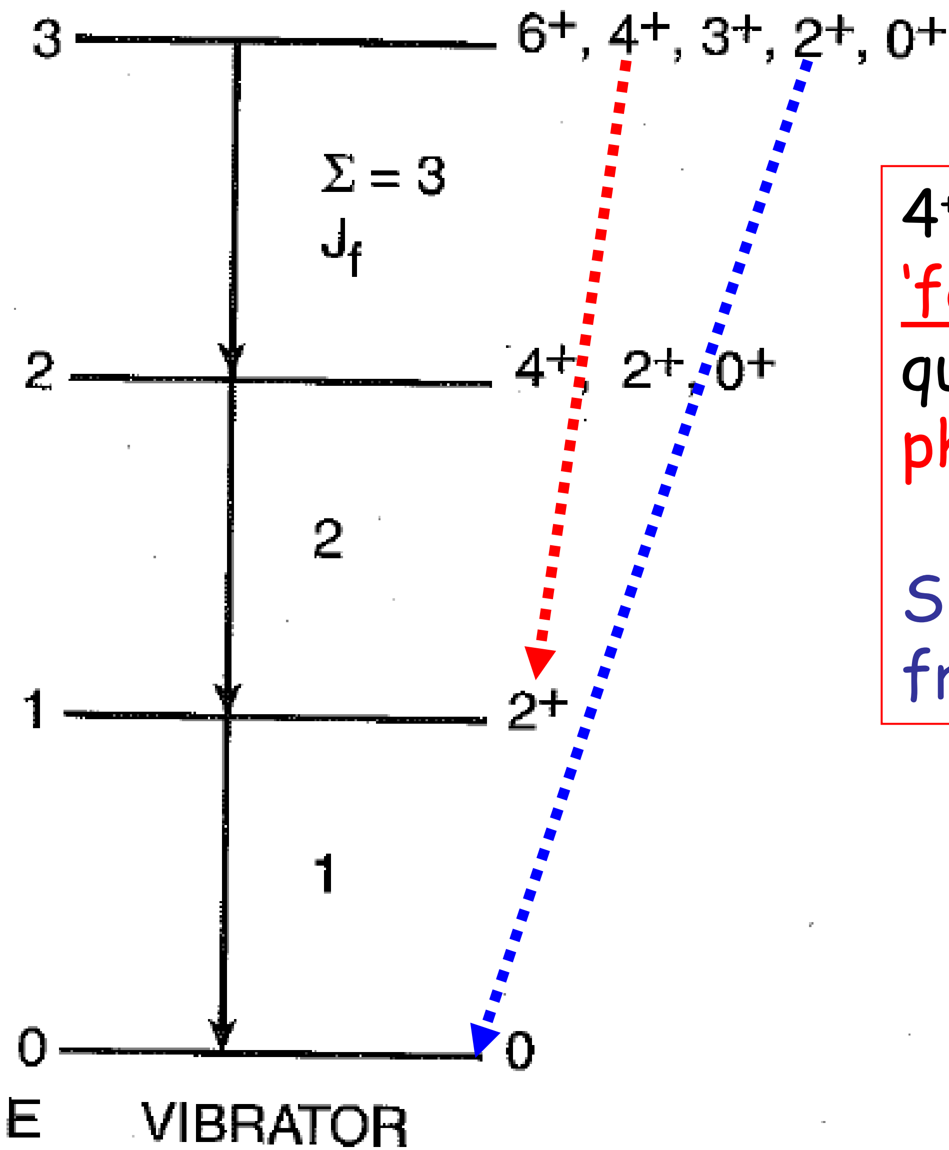


FIG. 6.2. Low-lying levels of the harmonic vibrator phonon model.

$4^+ \rightarrow 2^+$ E2 from n=3 →n=1 is 'forbidden' in an idealised quadrupole vibrator by phonon selection rule.

Similarly, E2 from $2^+ \rightarrow 0^+$ from n=3 →n=0 not allowed.

For an idealised quantum quadrupole vibrator, the phonon (='d-boson') selection rule is $\Delta n_p=1$

Collective (Quadrupole) Nuclear Rotations and Vibrations

- What are the (idealised) excitation energy signatures for quadrupole collective motion (in even-even nuclei) ?
  - (extreme) theoretical limits

Perfect, quadrupole (ellipsoidal), axially symmetric quantum rotor with a constant moment of inertia (I) has rotational energies given by (from $E_{class}(\text{rotor}) = L^2/2I$)

$$E_J = \frac{\hbar^2}{2\ell} J(J+1), \qquad \frac{E(4^+)}{E(2^+)} = \frac{4(5)=20}{2(3)=6} = 3.33$$

Collective (Quadrupole) Nuclear Rotations and Vibrations

- What are the (idealised) excitation energy signatures for quadrupole collective motion (in even-even nuclei) ?
  - (extreme) theoretical limits

Perfect, quadrupole (ellipsoidal), axially symmetric quantum rotor with a constant moment of inertia (I) has rotational energies given by (from $E_{class}(\text{rotor}) = \frac{1}{2} L^2/2I$)

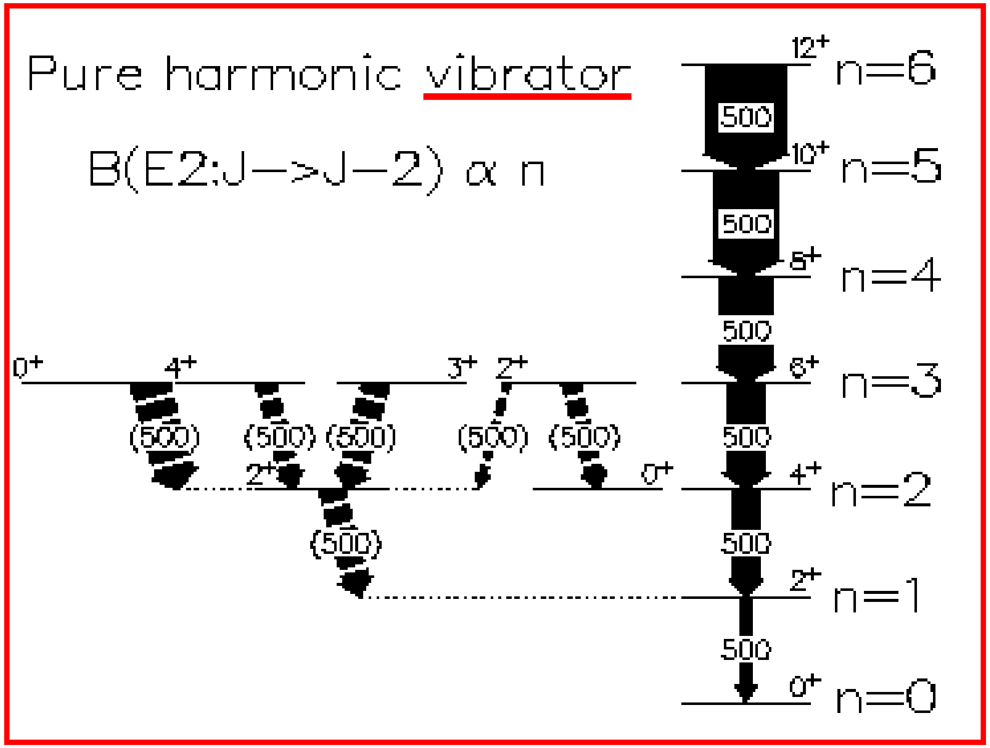
$$E_J = \frac{\hbar^2}{2\ell} J(J+1), \qquad \frac{E(4^+)}{E(2^+)} = \frac{4(5)=20}{2(3)=6} = 3.33$$

Perfect, quadrupole vibrator has energies given by the solution to the harmonic oscillator potential ($E_{classical} = {}^1/_2 k\Delta x^2 + p^2/2m$).

$$E_N = \hbar\omega N \qquad \frac{E(4^+)}{E(2^+)} = \frac{2}{1} = 2.00$$

Other Signatures of (perfect) vibrators and rotors

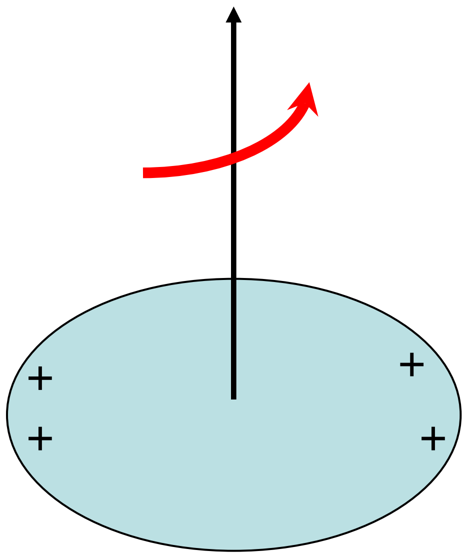
Decay lifetimes give B(E2) values.
Also selection rules important
(eg. $\Delta n=1$).



$E_\gamma=\hbar\omega$; $\Delta E\gamma\ (J\rightarrow J\text{-}2)=0$

For ('real') examples, see
J. Kern et al., Nucl. Phys. **A593** (1995) 21

Other Signatures of (perfect) vibrators and rotors

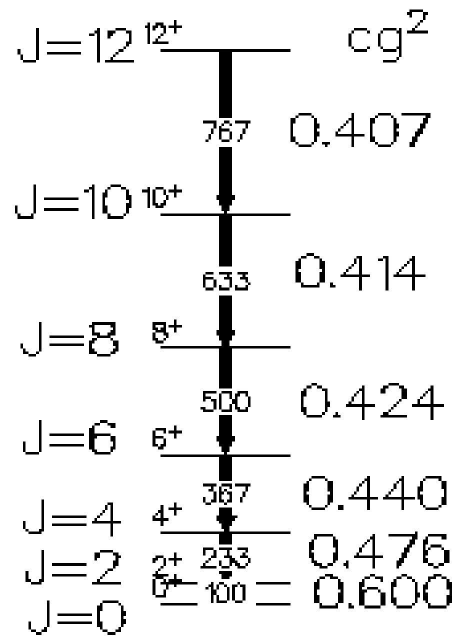


$E_x=(\hbar^2/_{2I})J(J+1)$

Perfect rotor

$$B(E2) = kQ^2 \langle J_i K 2 0 | j_f K \rangle^2$$

$$B(E2) \;\alpha\; \frac{3J(J-1)(J+1)}{(2J-2)(2J-1)(2J+1)} \quad \sim 3/8$$



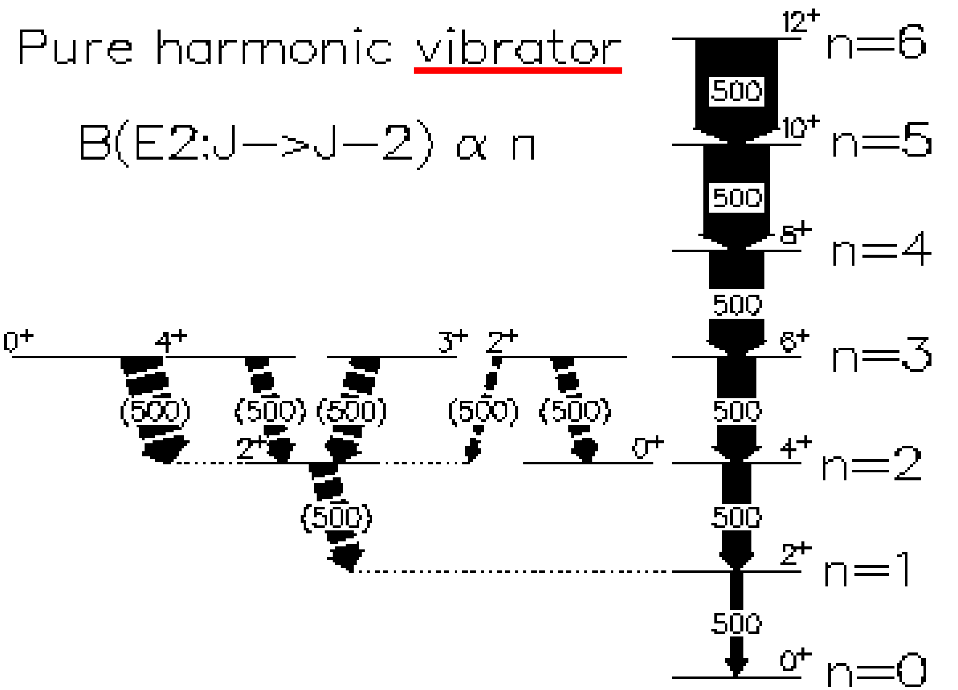
$E_x=(\hbar^2/_{2I})J(J+1)$, i.e., $E\gamma\ (J\rightarrow J-2)=(\hbar^2/_{2I})[J(J+1) - (J-2)(J-3)] = (\hbar^2/_{2I})(6J-6)$; $\Delta E\gamma=(\hbar^2/_{2I})*12=\text{const.}$

Other Signatures of (perfect) vibrators and rotors

Decay lifetimes give B(E2) values.
Also selection rules important
(eg. $\Delta n=1$).

Pure harmonic vibrator

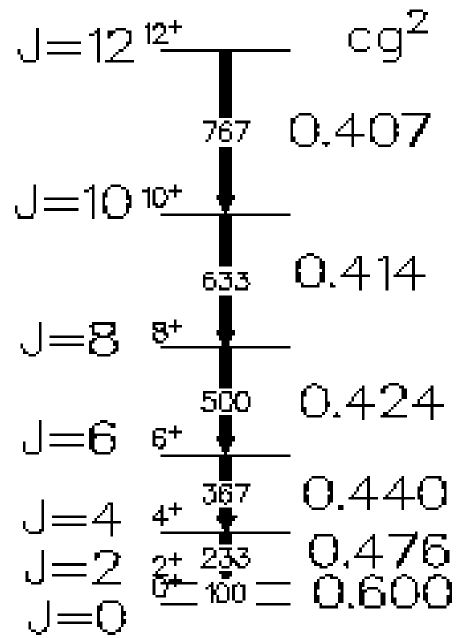
B(E2:J−>J−2) α n



Perfect rotor

$$B(E2) = kQ^2 \langle J_i K 2 0 | j_f K \rangle^2$$

$$B(E2) \; \alpha \; \frac{3J(J-1)(J+1)}{(2J-2)(2J-1)(2J+1)} \qquad \sim 3/8$$



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For ('real') examples, see
J. Kern et al., Nucl. Phys. **A593** (1995) 21

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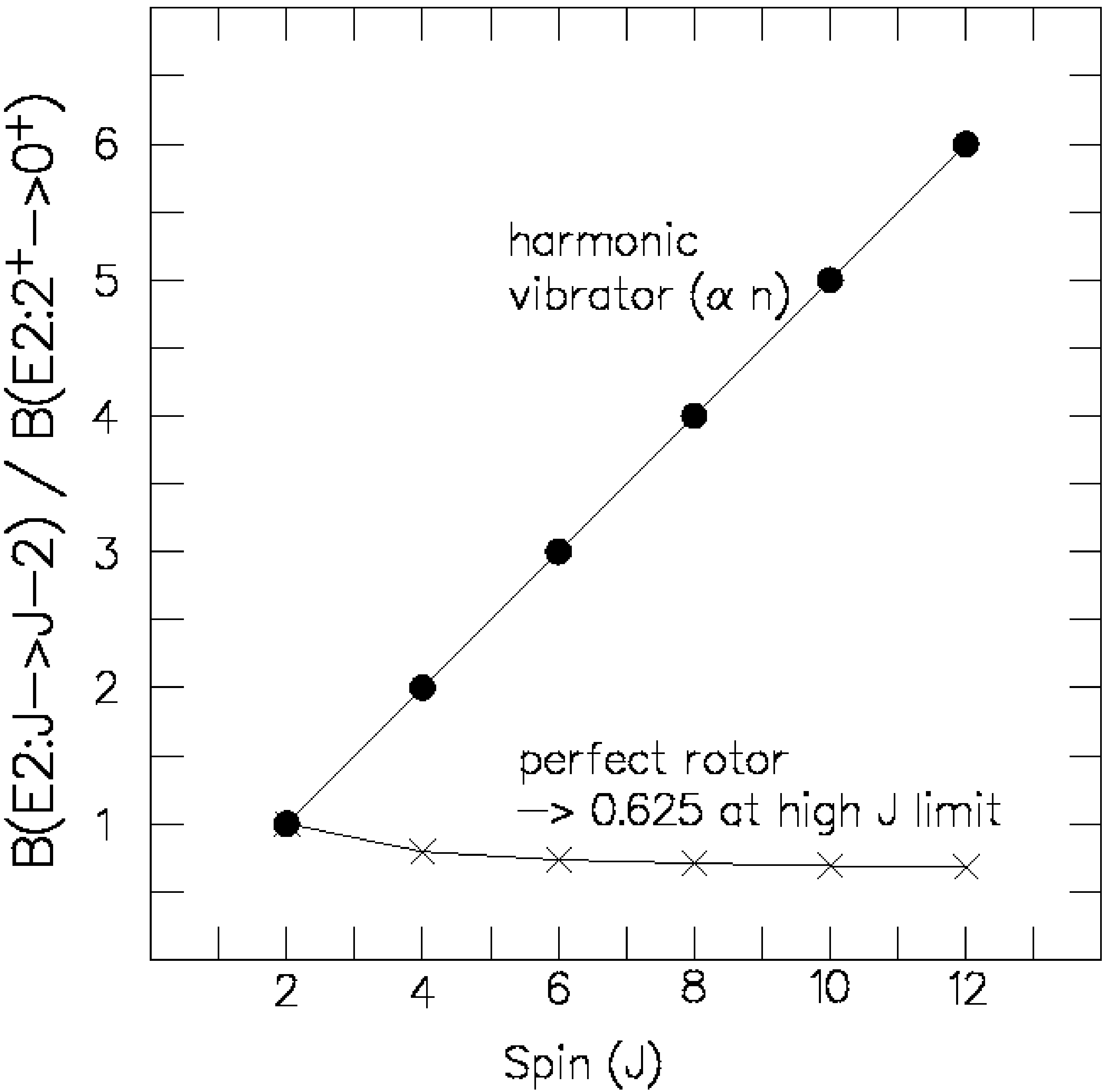
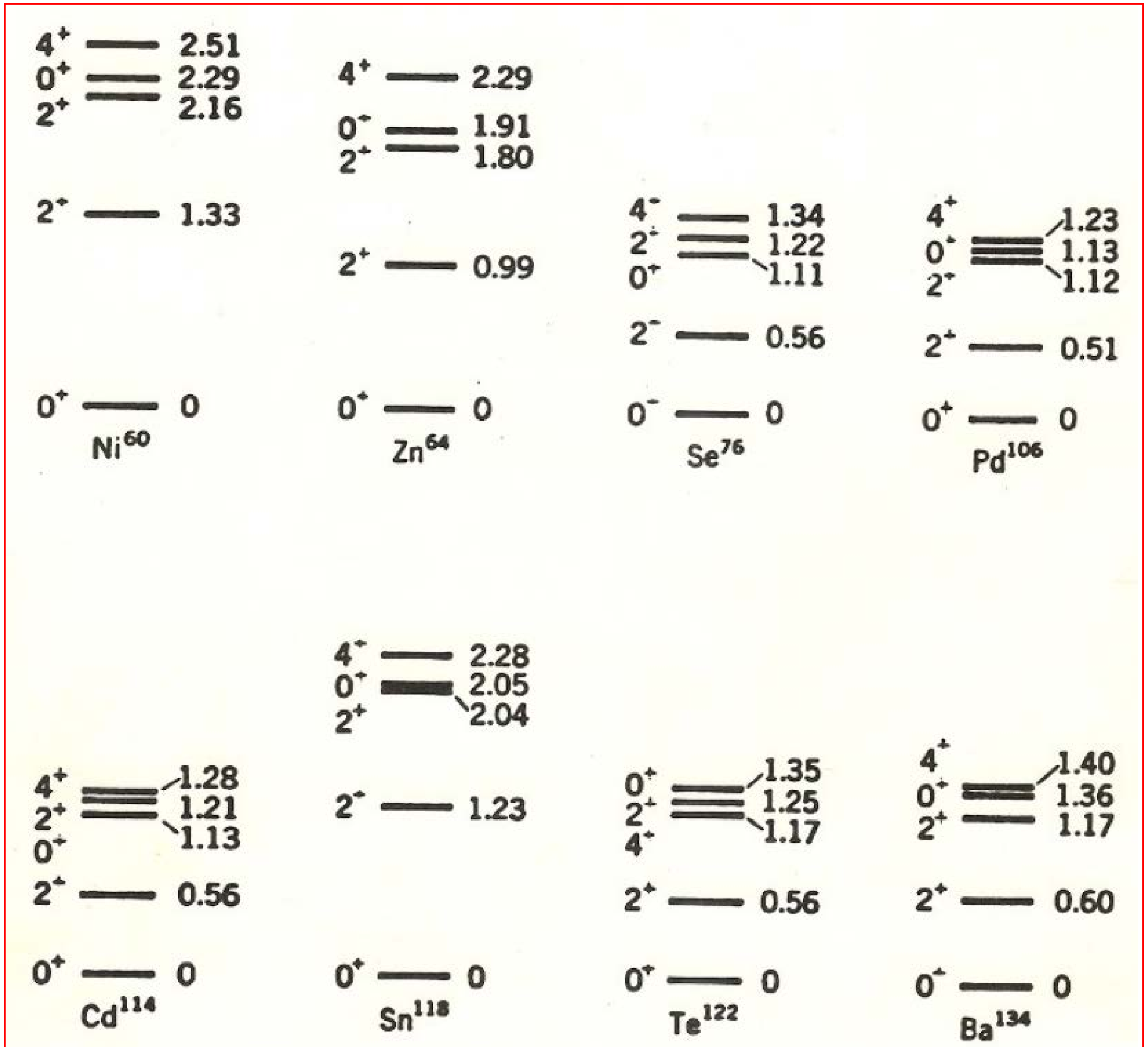
$(\hbar^2/_{2I})(6J-6)$; $\Delta E\gamma=(\hbar^2/_{2I})*12=\text{const.}$


B(E2:J−>J−2) / B(E2:2+−>0+)
Spin (J)
harmonic
vibrator (α n)
perfect rotor
−> 0.625 at high J limit
1
2
3
4
5
6
2
4
6
8
10
12

So, what about ‘real’ nuclei ?

Many nuclei with R(4/2)~2.0 also show $I^{\pi}=4^+,2^+,0^+$ triplet states at ~2E(2^+).



First Observation of a Near-Harmonic Vibrational Nucleus

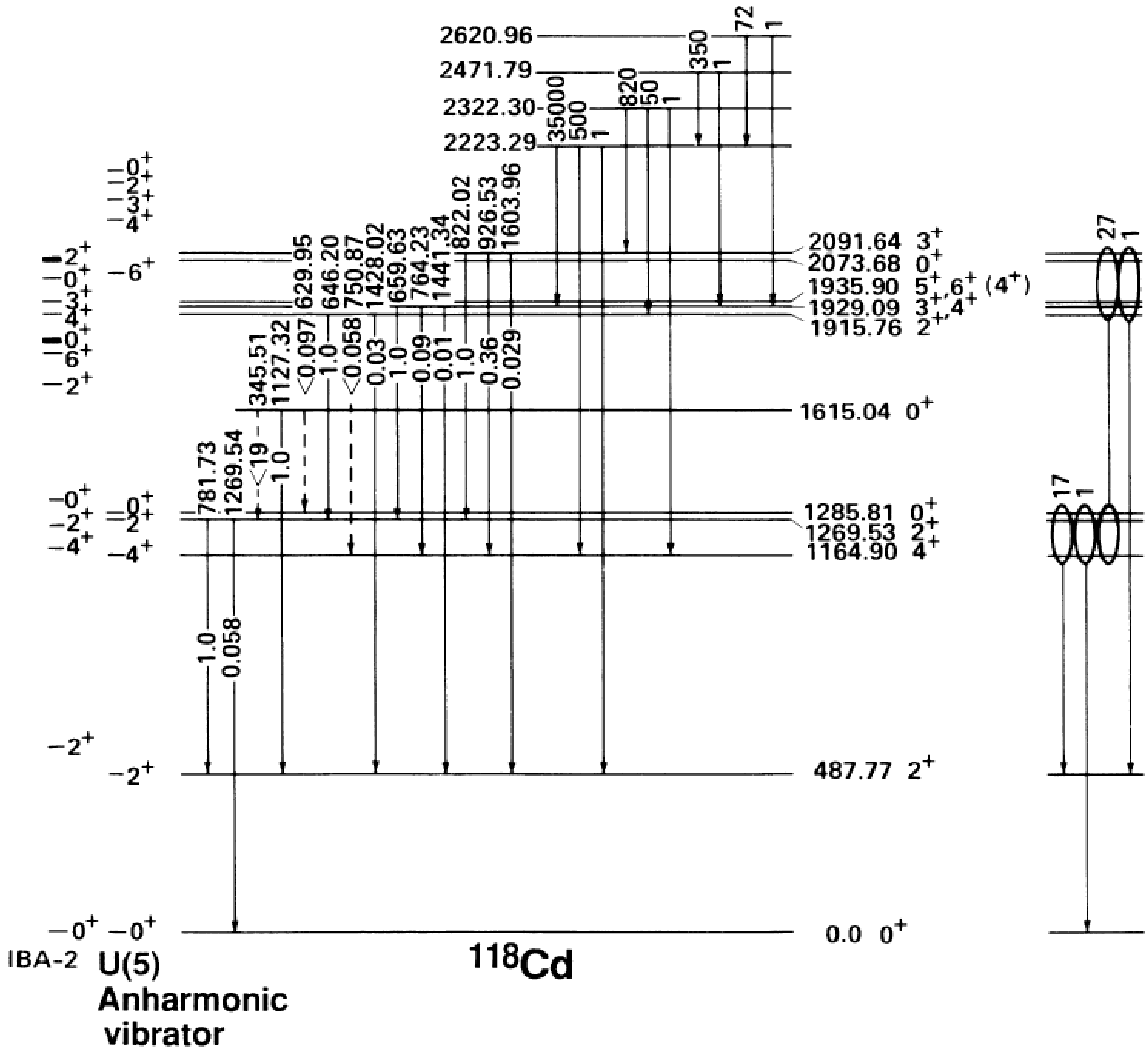
A. Aprahamian
Clark University, Worcester, Massachusetts 01610, and
Lawrence Livermore National Laboratory, Livermore, California 94550

D. S. Brenner
Clark University, Worcester, Massachusetts 01610

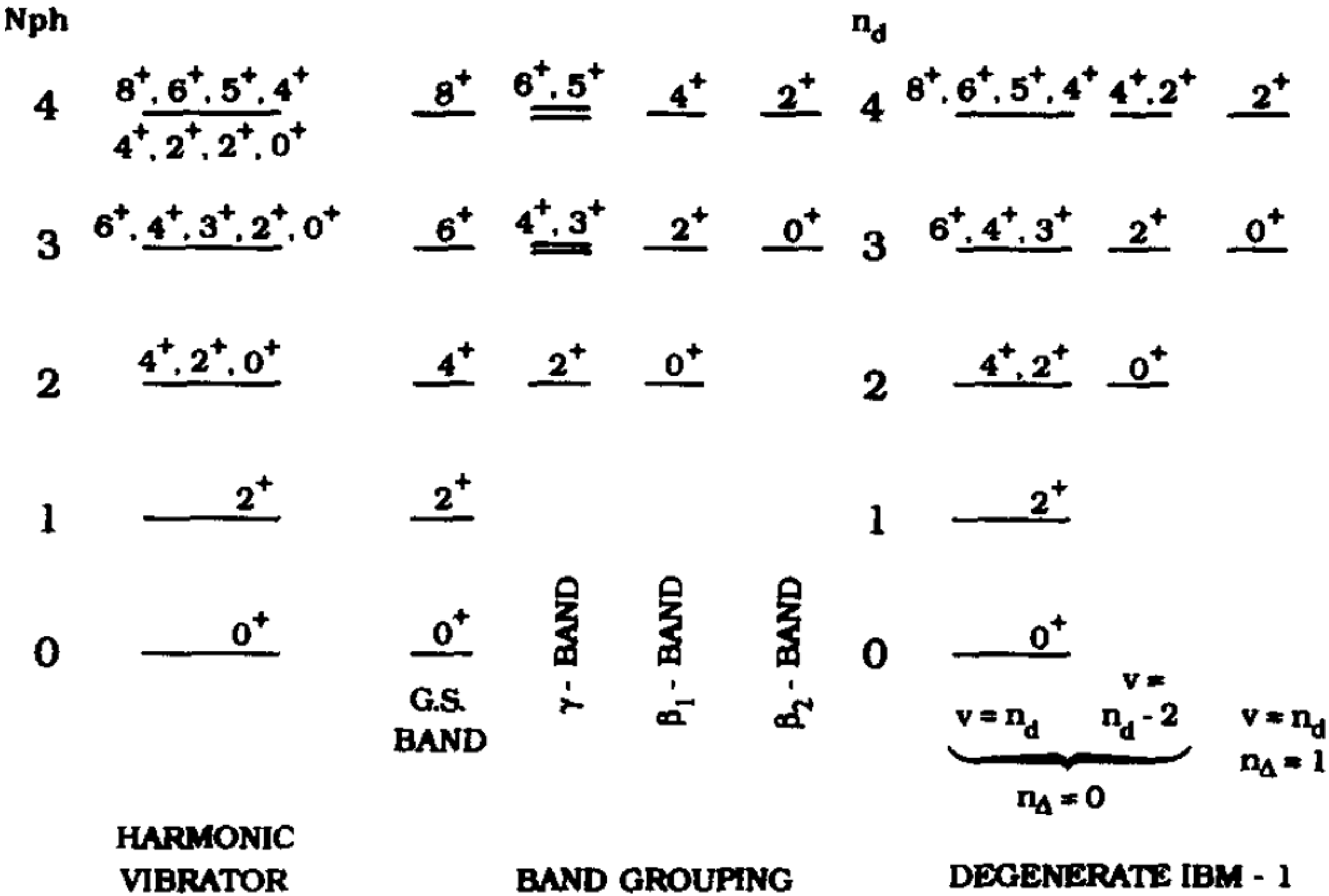
and

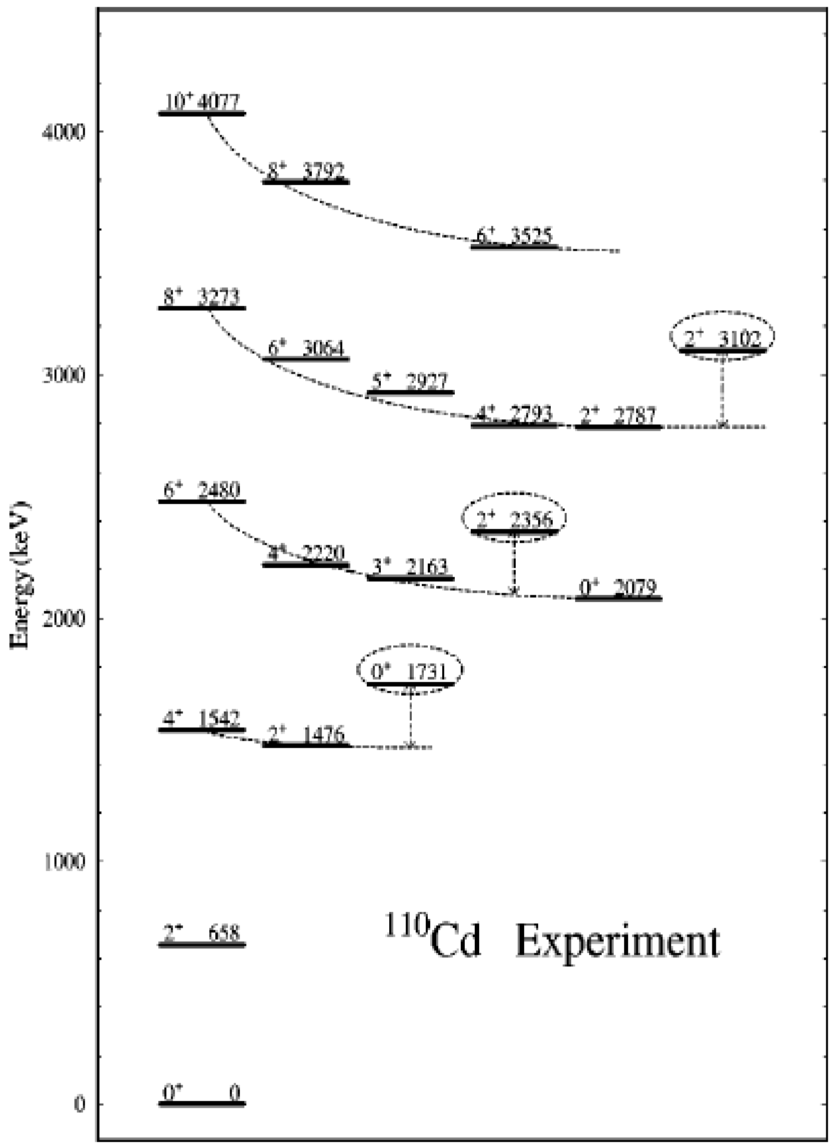
R. F. Casten, R. L. Gill, and A. Piotrowski[(a)]
Brookhaven National Laboratory, Upton, New York 11973
(Received 4 May 1987)

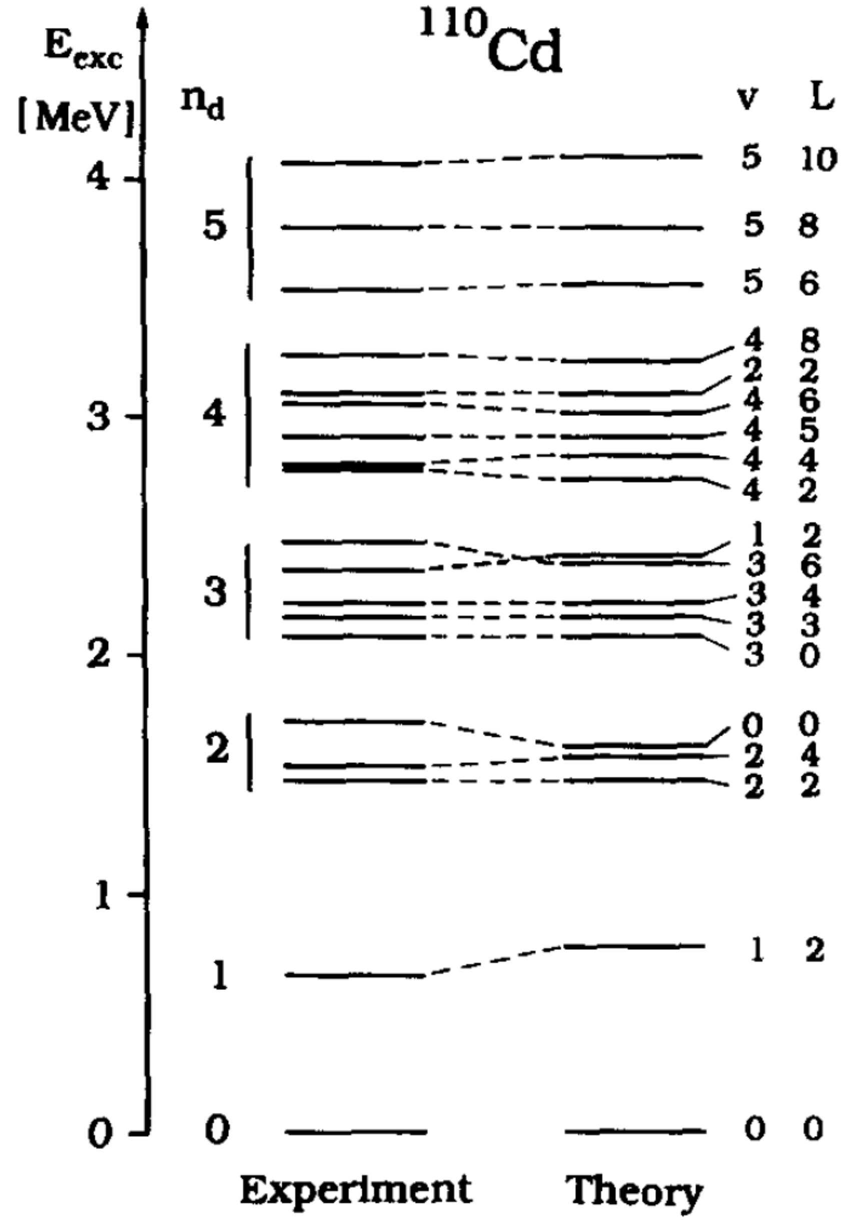
Evidence is presented for five closely spaced states in ^{118}Cd near $E_{ex} \simeq 2$ MeV, which are interpreted as a near-harmonic three-phonon quintuplet. Candidates for even higher-lying multiphonon states are also found. The experimental spectrum is compared with an anharmonic vibrator [or U(5) spectrum] and an interacting-boson-approximation calculation incorporating a two-particle, four-hole intruder configuration.



2620.96
2471.79
2322.30
2223.29
2091.64 3+
2073.68 0+
1935.90 5+,6+ (4+)
1929.09 3+,4+
1915.76 2+
1615.04 0+
1285.81 0+
1269.53 2+
1164.90 4+
487.77 2+
0.0 0+
IBA-2
U(5)
Anharmonic
vibrator
118Cd


Nph
nd
4
8+, 6+, 5+, 4+
4+, 2+, 2+, 0+
3
6+, 4+, 3+, 2+, 0+
2
4+, 2+, 0+
1
2+
0
0+
8+
6+, 5+
4+
2+
6+
4+, 3+
2+
0+
4+
2+
0+
2+
0+
G.S. BAND
γ - BAND
β1 - BAND
β2 - BAND
8+, 6+, 5+, 4+
4+, 2+
2+
6+, 4+, 3+
2+
0+
4+, 2+
0+
2+
0+
v = nd
v = nd - 2
nΔ = 0
v = nd
nΔ = 1
HARMONIC VIBRATOR
BAND GROUPING
DEGENERATE IBM - 1

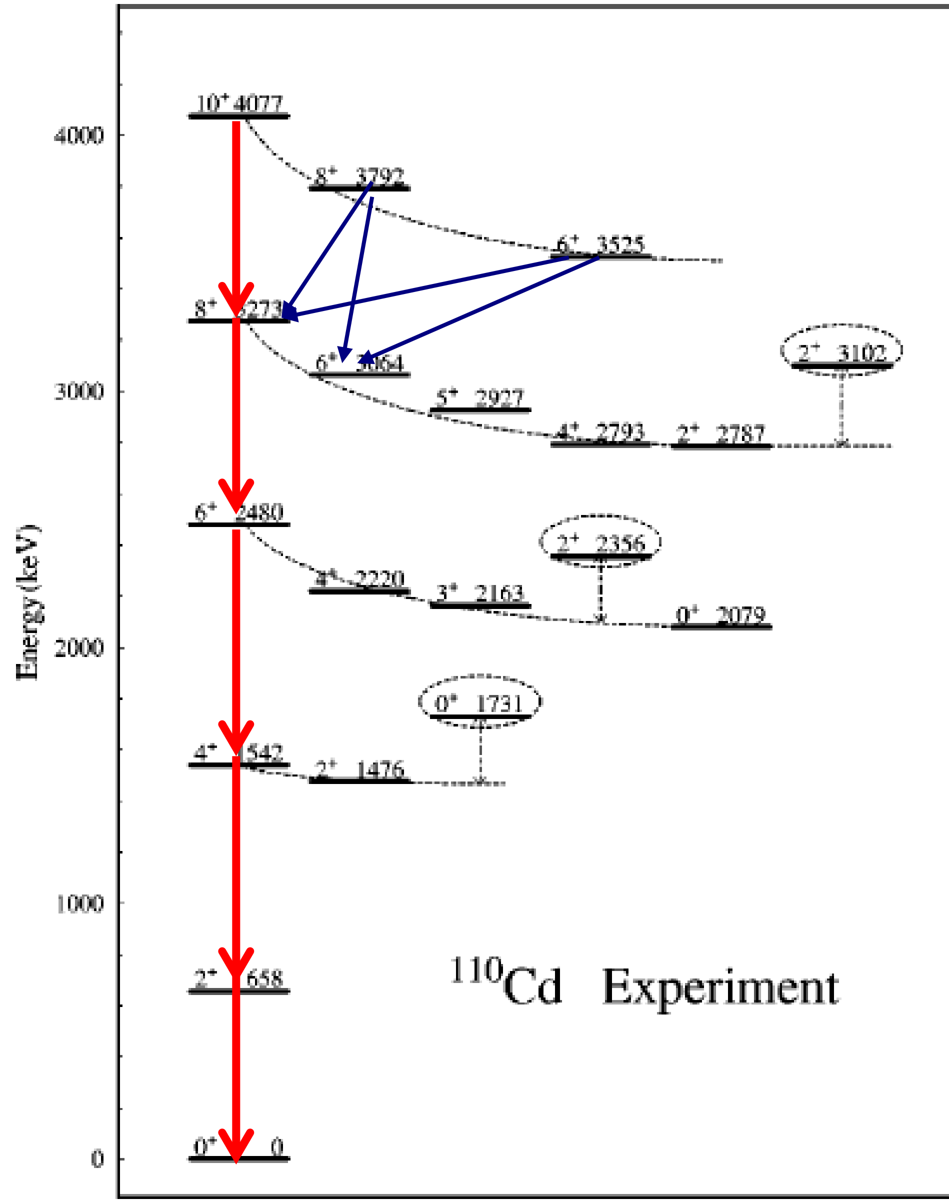

10+ 4077
8+ 3792
6+ 3525
8+ 3273
6+ 3064
5+ 2927
4+ 2793
2+ 2787
2+ 3102
6+ 2480
4+ 2220
3+ 2163
2+ 2356
0+ 2079
0+ 1731
4+ 1542
2+ 1476
2+ 658
0+ 0
Energy (keV)
110Cd Experiment


110Cd
Eexc
[MeV]
nd
v L
Experiment
Theory

Note on 'near-yrast feeding' for vibrational states in nuclei.

If 'vibrational' states are populated in very high-spin reactions (such as heavy ion induced fusion evaporation reactions), only the decays between the (near)-YRAST states are likely to be observed.

The effect is to (only?) see the 'stretched' E2 cascade from $J_{max} \rightarrow J_{max}$-2 for each phonon multiplet.



↓ = the 'yrast' stretched E2 cascade.

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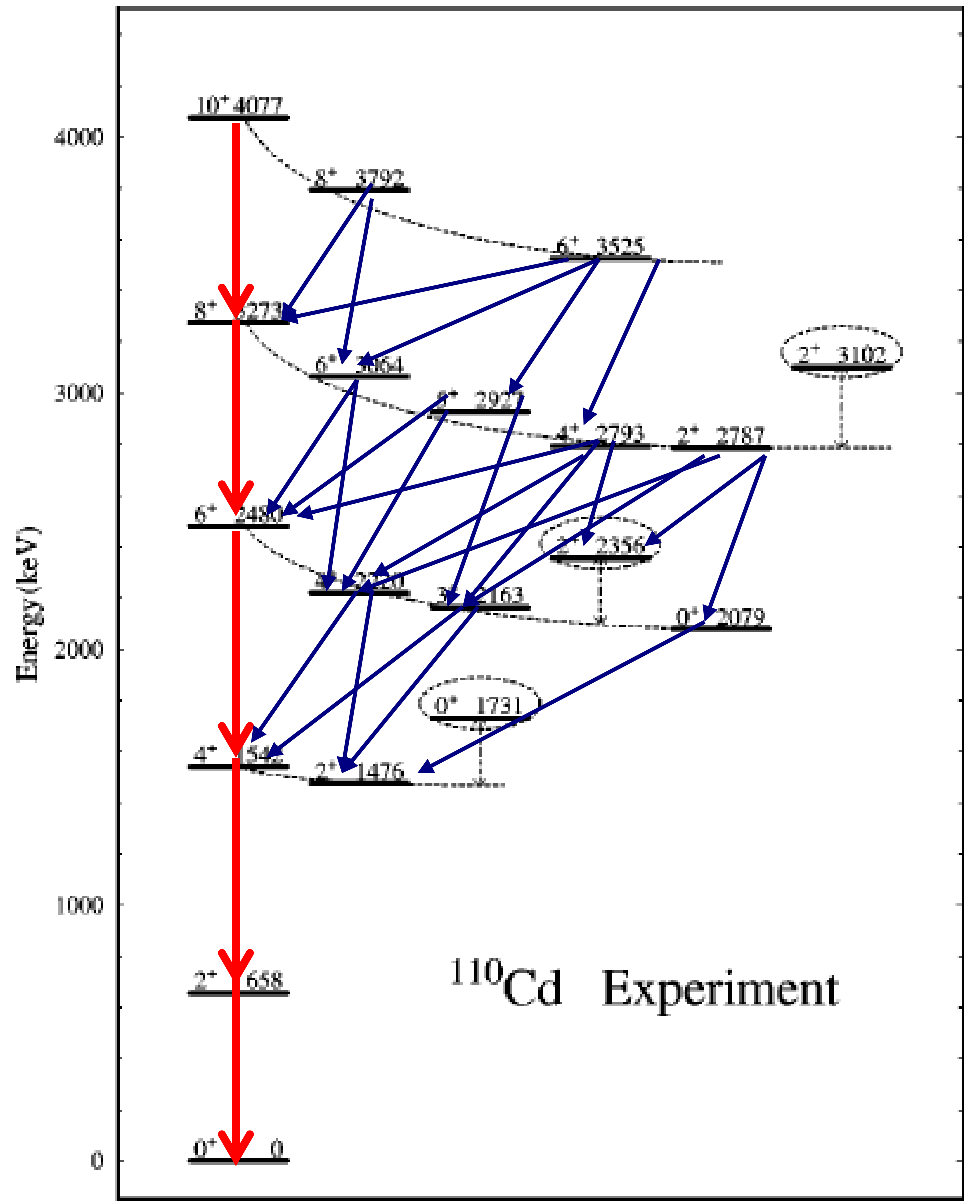
= the 'yrast' stretched E2 cascade.

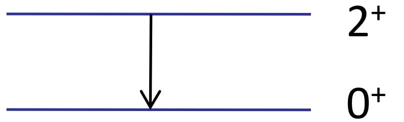


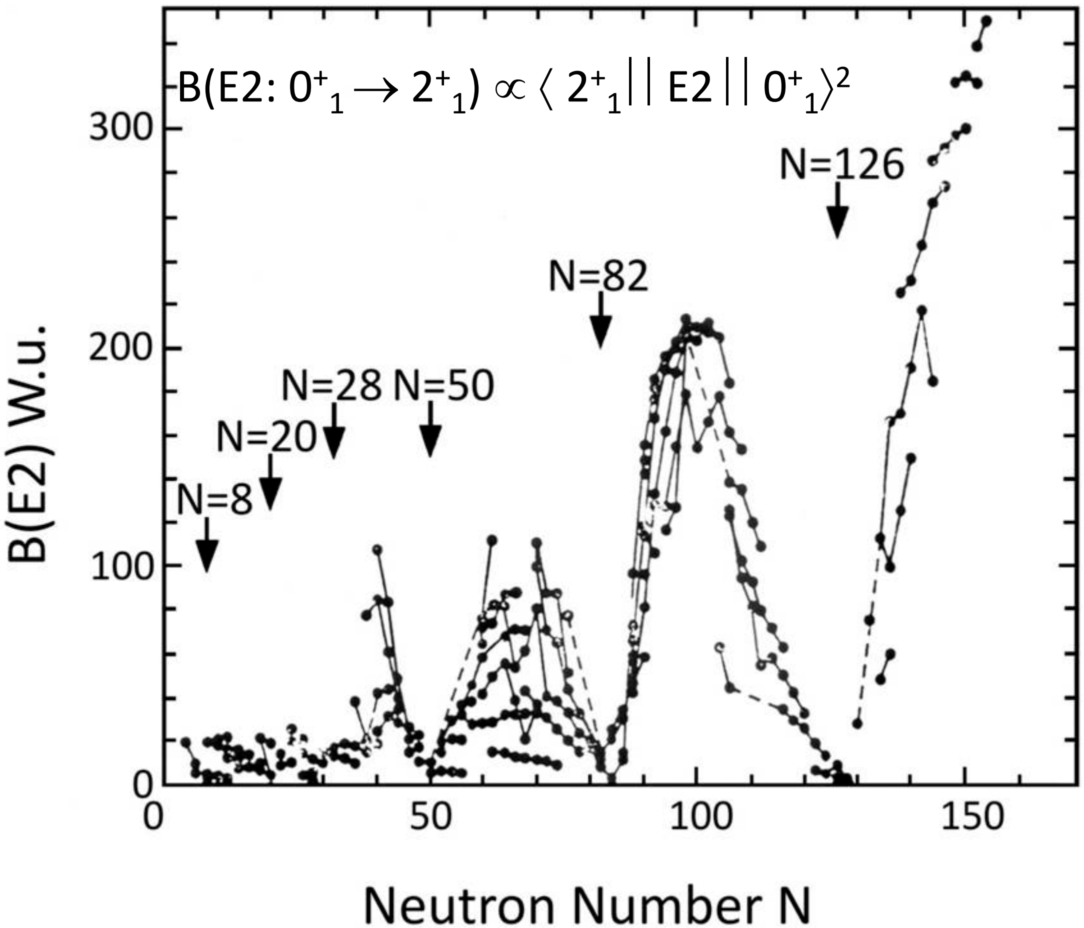
Nuclear Rotations and Static Quadrupole Deformation

$$T(E2) = 1.223 \times 10^9 E_\gamma^5 B(E2)$$

T (E2) = transition probability = 1/τ (secs);

E_γ = transition energy in MeV





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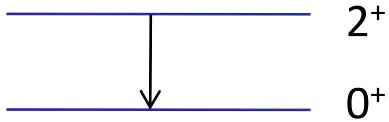
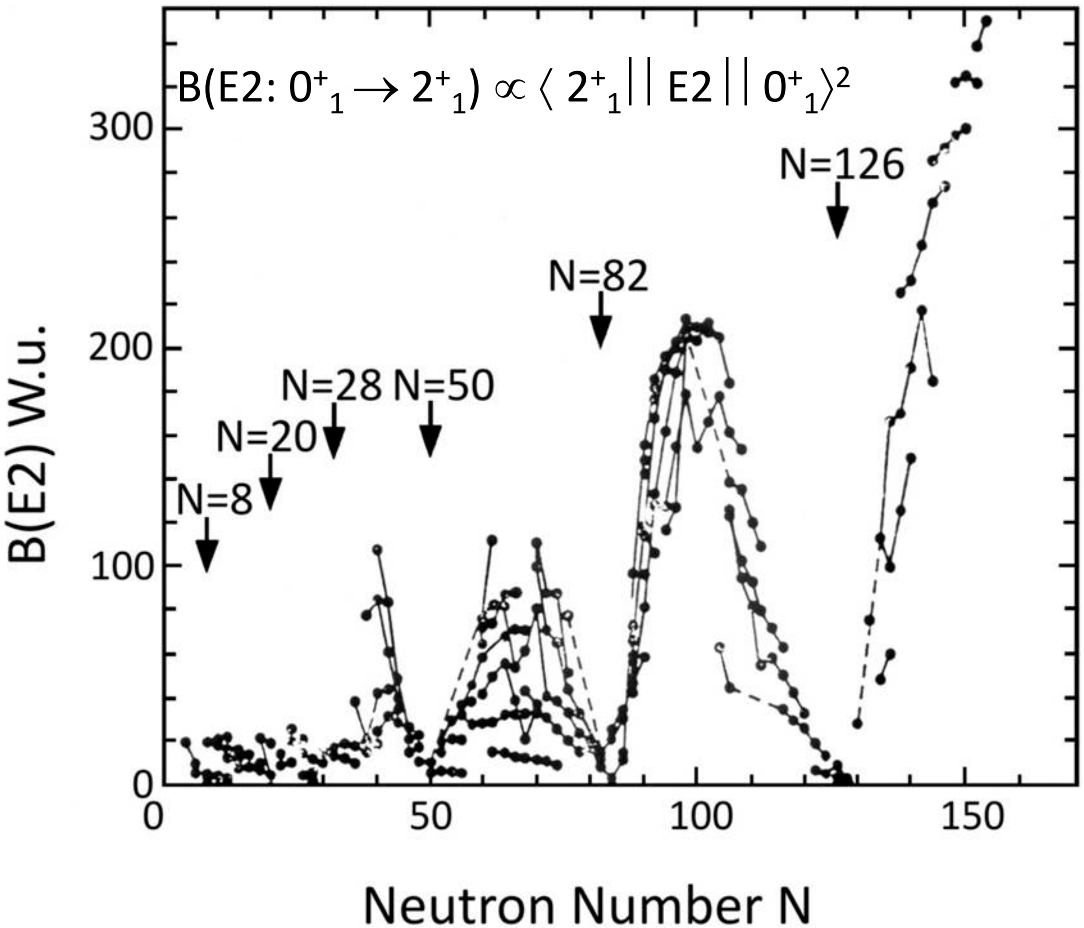




Qo = INTRINSIC (TRANSITION) ELECTRIC QUADRUPOLE MOMENT.

This is intimately linked to the electrical charge (i.e. proton) distribution within the nucleus.

Non-zero Qo means some deviation from spherical symmetry and thus some quadrupole 'deformation'.

Rotational model, B(E2: I→I-2) gives:

$$B(E2) = \frac{5}{16\pi} Q_o^2 \frac{3(I-K)(I-K-1)(I+K)(I+K-1)}{(2J-2)(2J-1)J(2J+1)}$$

Bohr and Mottelson, Phys. Rev. **90**, 717 (1953)

Isomer spin in ^{180}Hf, I^{π}>11 shown later to be $I^{\pi}=K^{\pi}=8^{-}$ by Korner et al. Phys. Rev. Letts. **27**, 1593 (1971)).

K-value very important in understanding isomers.

$E_x = (\hbar^2/_{2I})^*J(J+1)$

I = moment of inertia.
This depends on nuclear deformation and I~ kMR^2
Thus, I ~ kA$^{5/3}$
(since r_{nuc}=1.2A$^{1/3}$fm)

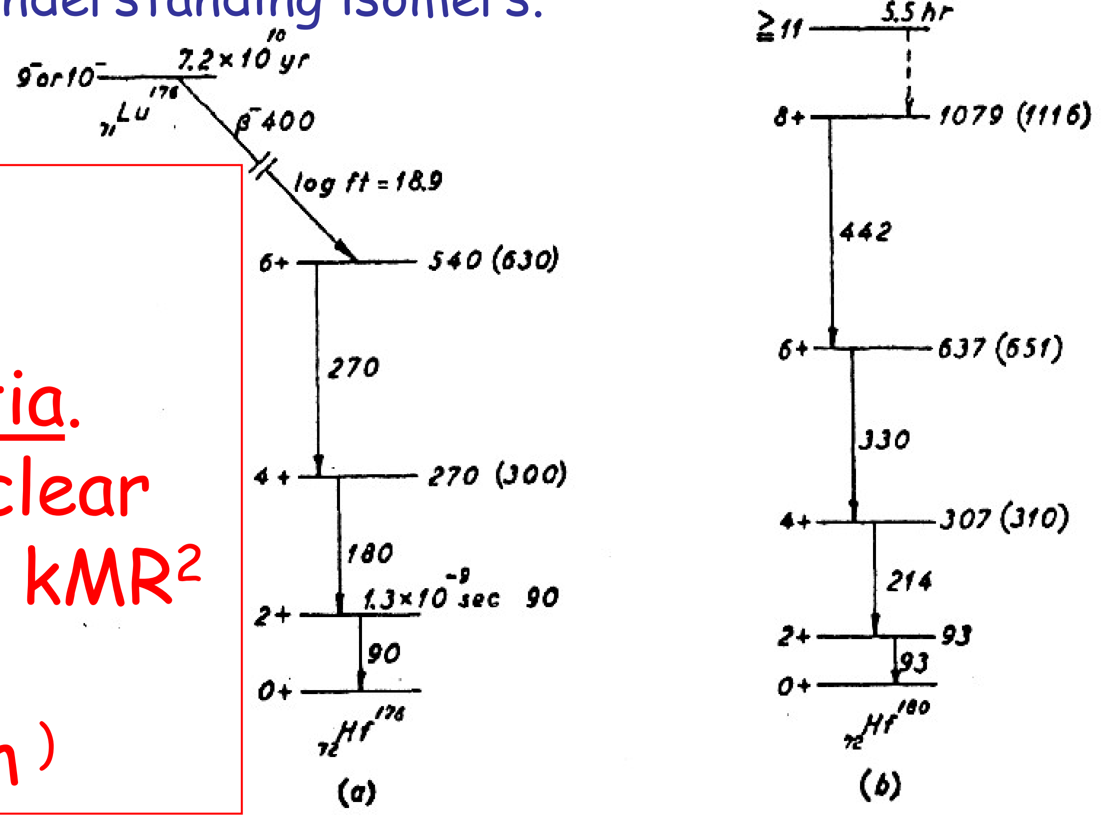
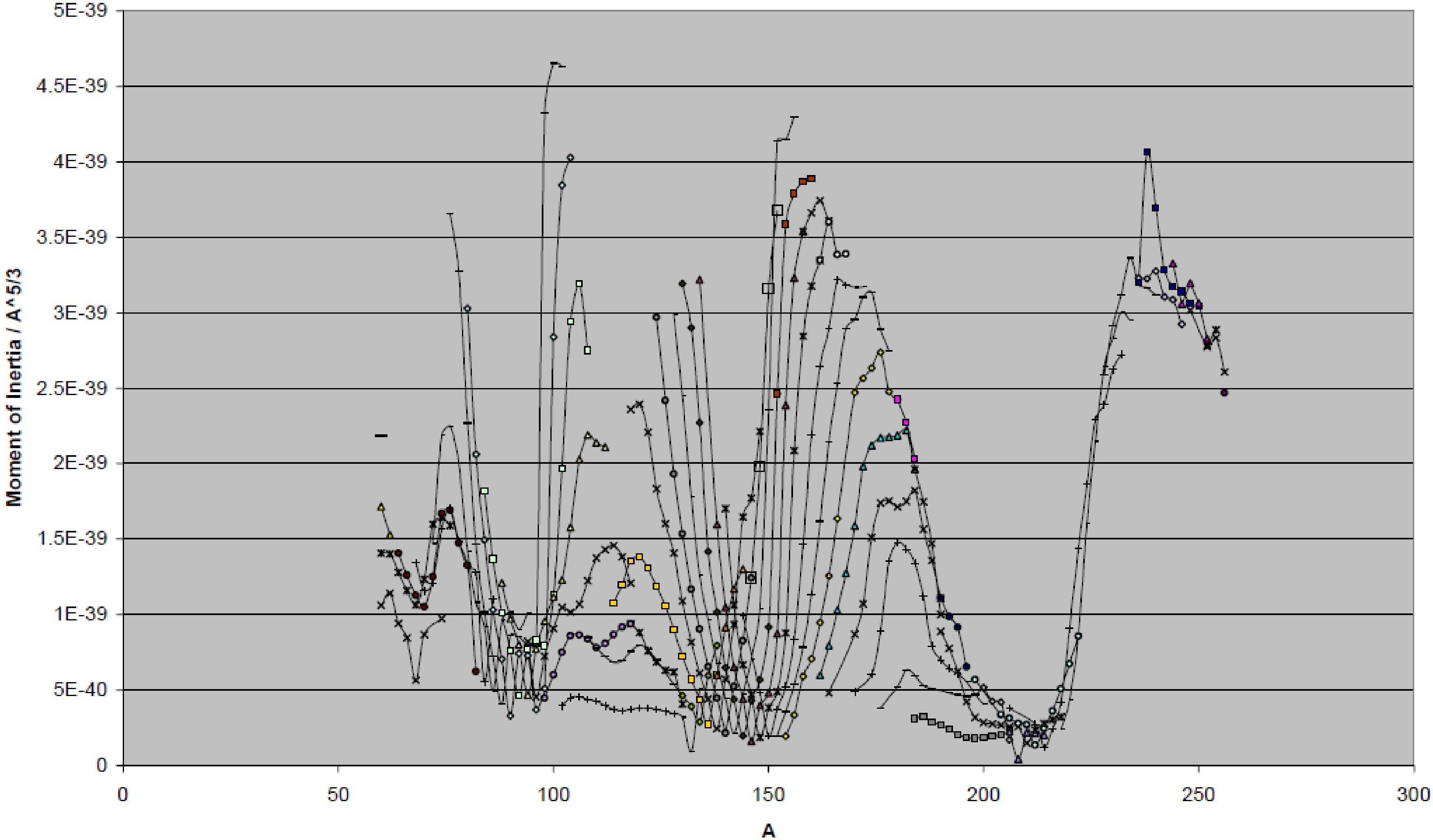
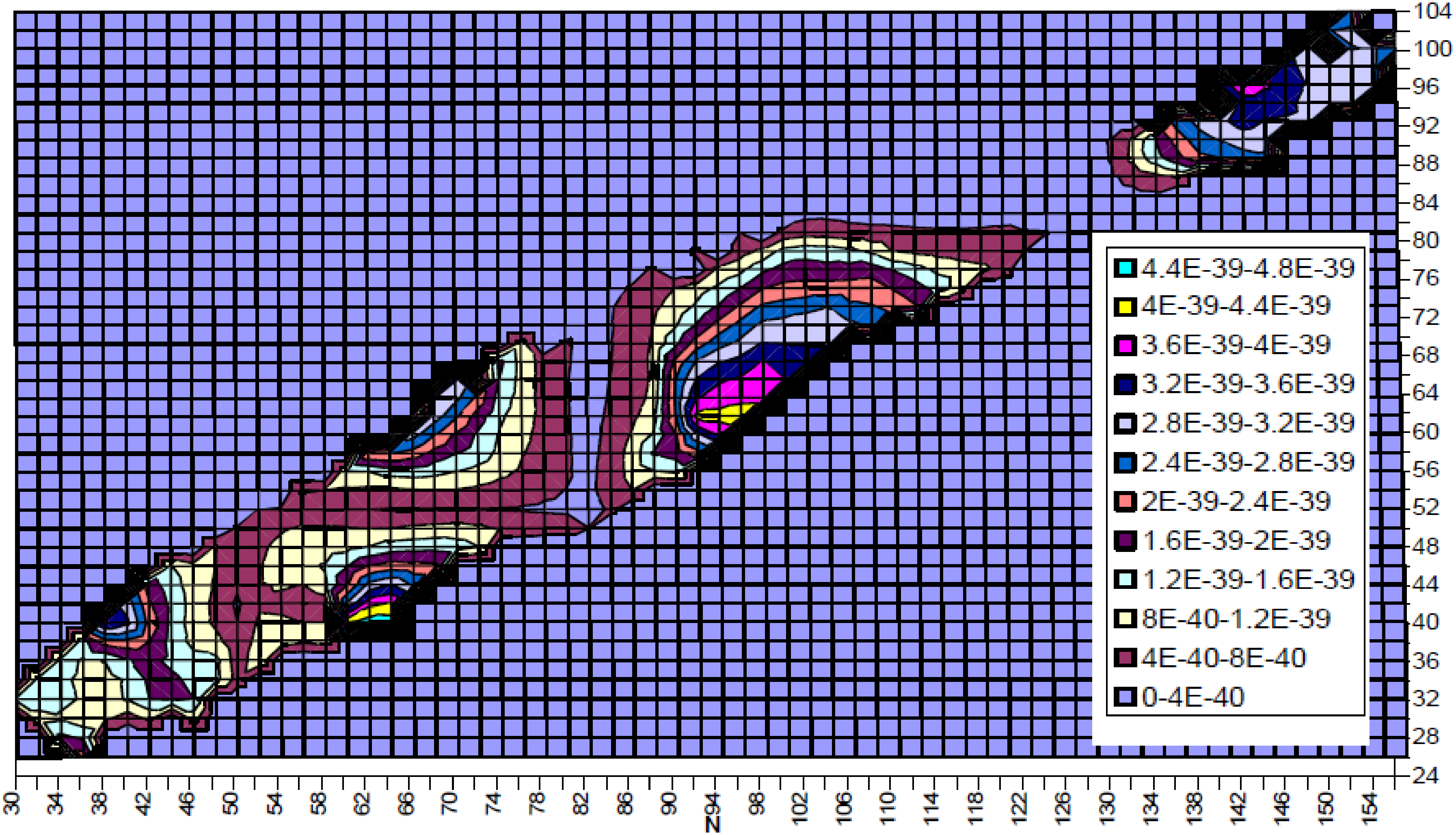


FIG. 2. Suggested decay schemes of $_{71}$Lu^{176} and $_{72}$Hf^{180}. The gamma-ray energies as well as the β-decay data of Lu^{176} are taken from reference 2. The excitation energies listed in parentheses are obtained from Eq. (1) adjusted to give the energy of the first excited state. All energies are given in kev.

Therefore, plotting the moment of inertia, divided by $A^{5/3}$ should give a comparison of nuclear deformations across chains of nuclei and mass regions….

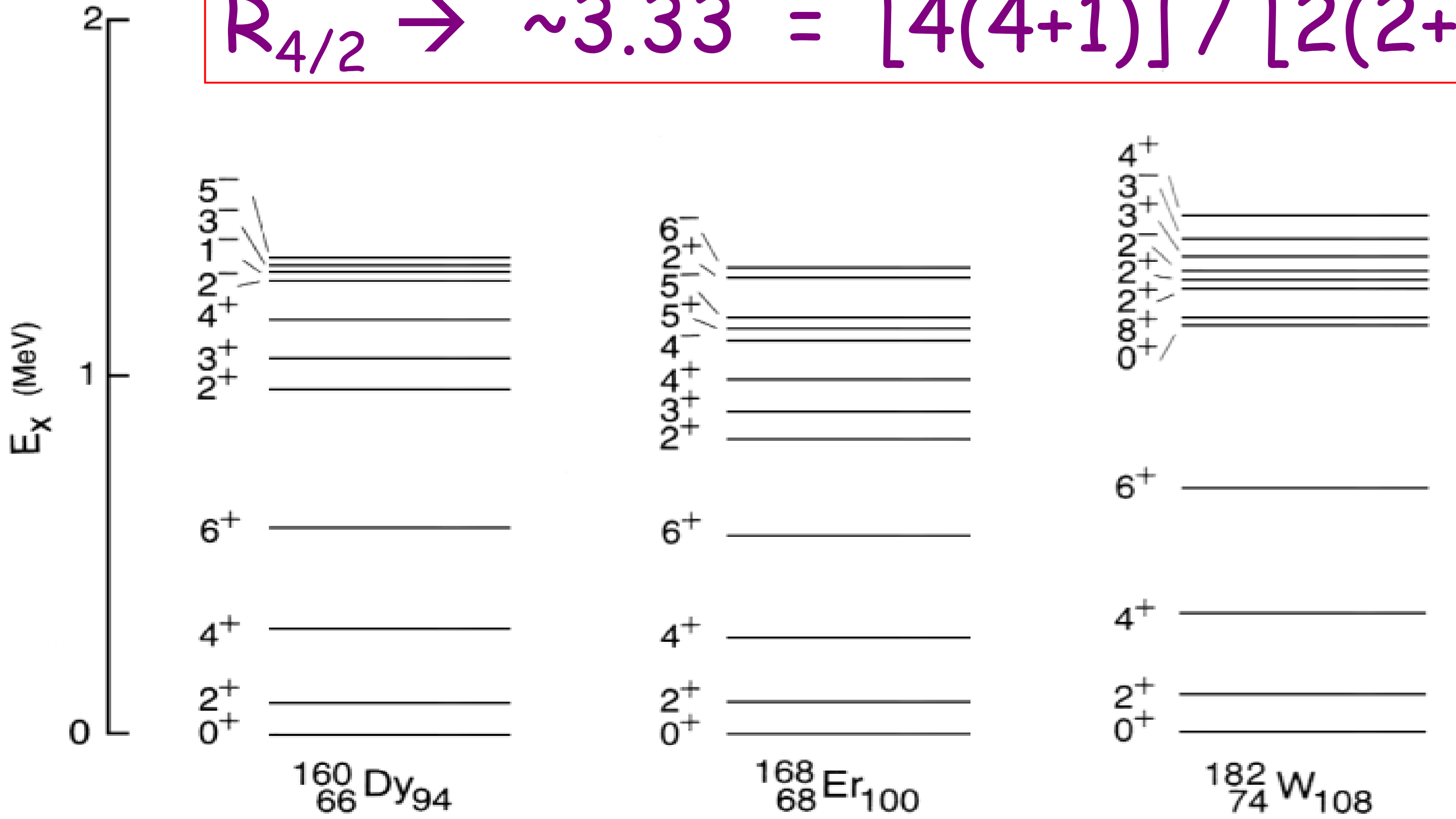


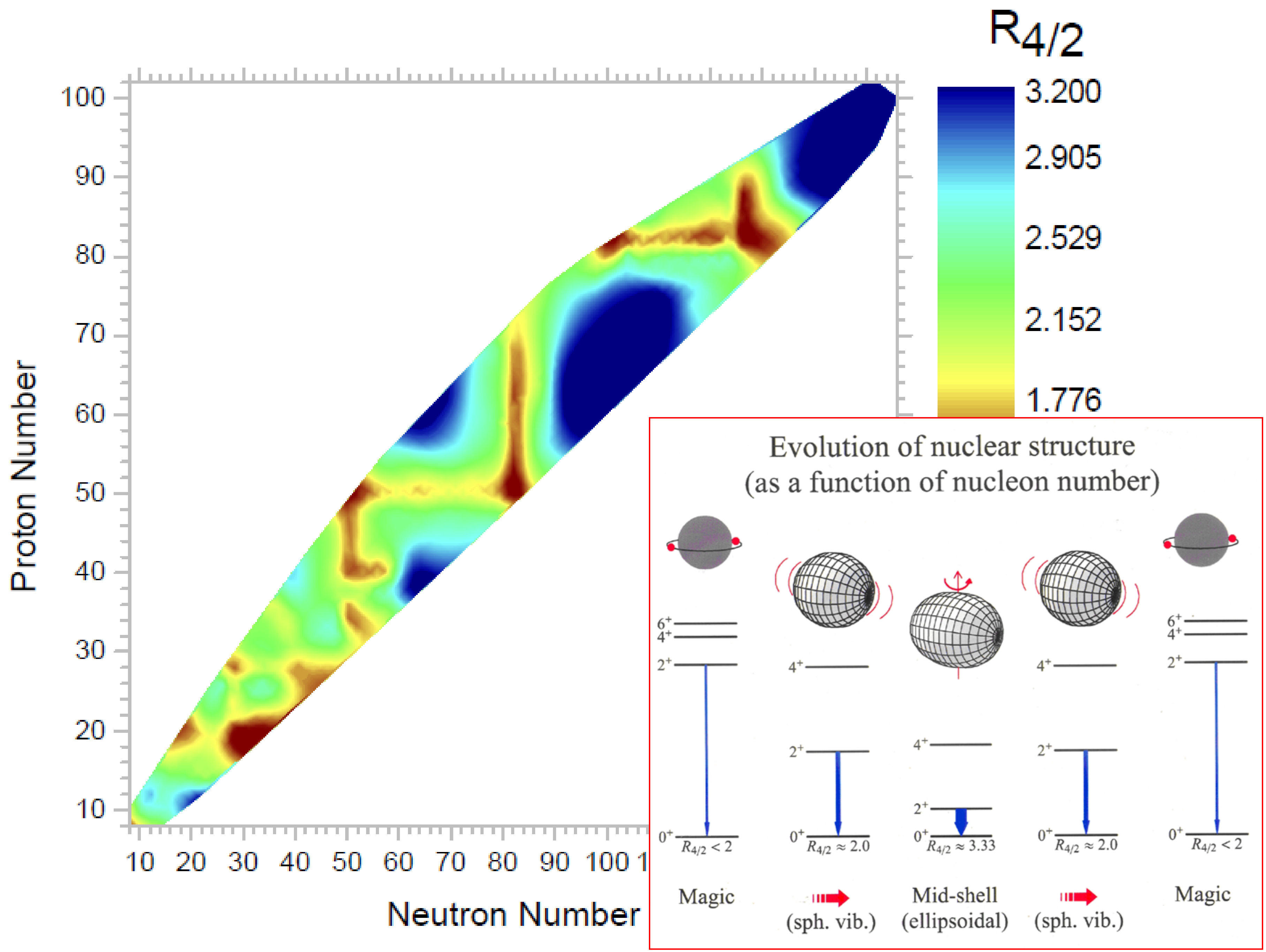


Nuclear static moment of inertia for E(2+) states divided by $A^{5/3}$ trivial mass dependence.
Should show regions of quadrupole deformation.

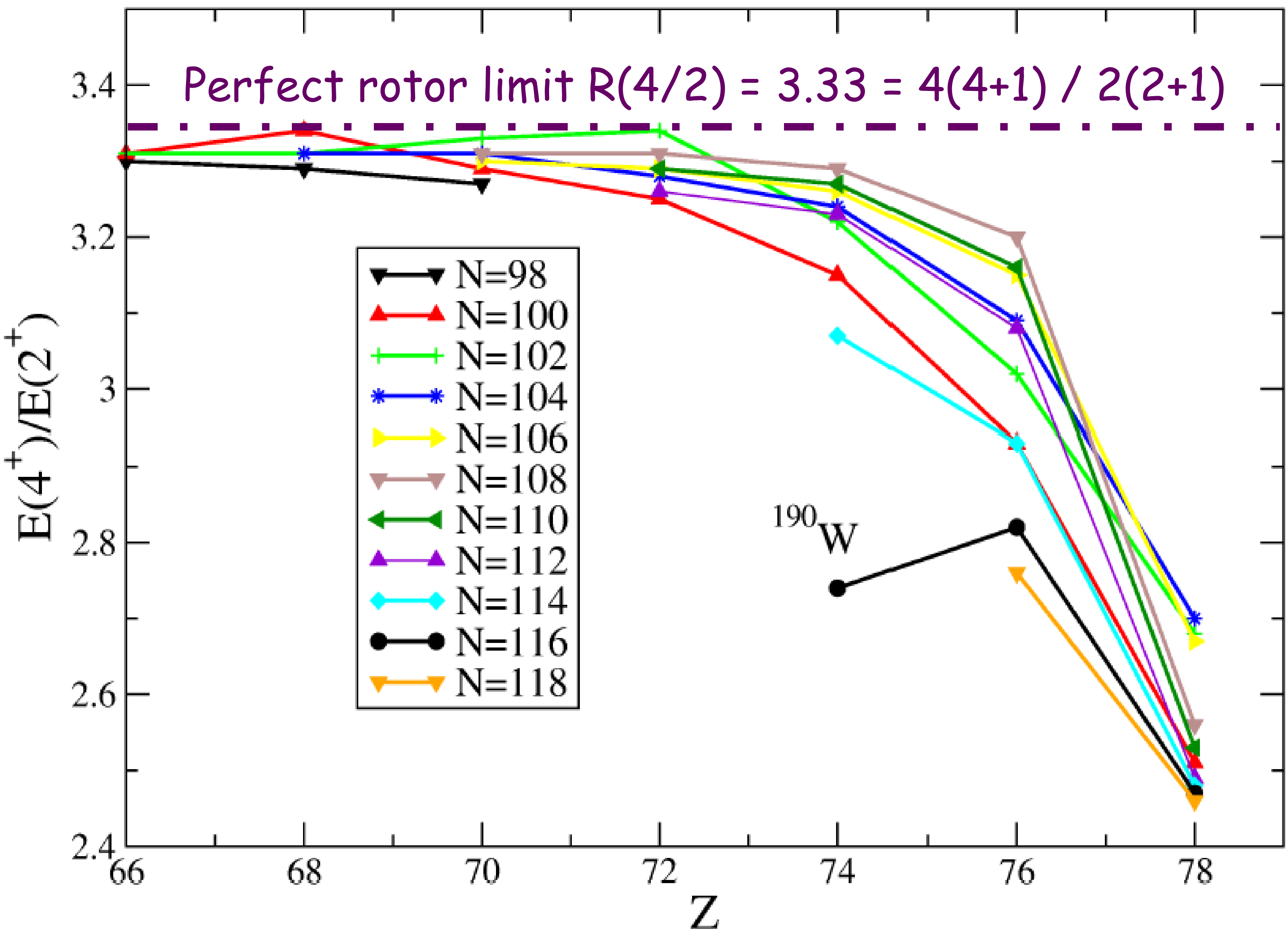
Lots of valence nucleons of both types: emergence of deformation and therefore rotation

$$R_{4/2} \rightarrow \sim 3.33 = [4(4+1)] / [2(2+1)]$$





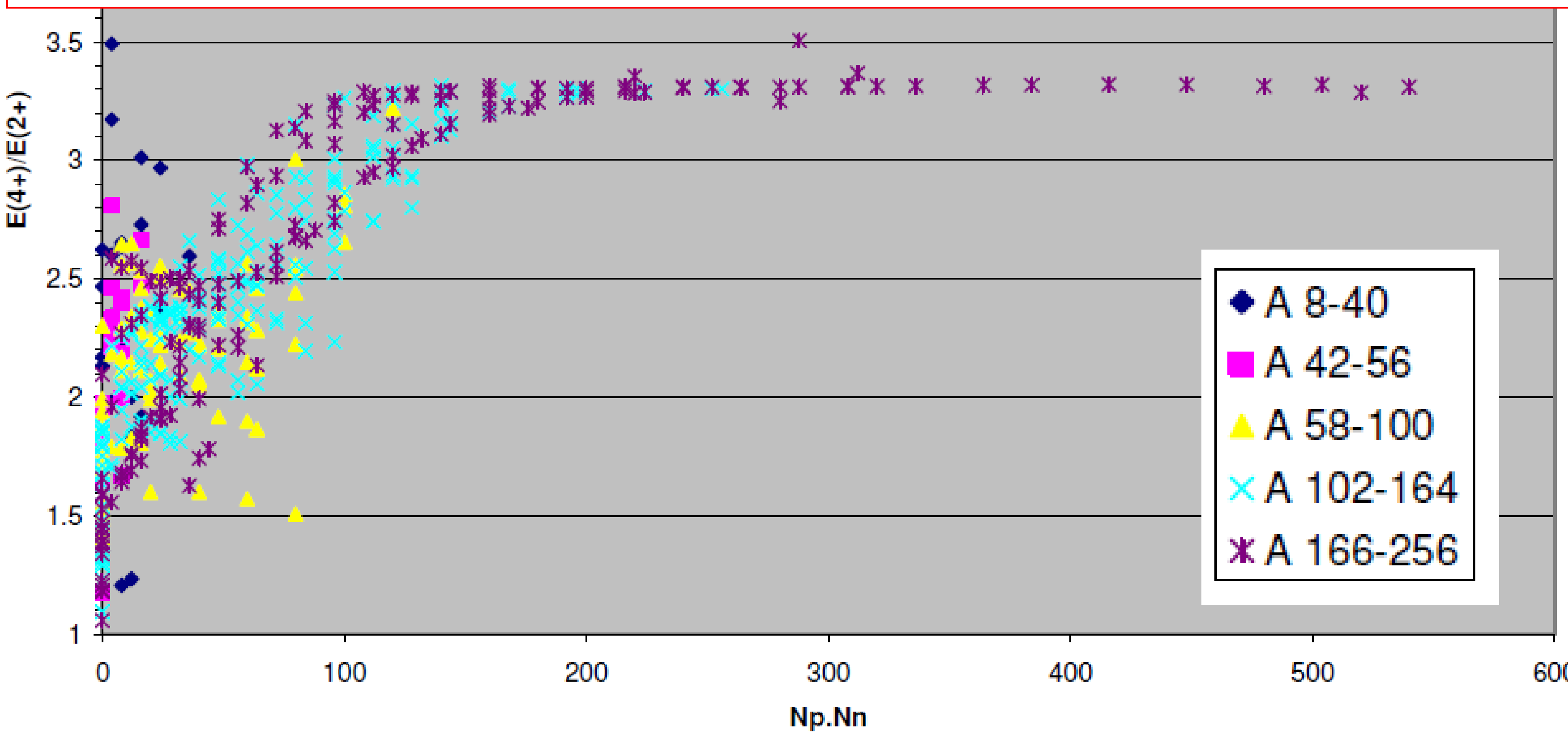
R4/2
3.200
2.905
2.529
2.152
1.776
Proton Number
Neutron Number
Evolution of nuclear structure
(as a function of nucleon number)
Magic
(sph. vib.)
Mid-shell
(ellipsoidal)
(sph. vib.)
Magic
R4/2 < 2
R4/2 ≈ 2.0
R4/2 ≈ 3.33
R4/2 ≈ 2.0
R4/2 < 2



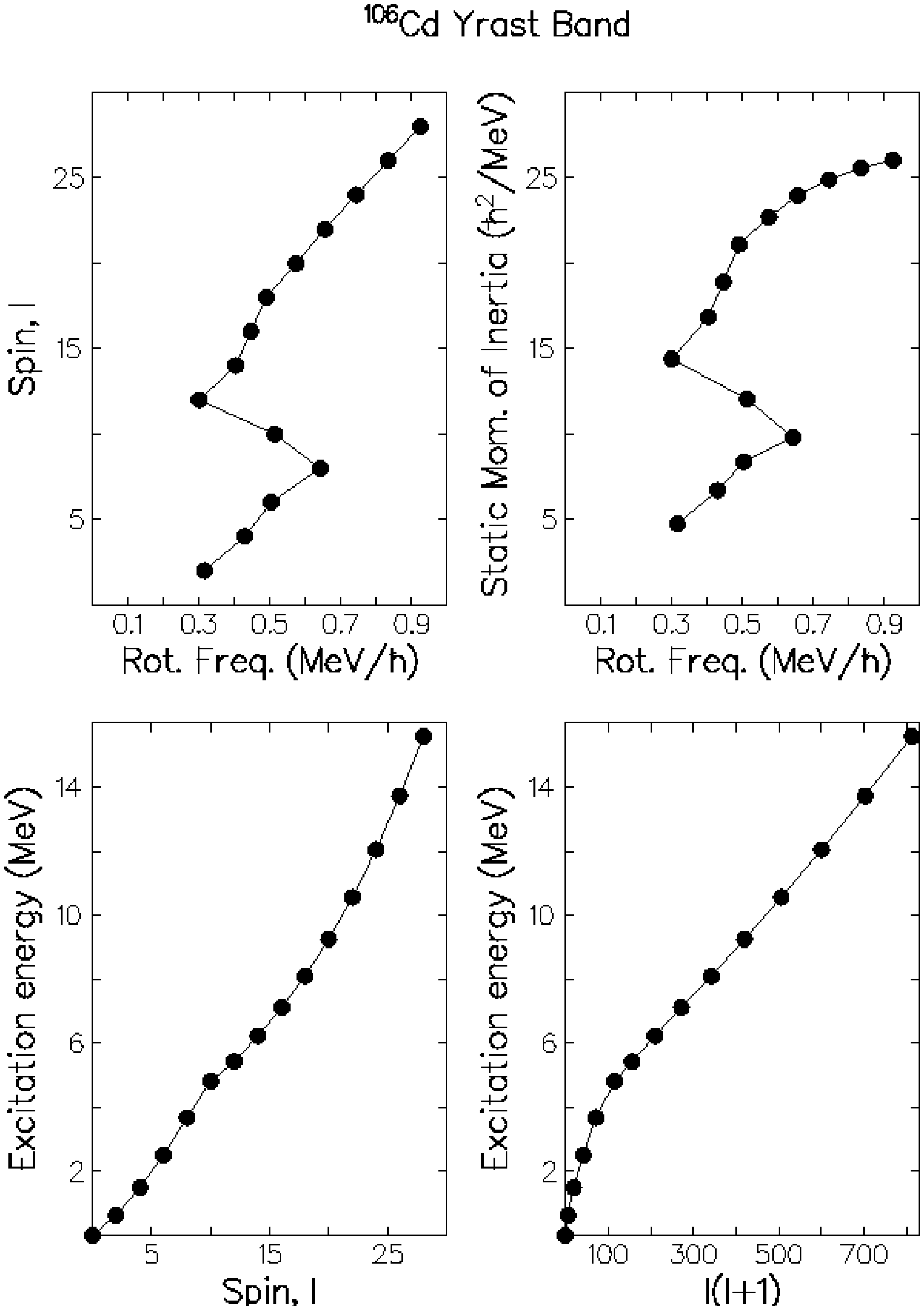
Perfect rotor limit R(4/2) = 3.33 = 4(4+1) / 2(2+1)
E(4+)/E(2+)
Z
N=98
N=100
N=102
N=104
N=106
N=108
N=110
N=112
N=114
N=116
N=118
190W
3.4
3.2
3
2.8
2.6
2.4
66
68
70
72
74
76
78

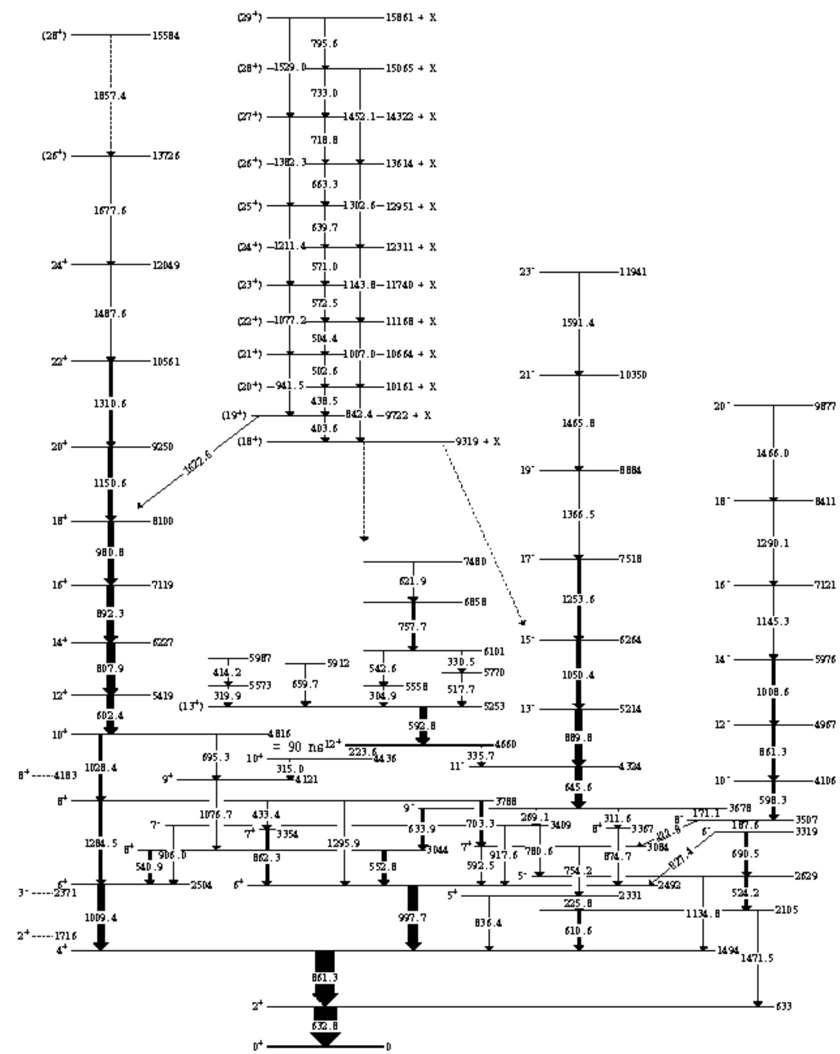
Best nuclear 'rotors' have largest values of Nπ.Nν

This is the product of the number of 'valence' protons, Nπ X the number of valence neutrons Nν



Alignments and rotational motion in ‘vibrational’ ^{106}Cd (Z=48, N=58),





PHR et al. Nucl. Phys. ***A586*** *(1995) p351*

Some useful nuclear rotational, 'pseudo-observables'…

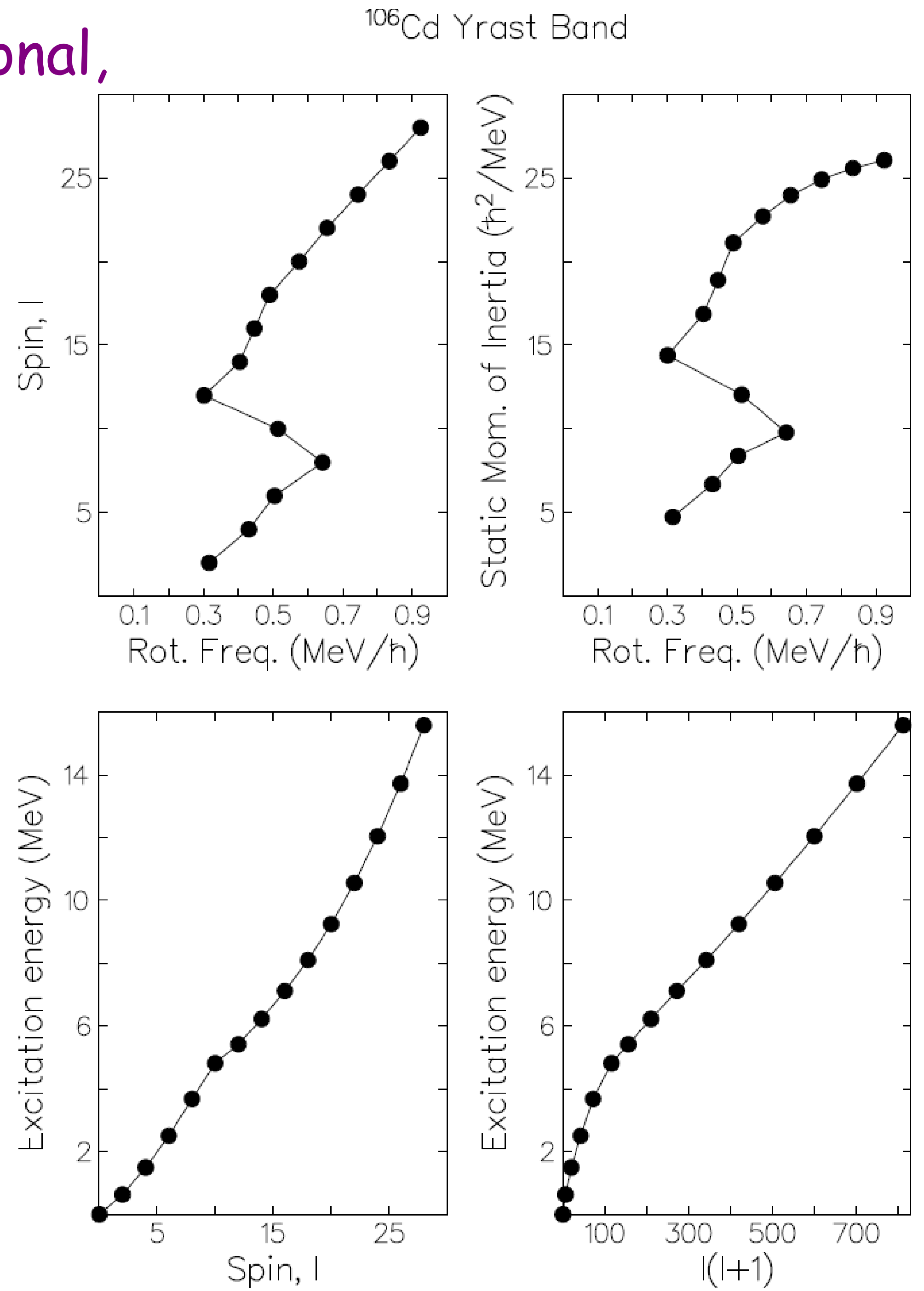
$$E_{rot}(I) = \frac{\hbar^2}{2\mathcal{I}^{(0)}(I)} I(I+1)$$

The kinematic moment of inertia is given by

$$\mathcal{I}^{(1)}(I) = \frac{I}{\omega}$$

while the dynamic moment is given by

$$\mathcal{I}^{(2)} = \frac{dI}{d\omega} \approx \frac{4\hbar}{\Delta E_\gamma}$$



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while the dynamic moment is given by

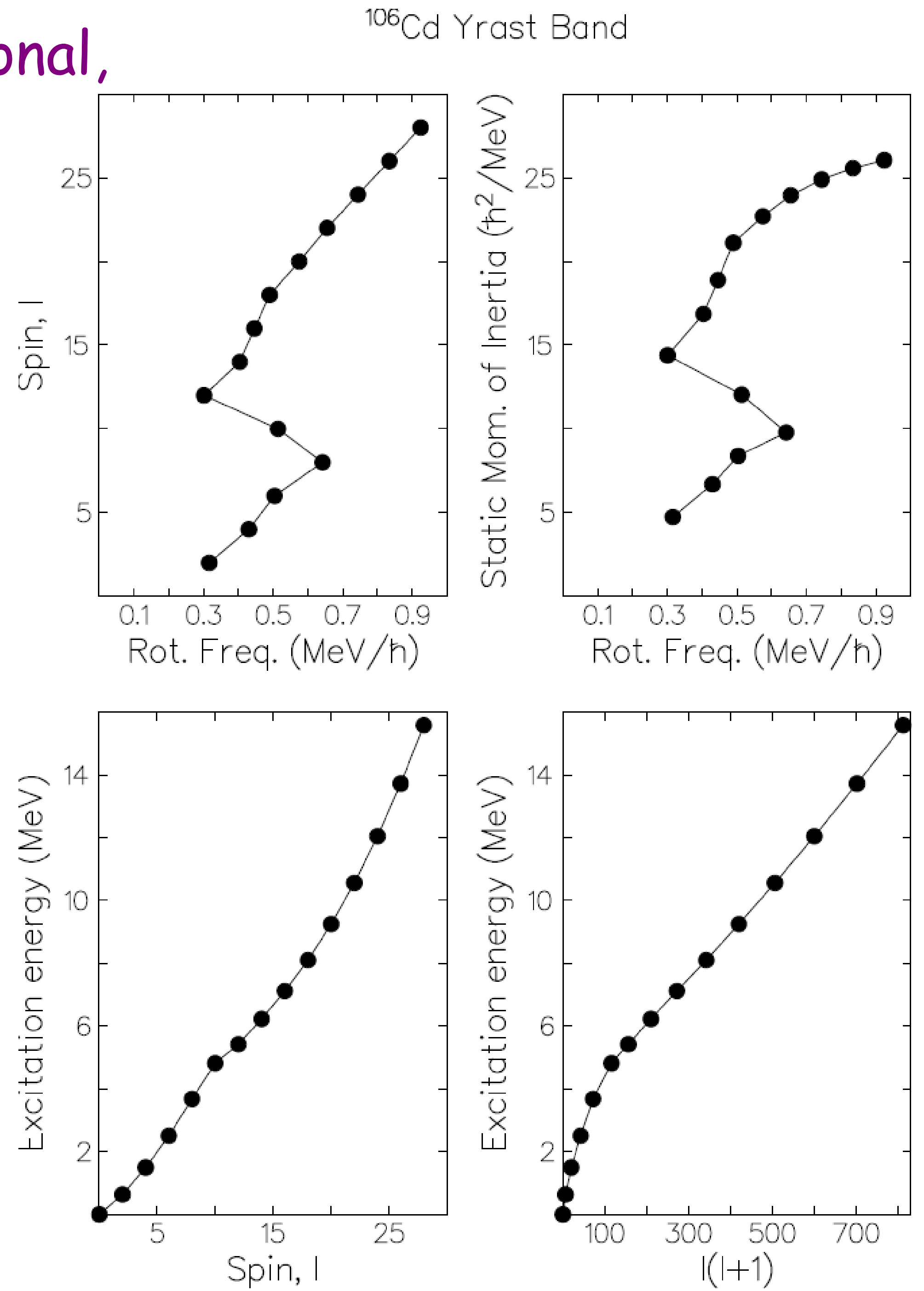
$$\mathcal{I}^{(2)} = \frac{dI}{d\omega} \approx \frac{4\hbar}{\Delta E_\gamma}$$

Rotational 'frequency', ω given by,

$$\omega = \frac{dE(I)}{dI_x(I)} \approx \frac{E(I+1) - E(I-1)}{I_x(I+1) - I_x(I-1)}$$

$$I_x(I) = \sqrt{I(I+1) - K^2} \approx \sqrt{\left(I + \frac{1}{2}\right)^2 - K^2}$$

$$\omega \approx \frac{E_\gamma}{\sqrt{(I+\frac{3}{2})^2 - K^2} - \sqrt{(I-\frac{1}{2})^2 - K^2}}$$



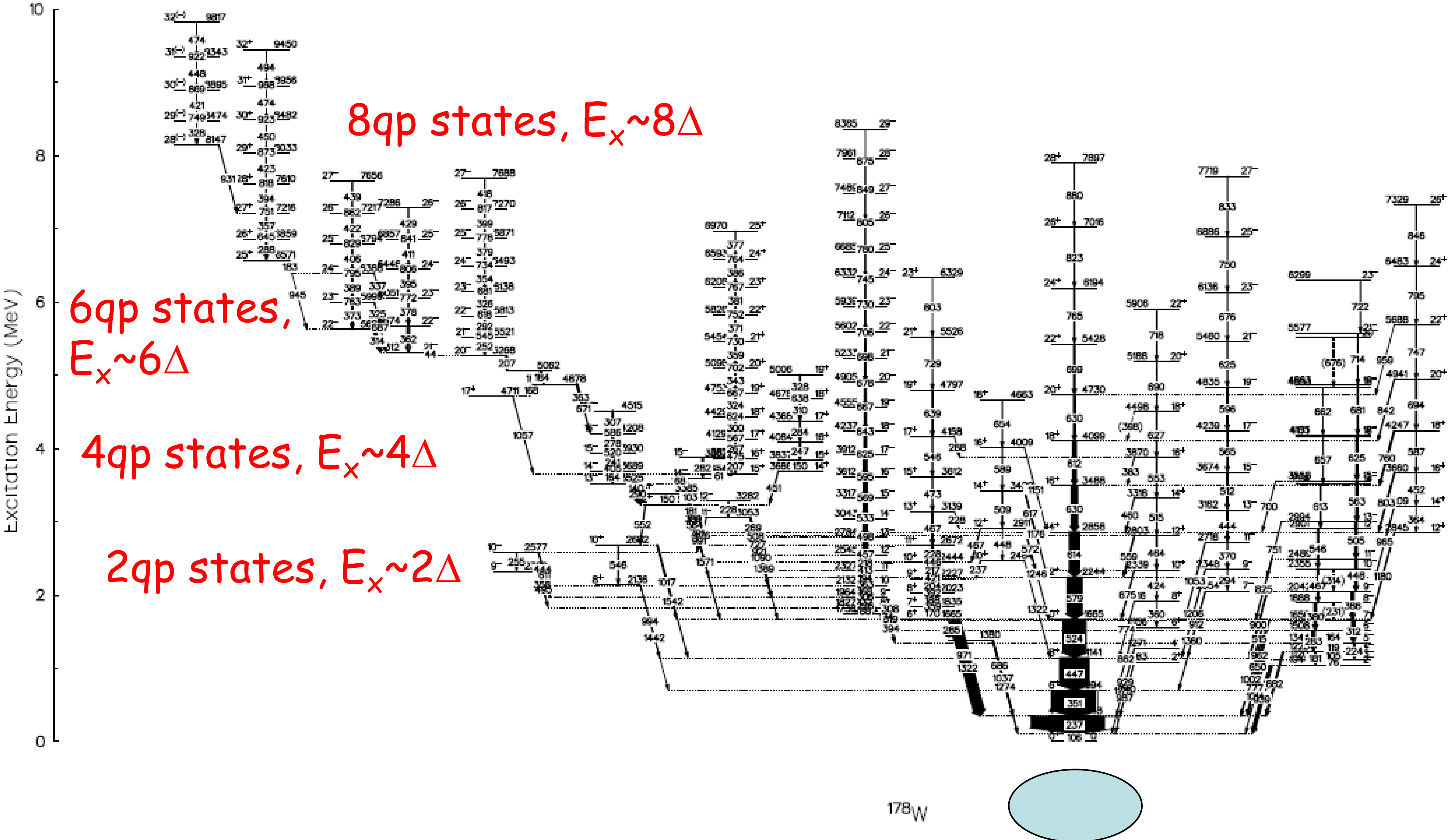
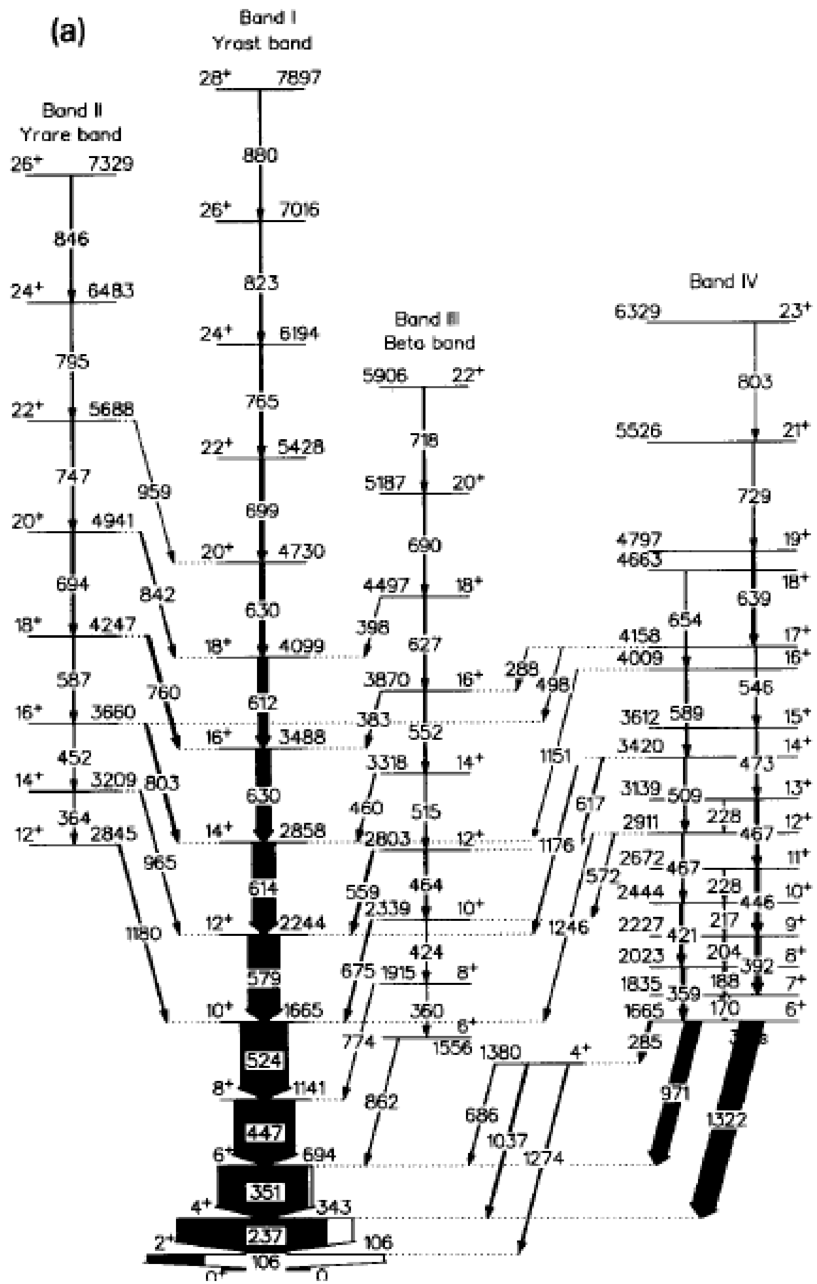


Figure 2.21: Partial decay scheme for ^{178}W showing 0, 2, 4, 6 and 8 quasi-particle structures [9].

C.S.Purry et al., Nucl. Phys. A632 (1998) p229


(a)
Band I
Yrast band
Band II
Yrare band
Band III
Beta band
Band IV
28+ 7897
880
26+ 7016
823
24+ 6194
765
22+ 5428
699
20+ 4730
630
18+ 4099
612
16+ 3488
630
14+ 2858
614
12+ 2244
579
10+ 1665
524
8+ 1141
447
6+ 694
351
4+ 343
237
2+ 106
106
0+ 0
26+ 7329
846
24+ 6483
795
22+ 5688
747
20+ 4941
694
18+ 4247
587
16+ 3660
452
14+ 3209
364
12+ 2845
959
842
760
803
965
1180
5906 22+
718
5187 20+
690
4497 18+
627
3870 16+
552
3318 14+
515
2803 12+
464
2339 10+
424
1915 8+
360
1556 6+
398
383
460
559
675
774
862
1380 4+
686
1037
1274
288
498
1151
1176
617
572
1246
6329 23+
803
5526 21+
729
4797 19+
4663 18+
639
654
4158 17+
4009 16+
546
589
3612 15+
3420 14+
473
3139 13+
509
2911 12+
228
467
2672 11+
467
2444 10+
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170
285
971
1322

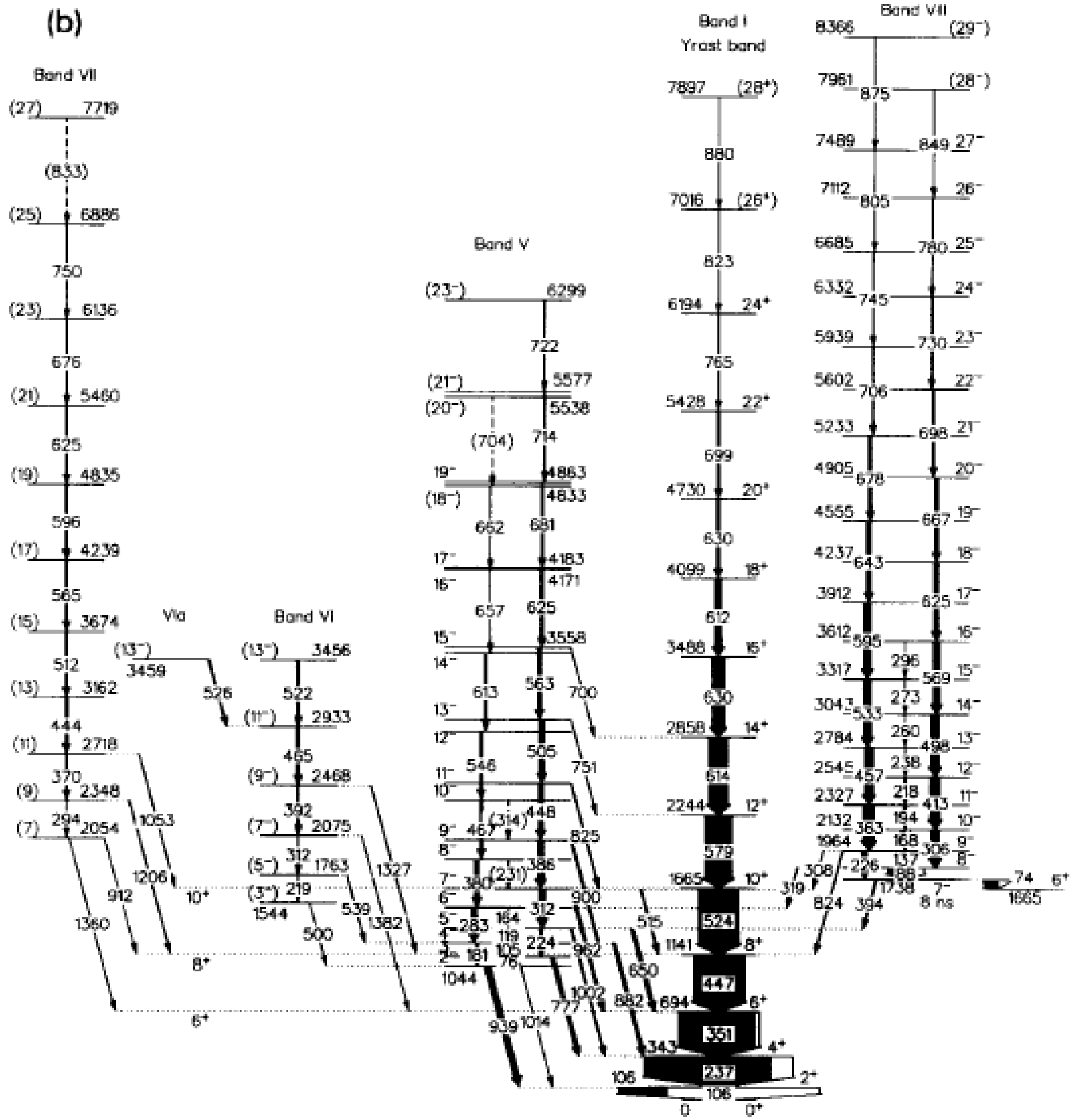


Fig. 1 — continued.

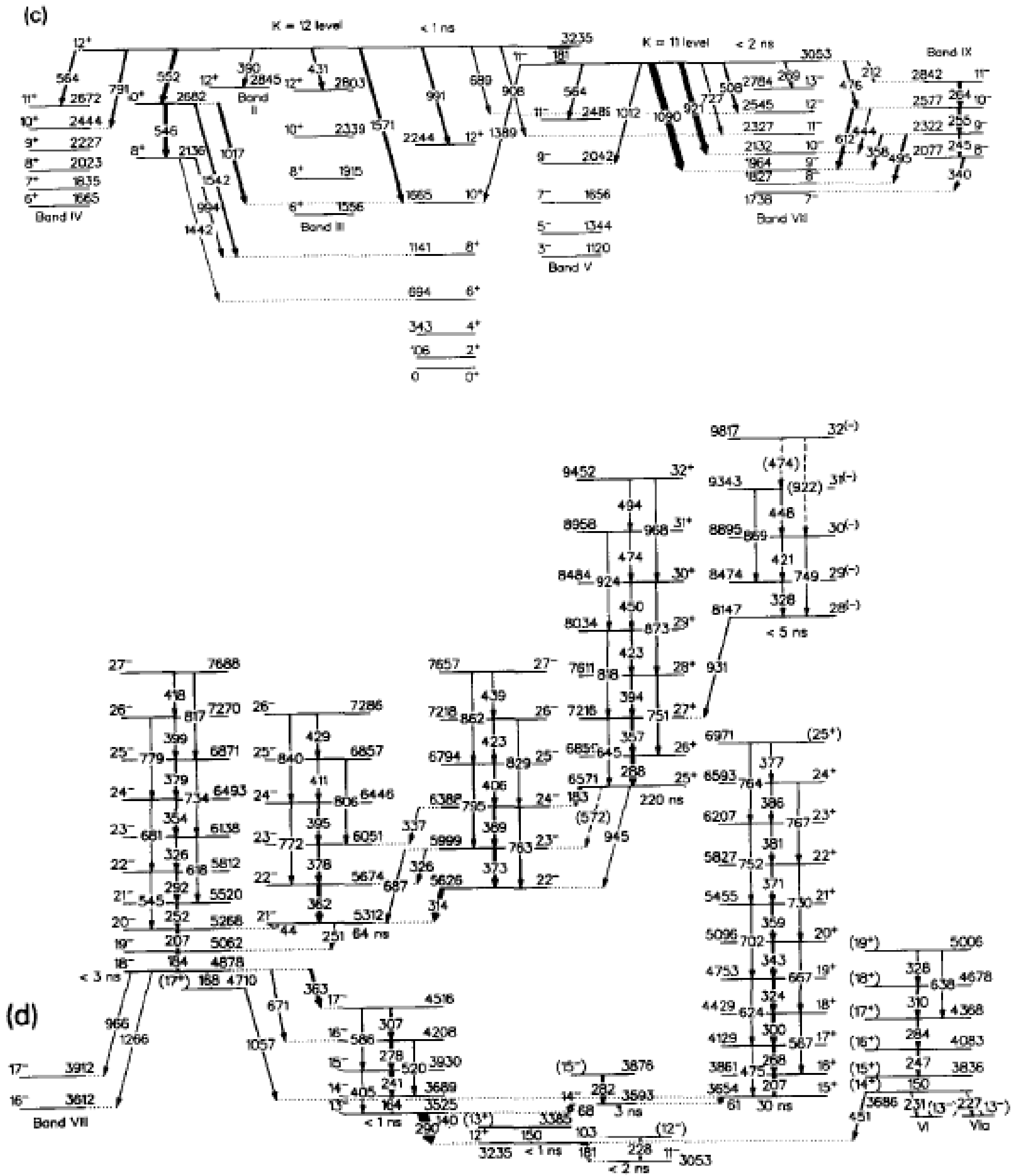


Fig. 1 — continued.

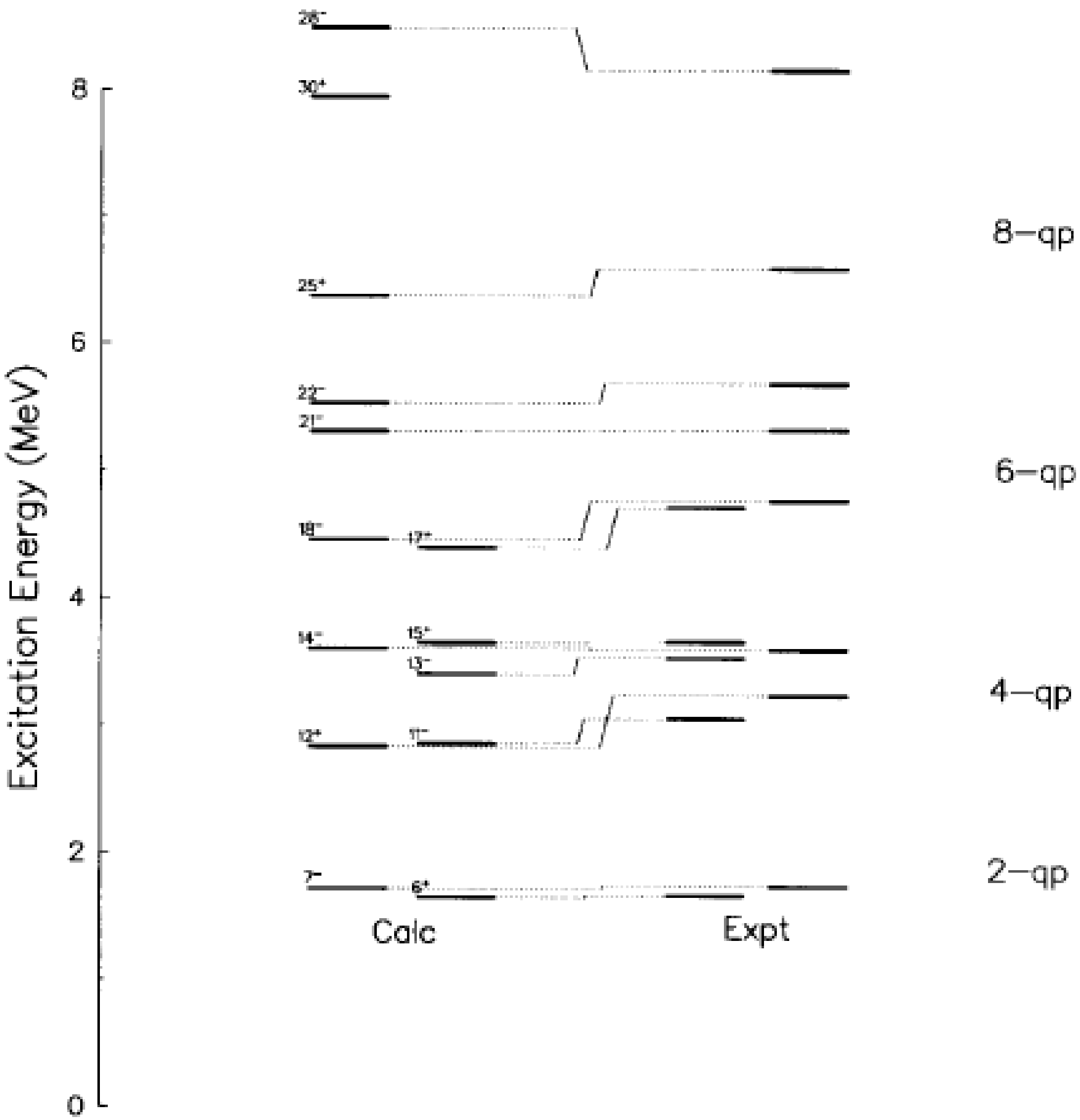


Fig. 14. Experimental and calculated bandhead energies.

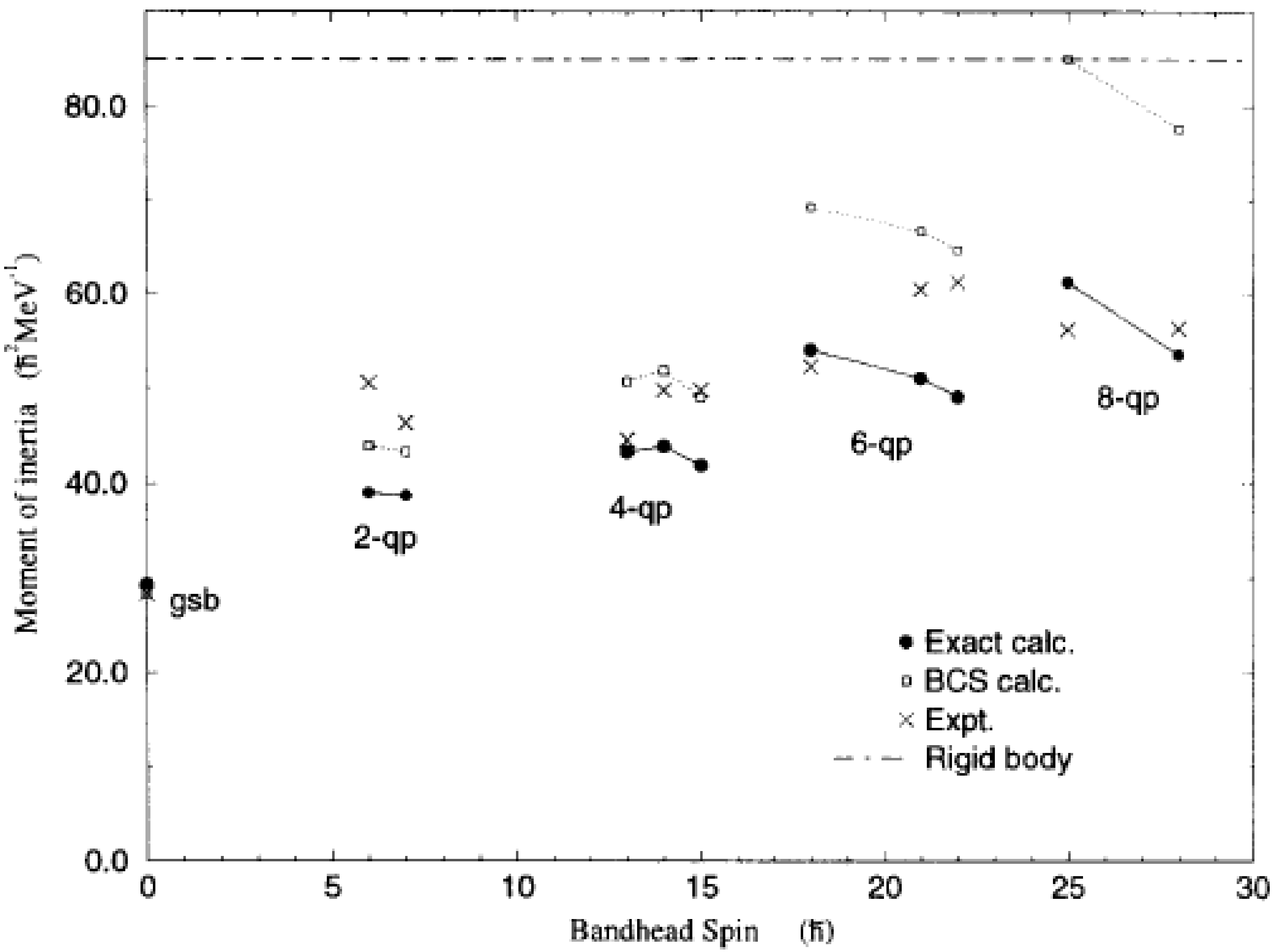


Fig. 17. Comparison of experimental and calculated moments of inertia.

Transitions from Vibrator to Rotor?

PHR, Beausang, Zamfir, Casten, Zhang et al., ***Phys. Rev. Lett.*** **90** (2003) 152502

$$\text{Vibrator}: \quad E_n = n\hbar\omega = \frac{J}{2}\hbar\omega, \quad E_\gamma = \hbar\omega$$

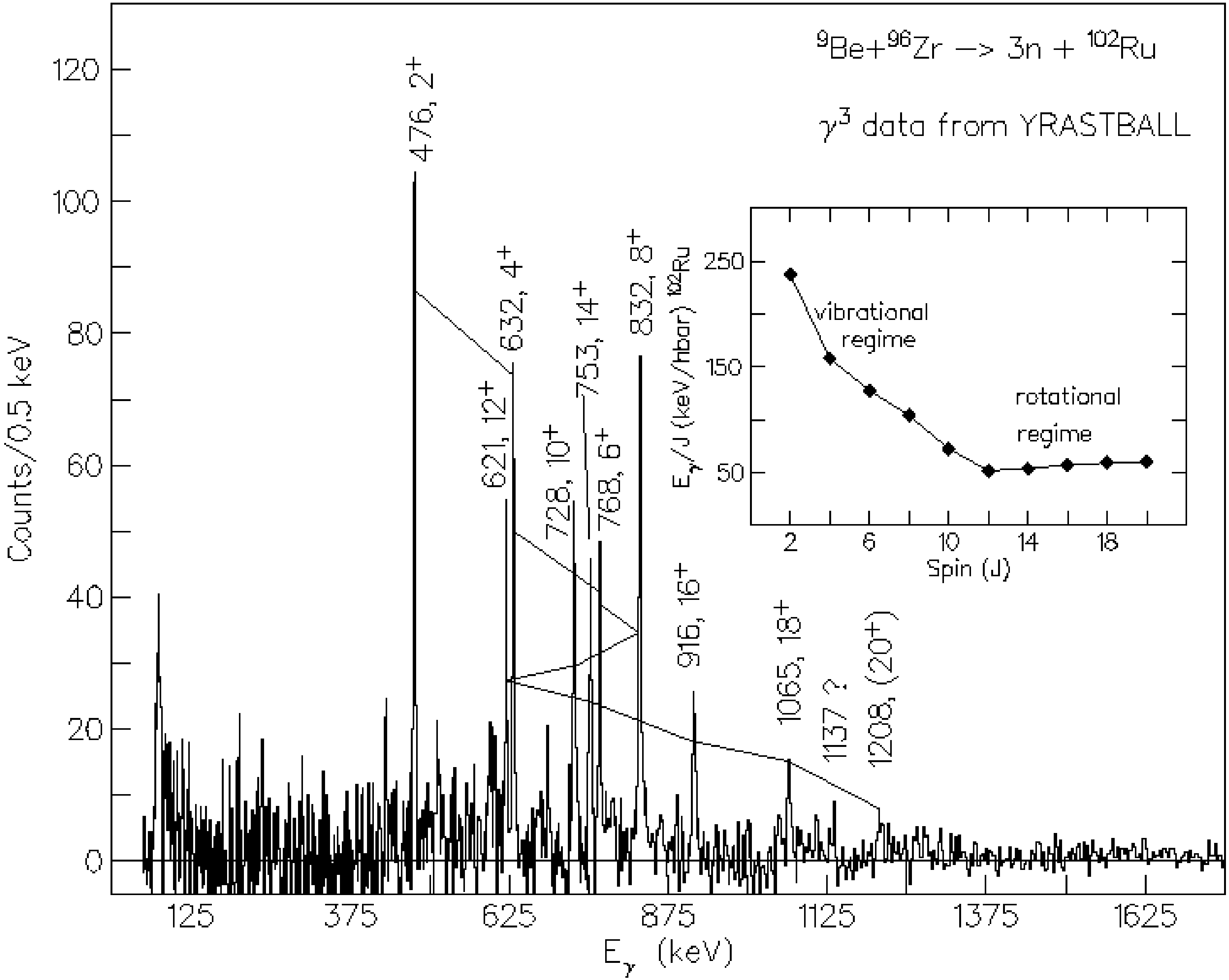
$$\text{Rotor}: \quad E_J = \frac{\hbar^2}{2\ell} J(J+1), \quad E_\gamma = \frac{\hbar^2}{2\ell}(4J-2)$$

$$\boxed{R = \frac{E_\gamma(J \to J-2)}{J}}$$

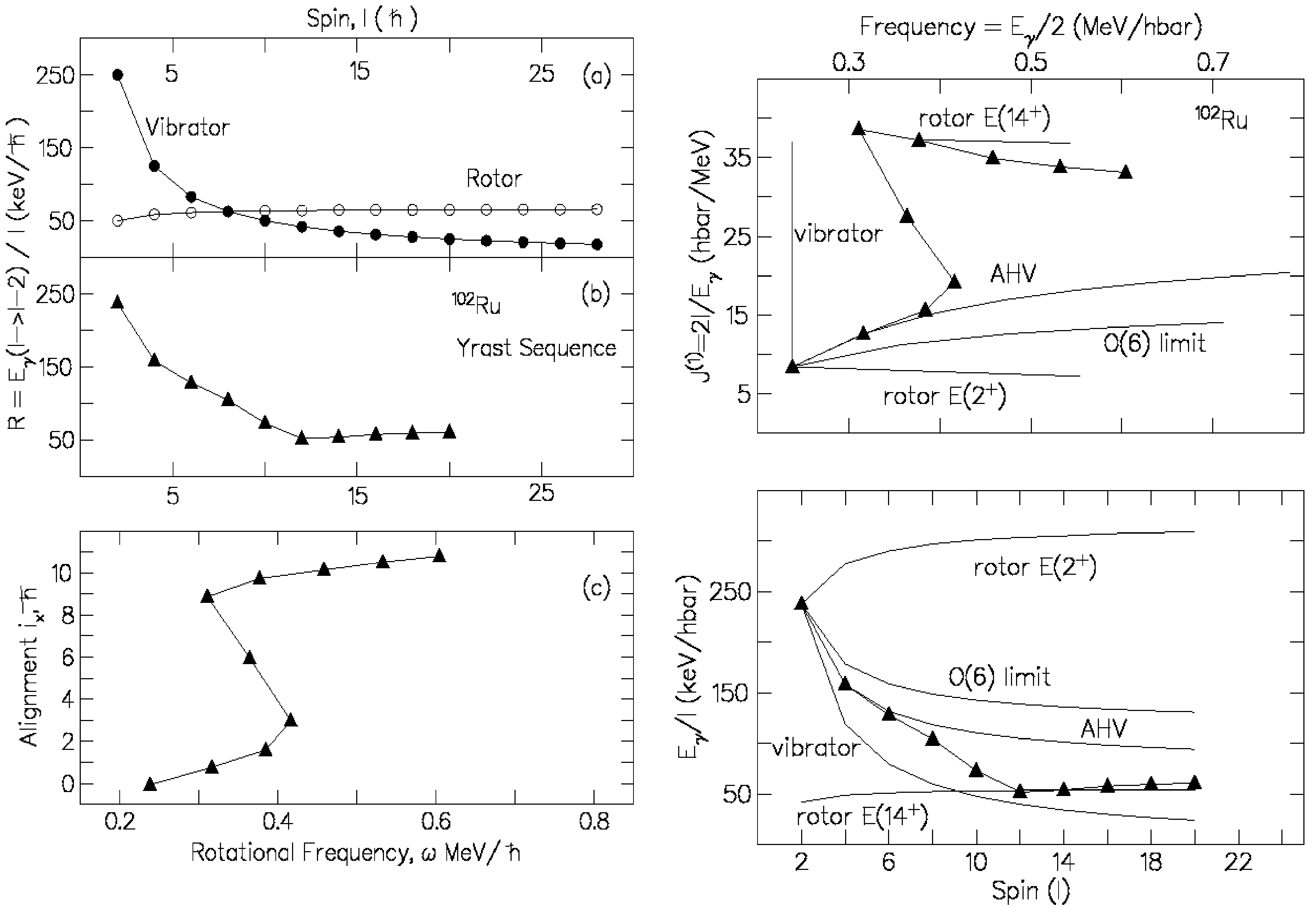
$$\text{Vibrator} \quad : R = \frac{\hbar\omega}{J} \xrightarrow{J \to \infty} 0$$

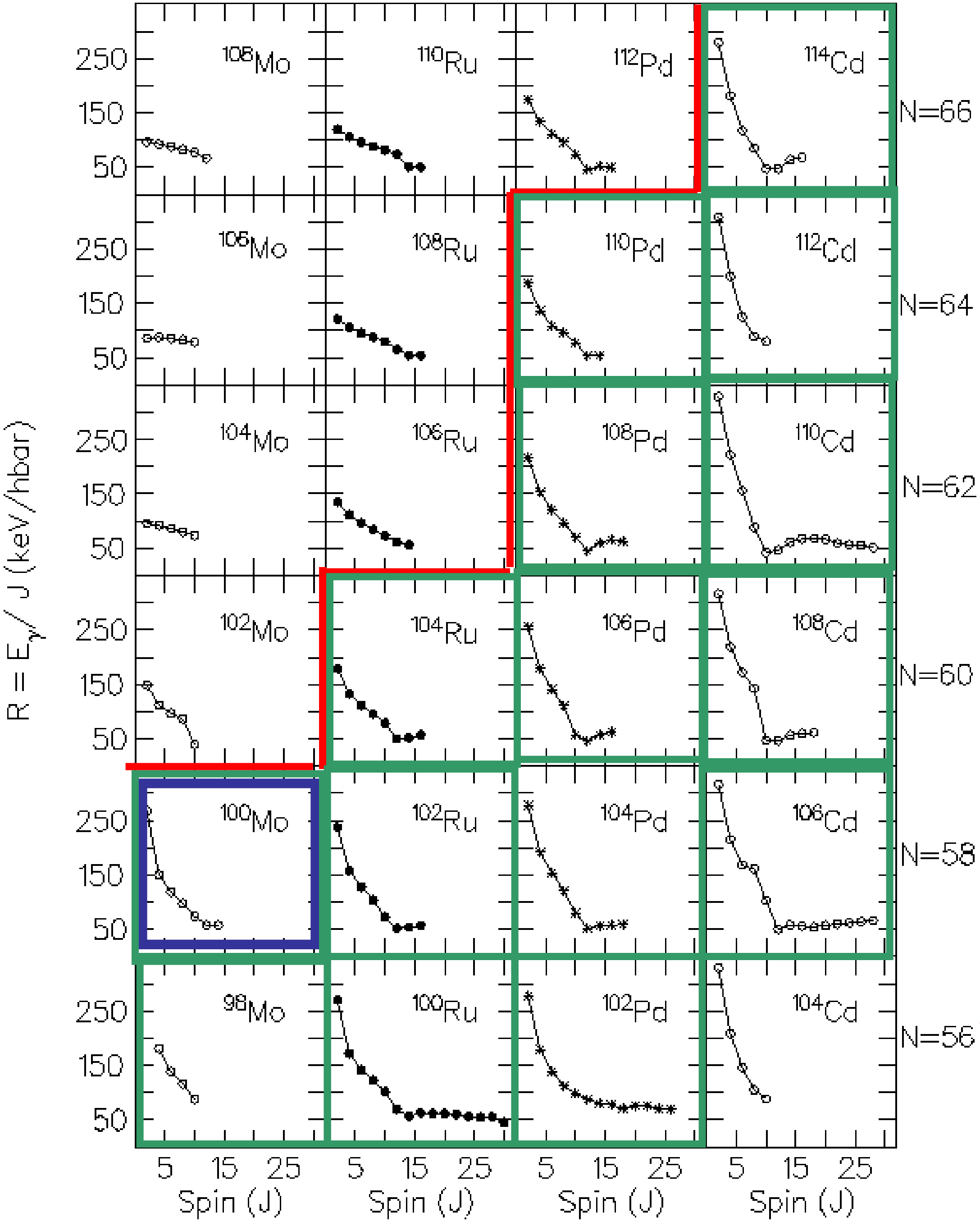
$$\text{Rotor} \quad : R = \frac{\hbar^2}{2\ell}\left(4 - \frac{2}{J}\right) \xrightarrow{J \to \infty} 4\left(\frac{\hbar^2}{2\ell}\right)$$

PHR, Beausang, Zamfir, Casten, Zhang et al., ***Phys. Rev. Lett.*** **90** (2003) 152502



PHR, Beausang, Zamfir, Casten, Zhang et al., ***Phys. Rev. Lett.* 90** (2003) 152502





Vibrator-Rotator phase change is a feature of near stable (green) A~100.

'Rotational alignment' can be a crossing between quasi-vibrational GSB & ***deformed rotational*** sequence. (stiffening of potential by population of high-j, equatorial ($h_{11/2}$) orbitals).

PHR, Beausang, Zamfir, Casten, Zhang et al., ***Phys. Rev. Lett.*** **90** (2003) 152502