Chapter 11

The Force Between Nucleons

Note to students and other readers: This Chapter is intended to supplement Chapter 4 of Krane's excellent book, "Introductory Nuclear Physics". Kindly read the relevant sections in Krane's book first. This reading is supplementary to that, and the subsection ordering will mirror that of Krane's, at least until further notice.

Imagine trying to understand chemistry before knowing Quantum Mechanics...

The understanding of the physical world was accomplished through many observations, measurements, and then trying to abstract them in a *phenomenological theory*, a "theory" that develops empirical relationships that "fit" the data, but do not explain why nature should behave this way. For example, Chemistry had organized the periodic table of the elements long before Quantum Mechanics was discovered.

Then, the Schrödinger equationwas discovered. Now we had a well-defined theory on which to base the basic understanding of atomic structure. With a solid fundamental theory, we now have a well-defined systematic approach:

- Solve the Schrödinger equationfor the H atom, treating p, e^- as fundamental point-charge particles, a good approximation since the electron wavefunctions overlap minimally with the very small, but finite-sized proton.
- Build up more complex atoms using the rules of Quantum Mechanics. Everything is treated theoretically, as a perturbation, since the Coulomb force is relatively weak. This is expressed by the smallness of the fine-structure constant:

$$\alpha = \frac{e^2}{4\pi\epsilon_0\hbar c} = 7.297\,352\,5376(50) \times 10^{-3} = \frac{1}{137.035\,999\,679(94)} \tag{11.1}$$

- The interaction between atoms is studied theoretically by *Molecular Theory*. Molecular Theory treats the force between atoms as a *derivative force*, a remnant of the more basic

Coulomb force that binds the atom. *Covalent bonds*, *ionic bonds*, *hydrogen bonds*, *Van der Wall forces* are quasi-theoretic, quasi-phenomenological derivatives of the desire of Quantum Mechanics to close shells by sharing electrons (*covalent bonding*, stealing electrons (*ionic bonds*, simple electrostatic attraction (*hydrogen bonds*), or dipole-dipole and higher multipole arrangements (*generalized Van der Wall forces*).

It is tempting to attempt the same procedure in nuclear physics. In this case, the simplest bound system is the deuteron. In atomic physics, the H-atom was the ideal theoretical laboratory for our studies. In nuclear physics, the simplest bound system is the deuteron, a neutron and a proton bound by the strong force. However, some physical reality imposes severe restrictions on the development of an analysis as elegant as in atomic or molecular physics:

- n and p are tightly bound, practically "in contact" with each other. In other words, their wavefunctions overlap strongly.
- n and p are not point-like particles. They have internal structure. Because their wavefunctions overlap so strongly, the internal structure of one, has an important effect on the other.
- The forces binding the nucleons is *strong*. The *strong-coupling constants* are of the order unity, whereas the fine-structure constant (for Coulomb forces) is small, about 1/137. Hence, perturbation methods, so successful for atomic and molecular theory, are almost completely ineffective.

The internal structure of a nucleon

The proton is comprised of 2 "up" (u) quarks, and one "down (d) quark, in shorthand, $p(uud)$. The neutron is $n(ddu)$. Here is a table of the known properties of n, p, u, d.

	mass [MeV]/c^2	charge (e)	total angular momentum ($\hbar$)
p	938.272013(23)	+1	1/2
n	939.565560(81)	0	1/2
u	1.5–3.3	+2/3	1/2
d	3.5–6.0	−1/3	1/2

The above table explains the observed charge of the n and p. One can also argue that the spins of the similar quarks in each nucleon "anti-align" (Invoke the Pauli Exclusion Principle.) to result in an overall spin-1/2 for the n and p. However, the mass discrepancy is phenomenal! It is explained as follows:

Quarks are so tightly bound to one another; so tightly bound, in fact, that if one injects enough energy to liberate a quark, the energy is used up in creating quark-antiquark pairs that bind, and prevent the observation of a "free quark". The "exchange particle" that binds the quarks is called the *gluon*. (The exchange particle for the Coulomb force is the photon.) Most of the mass of the nucleons come from the "cloud" of gluons that are present in the frenetic environment that is the inside of a nucleon. Recall that a nucleus was likened to a "mosh pit", in the previous chapter. The mayhem inside a nucleon defies description or analogy.

Since quarks are very tightly bound, covalent or ionic bonds between nucleons do not exist. All the forces are derivative forces, analogs to dipole-dipole (and higher multipolarity) interactions.

The "derivative" nucleon-nucleon force

This "derivative" nucleon-nucleon force is also called the *Meson Exchange Model*. The exchange particle, in this case, is thought of as being two mesons. A "vector" meson, mass $m_{\rm V}$, so named because it has spin-1, provides the short-ranged repulsive force that keep nucleons from collapsing together. A "scalar" spin-0 meson, mass $m_{\rm S}$, provides the slightly longer ranged attractive force. Both mesons are 100's of MeV, but $m_{\rm S} < m_{\rm V}$, with the result that a potential well is formed. This is apparent in the form of the potential given in (11.2), as well as in the sketch in Fig. 11.1.

$$V_{nn} = \frac{\hbar c}{r}\left\{\alpha_{\rm V}\exp[-(m_{\rm V}c/\hbar)r] - \alpha_{\rm S}\exp[-(m_{\rm S}c/\hbar)r]\right\} \tag{11.2}$$

The vector and scalar coupling constants, $\alpha_{\rm V}$ and $\alpha_{\rm S}$ are order unity. Note the similarity of this potential with the attractive or repulsive Coulomb potential, that may be written:

$$V_{\rm C} = \pm\frac{\hbar c\alpha}{r} \ ,$$

where α is the fine-structure constant expressed in (11.1). We immediately draw the conclusion that it is the mass of the exchange particle which determines the range of the force. The photon is massless, hence the Coulomb force is long.

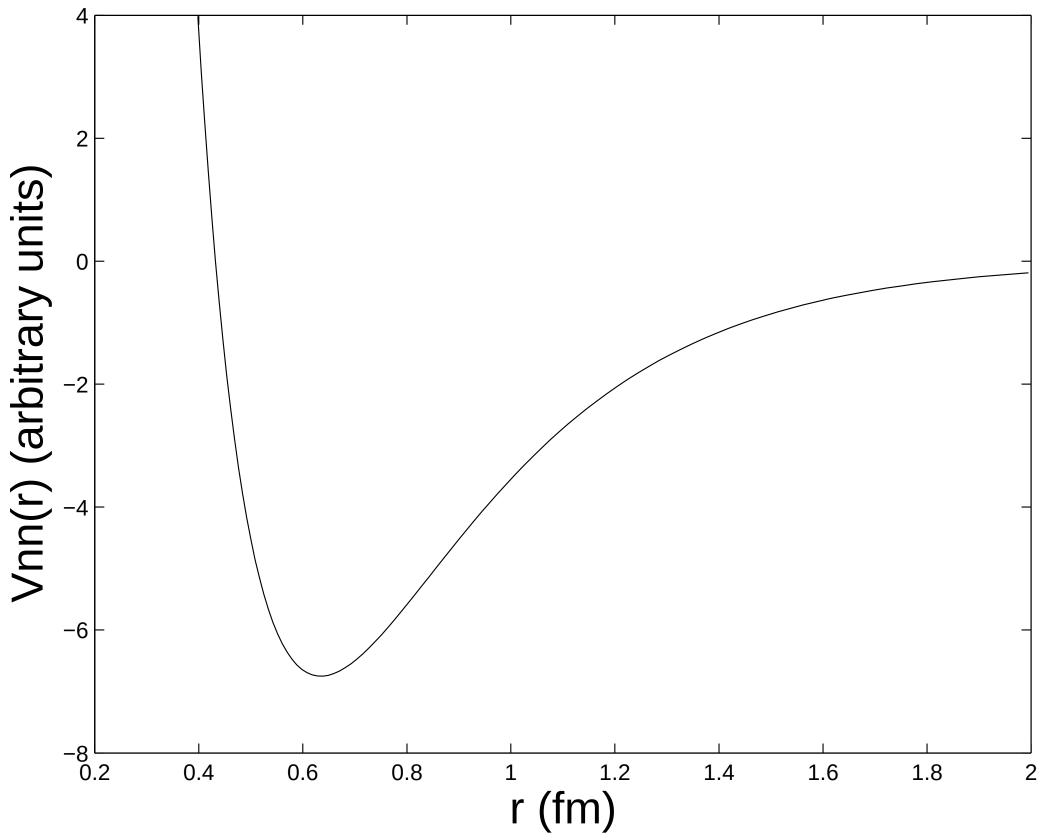

Figure 11.1: A sketch of the central part $V_{nn}(r)$ of the nucleon-nucleon potential.

A few noteworthy mentions for the strong force:

- The particles that feel the strong force are called "hadrons". Hadrons are bound states of quarks. The hadrons divide into 2 different types:
  - "Baryons" are 3-quark combinations, *e.g.* $p(uud)$, $n(ddu)$ ...
  - "Mesons" are combinations of two quarks, a quark-antiquark pair. *e.g.* the π-mesons, $\pi^+(u\overline{d})$, $\pi^-(d\overline{u})$, and the $\pi^0([d\overline{d}+u\overline{u}]/\sqrt{2})$ mesons, being the most "common". Note that the μ particle has been called the μ-meson. However, this usage has been deprecated since the discovery of quarks. The μ is a *lepton*, in the same class as electrons and neutrinos.
- The nuclear force is "almost" independent of nucleon "flavor", that is, V_{nn}, V_{np}, V_{pp} are almost the same. The *p*-*p* system also has a separate Coulomb repulsive force.
- The nuclear force depends strongly on the alignment of nucleon spins.
- As a consequence of the above, the nucleon-nucleon force is non-central, that is, $V(\vec{x}) \neq V_{nn}(r)$. The impact of this is that the orbital angular momentum is not a conserved quantity, although the total angular momentum, $\vec{I} = \vec{L} + \vec{S}$ **is** conserved.
- Including the non-central and Coulomb components, the complete nucleon-nucleon potential may be written:

$$V_N(\vec{x}) = V_{nn}(r) + V_{\rm C}(r) + V_{\rm so}(\vec{x}) + V_{\rm ss}(\vec{x}) \ , \tag{11.3}$$

  where $V_{nn}(r)$ is the central part of the strong force, $V_{\rm C}(r)$ is the Coulomb repulsive potential that is switched on if both nucleons are protons, $V_{\rm so}(\vec{x})$ is the "spin-orbit" part that involves the coupling of $\vec{l}$'s and $\vec{s}$'s, and $V_{\rm ss}(\vec{x})$ is the "spin-spin" part that involves the coupling of intrinsic spins.

11.1 The Deuteron

The deuteron, ^{2}H, also known as D, is made up of one neutron and one proton. It is relatively weakly bound (2.22452(20) MeV), but stable, and has a relative 0.015% natural isotopic abundance. Properties of the isotopes of hydrogen are given below:

^AX	abundance or $t_{1/2}$	I^π
^{1}H	0.99985%	$\frac{1}{2}^+$
^{2}H, or D	0.00015%	1^+
^{3}H, or T	12.3 y (β^--decay)	$\frac{1}{2}^+$

The deuteron is the only bound dinucleon. It can be easily understood that the diproton would be rendered unstable by Coulomb repulsion. Dineutrons do not exist in nature, not even as a short-lived unstable state. This is most likely due to the Pauli Exclusion Principle applied at the quark level — 3 up quarks and 3 down quarks (all the quarks in a deuteron) are more likely to bind than 4 downs and 2 ups (the quark content of a dineutron). The spin-1 nature of the deuteron is explained by the spin-spin interaction of the neutron and proton. Their magnetic moments have opposite signs to one another, hence the alignment of spins tends to antialign the magnetic dipoles, a more energetically stable configuration.

Cultural aside:
Deuterium is a form of water. It is extracted from natural water using electrolysis or centrifugal techniques, producing DOH or D_2O, also called "heavy water". Although heavy water is very similar to H_2O, there are small differences, and these can impact biological systems. It is estimated that drinking nothing but heavy water for 10–14 days is lethal. Reactor-grade heavy water costs about $600–$700 per kilogram!

The deuteron is the most ideal "laboratory" to study the nucleon-nucleon force. There are no bound states of the deuteron.

Binding energy of the deuteron

Using the mass-binding energy relation, (**??**), that is

$$B_N(Z,A) = [Zm(^1H) + Nm_n - m(^AX)]c^2 \ ,$$

we can evaluate the binding energy of the deuteron,

$$B_N(1,2) = [m(^1H) + m_n - m(D)]c^2 \ .$$

Krane gives 2.2463(4) MeV. Modern data gives the binding energy as 0.002388169(9) u, or 2.224565(9) MeV. The binding energy inferred from H(n,γ)D scattering experiments is 2.224569(2) MeV, and from D(γ,n)H experiments, 2.224(2) MeV. (Neutron spectroscopy is much less accurate than γ-spectroscopy.)

Why does the deuteron have only one bound state?

In NERS311 we discussed the s-state solutions to the 3D-Schrödinger equation. We discovered that, unlike the 1D-square well, which always has a bound state no matter the depth of the well, the 3D requires the potential depth to to satisfy the following:

$$|V_0| > \frac{\pi^2 \hbar^2}{8MR^2} \ .$$

In this case, M is the reduced mass, and R is the radius of the deuteron.

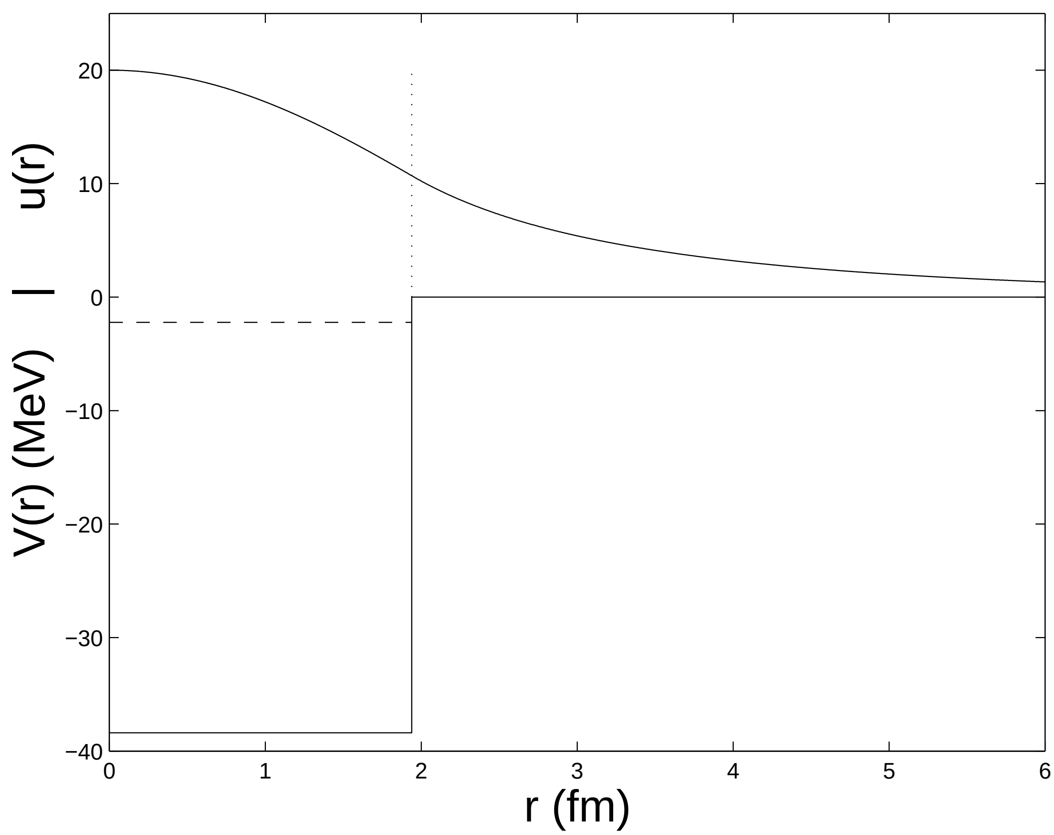

Figure 11.2: square-well potential model for the deuteron. The single bound level is shown at $E = -2.224$ MeV. The loose binding of the deuteron is indicated by the long exponential tail outside of the radius of the deuteron.

The spin and parity of the deuteron

The observed I^π of the deuteron is 1^+. We recall that the total spin is given by $\vec{I} = \vec{s}_n + \vec{s}_p + \vec{l}$. Therefore, with $I = 1$, applying the quantum mechanical rules for adding spins, we can only have 4 possible ways obtaining $I = 1$, namely:

S	l	I	π	?
$\uparrow\uparrow$	0	1	+1	yes
$\uparrow\uparrow$	1	1	-1	no
$\downarrow\uparrow$	1	1	-1	no
$\uparrow\uparrow$	2	1	+1	yes

We can rule out the $l = 1$ states due to the parity relation, $\pi = (-1)^k$. Without further knowledge, we can not rule out the $l = 2$ state. We can use experiments to determine which l-state contributes.

This can be done in two ways:

The magnetic dipole of the deuteron

If $l = 0$, only S can contribute to the magnetic moment. Following the discussion in Chapter 10, the magnetic moment would be given by:

$$\mu_{\mathrm{D}} = \frac{\mu_N}{2}(g_{sn} + g_{sp}) \ . \tag{11.4}$$

Using the data $g_{sn} = -3.82608545(46)$ and $g_{sp} = 5.585694713(90)$, we evaluate $(g_{sn} + g_{sp}) = 1.75960926(10)$, while experiment gives 1.714876(2), a difference of 0.044733(2). The difference is small but significant, indicating that the deuteron can not be a pure s state!

Since part of the nucleon-nucleon potential is non-central, bound states that do not have a unique $\langle\vec{l}\rangle$ are permissible. Hence if we consider that the deuteron is a mixture of s and d states, we can write:

$$\psi_{\mathrm{D}} = a_s\psi_s + a_d\psi_d \ , \tag{11.5}$$

where $|a_s|^2 + |a_d|^2 = 1$. Taking expectation values:

$$\mu = |a_s|^2\mu(l = 0) + |a_d|^2\mu(l = 2) \ , \tag{11.6}$$

with $\mu(l = 0)$ as given in (11.4) and $\mu(l = 2) = (3 - g_{sn} - g_{sp})\mu_N/4$ leads us to conclude[1] that the deuteron is 96% s and 4% d. It is a small correction but it is an interesting one, and a clear example of the non-conservation of orbital angular momentum when non-central forces and spins are involved.

[1] This calculation of $\mu(l = 2)$ would take several hours to explain. For more information see: www.physics.thetangentbundle.net/wiki/Quantum_mechanics/magnetic_moment_of_the_deuteron

The quadrupole moment of the deuteron

Another demonstration that isolates the d component of the deuteron wave function, is the measurement of the quadrupole moment. An s state is spherical, and hence its quadrupole moment (Q) vanishes. The Q of the deuteron is measured to be $Q = 0.00288(2)$ b. A "b" is a "barn". A barn is defined as 10^{28} m^2, about the cross sectional area of a typical heavy nucleus. This unit of measure is a favorite among nuclear physicists.

Using (11.5) in the definition of the quadrupole moment given by (**??**), that is,

$$Q = \int \mathrm{d}\vec{x}\ \psi_N^*(\vec{x})(3z^2 - r^2)\psi_N(\vec{x}) \ ,$$

results in

$$Q = \frac{\sqrt{2}}{10} a_s^* a_d \langle R_s|r^2|R_d\rangle - \frac{1}{20}|a_d|^2\langle R_d|r^2|R_d\rangle \ , \tag{11.7}$$

where R_s and R_d are the radial components of the deuteron's s and d wavefunctions. For consistency, a_s appears in (11.7) as its complex conjugate. However, the a_s and a_d constants may be chose to be real for the application, and thus we can replace $a_s^* = a_s$ and $|a_d|^2 = a_d^2$.The deuteron's radial wavefunctions are unknown and unmeasured. However, reasonable theoretical approximations to these can be formulated[2], and yield results consistent with the 96%/4% s/d mixture concluded from the deuteron's magnetic moment measurements.

11.2 Nucleon-nucleon scattering

This material will not be covered in NERS312. The section heading is included here as a "stub" to keep the subsection numbering in Krane and these notes aligned.

11.3 Proton-proton and neutron-neutron interactions

Ditto the italicized stuff above.

The NERS Department played an important role in the development of neutron spectroscopy and scattering. Please visit:

`http://www.ur.umich.edu/0708/Sep24_07/obits.shtml`

[2]Extra credit to any student who comes up with a decent approximation to the deuteron's wavefunctions, and calculates consistent results!

That link is an obituary of Professor John King, who was a pioneer in this area. His contributions to this research are outlined there.

11.4 Properties of the nuclear force

Described in the beginning of this chapter

11.5 The exchange force model

Also described elsewhere throughout this chapter