Some (more) Nuclear Structure

Lecture 2
Low-energy Collective Modes and Electromagnetic Decays in Nuclei

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Outline of Lectures 1& 2

• 1) Overview of nuclear structure ‘limits’
  - Some experimental observables, evidence for shell structure
  - Independent particle (shell) model
  - Single particle excitations and 2 particle interactions.

• 2) Low Energy Collective Modes and EM Decays in Nuclei.
  - Low-energy quadrupole vibrations in nuclei
  - Rotations in even-even nuclei
  - Vibrator-rotor transitions, E-GOS curves
What about 2 nucleons outside a closed shell?
Residual Interactions?

- We need to include any addition changes to the energy which arise from the interactions between valence nucleons.

- This is in addition the mean-field (average) potential which the valence proton/neutron feels.

- Hamiltonian now becomes $H = H_0 + H_{\text{residual}}$

- 2-nucleon system can be thought of as an inert, doubly magic core plus 2 interacting nucleons.

- Residual interactions between these two 'valence' nucleons will determine the energy sequence of the allowed spins / parities.
What spins can you make?

• If two particles are in identical orbits ($j^2$), then what spins are allowed?

Two possible cases:

• Same particle, e.g., 2 protons or 2 neutrons = even-even nuclei like $^{42}\text{Ca}$, 2 neutrons in $f_{7/2} = (\nu f_{7/2})^2$
  We can couple the two neutrons to make states with spin/parity $J^\pi=0^+, 2^+, 4^+$ and $6^+$
  These all have $T=1$ in isospin formalism, intrinsic spins are anti-aligned with respect to each other.

• Proton-neutron configurations (odd-odd)
  e.g., $^{42}\text{Sc}$, 1 proton and 1 neutron in $f_{7/2}$
  We can couple these two make states with spin/parity $0^+, 1^+, 2^+, 3^+, 4^+, 5^+, 6^+$ and $7^+$.

Even spins have $T=1$ ($S=0$, intrinsic spins anti-aligned);
Odd spins have $T=0$ ($S=1$, intrinsic spins aligned)
m - scheme showing which $J_{\text{tot}}$ values are allowed for $(f_{7/2})^2$ coupling of two identical particles (2 protons or 2 neutrons).

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<tr>
<th>$j_1 = 7/2$</th>
<th>$j_2 = 7/2$</th>
<th>$M$</th>
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Note, that only even spin states are allowed.
Schematic for \((f_{7/2})^2\) configuration.
4 degenerate states if there are no residual interactions.

**Residual interactions** between two valence nucleons give additional binding, lowering the (mass) energy of the state.
Fig. 4.8. Definition and schematic illustration of some of the ideas used in the geometrical analysis of short-range residual interactions.
Residual Interactions—Diagonal Effects

Consider 2 particles, in orbits $j_1, j_2$ coupled to spin $J$, and interacting with a residual interaction, $V_{12}$.

2 Identical Nucleons

\[ |j_1, j_2, J\rangle \quad \text{NO RESIDUAL INTERACTION} \]

\[ |j_1, j_2, J\rangle \quad \text{RESIDUAL INTERACTION} \]

Differ in angle between orbital planes
Geometric Interpretation of the $\delta$ Residual Interaction for a $j^2$ Configuration Coupled to Spin $J$

Use the cosine rule and recall that the magnitude of the spin vector of spin $j = [j (j+1)]^{-1/2}$

\[
J^2 = j_1^2 + j_2^2 - 2 j_1 j_2 \cos(\theta)
\]

\[
therefore J(J+1) = j_1(j_1+1) + j_2(j_2+1) - \sqrt{j_1(j_1+1)}\sqrt{j_2(j_2+1)} \cos(\theta)
\]

\[
\therefore \text{for } j_1 = j_2 = j \quad \cos^{-1} \left[ \frac{J(J+1) - 2j(j+1)}{j(j+1)} \right]
\]
\[ \delta \text{-interaction gives nice simple geometric rationale for} \]

**Seniority Isomers** from

\[ \Delta E \sim -V_0 F_r \tan \left( \frac{\theta}{2} \right) \]

for \( T=1 \), even \( J \)

e.g. \( J^\pi = (h_{9/2})^2 \) coupled to \( 0^+, 2^+, 4^+, 6^+ \) and \( 8^+ \).
δ − interaction gives nice simple geometric rationale for Seniority Isomers from \( \Delta E \sim -V_0 F_r \tan \left( \frac{\theta}{2} \right) \) for \( T=1 \), even \( J \).

See e.g., Nuclear structure from a simple perspective, R.F. Casten Chap 4.)
Note, 2 neutron or 2 proton holes in doubly magic nuclei show spectra like 2 proton or neutron particles.
Basic EM Selection Rules?

\[ \left| I_i + I_f \right| \geq L \geq \left| I_i - I_f \right| \]

The transition probability for a state decaying from state \( J_i \) to state \( J_f \), separated by energy \( E_\gamma \), by a transition of multipole order \( L \) is given by [1, 7]

\[
T_{fi}(\lambda L) = \frac{8\pi(L + 1)}{\hbar L((2L + 1)!!)^2} \left( \frac{E_\gamma}{\hbar c} \right)^{2L+1} B(\lambda L : J_i \rightarrow J_f) \quad (1.1.2)
\]

where \( B(\lambda L : J_i \rightarrow J_f) \) is called the reduced matrix element.
\[ T_{fi}(\lambda L) = \frac{8\pi (L + 1)}{hL ((2L + 1)!!)^2} \left( \frac{E_\gamma}{hc} \right)^{2L+1} B(\lambda L : J_i \rightarrow J_f) \quad (5.0.3) \]

where \( B(\lambda L : J_i \rightarrow J_f) \) is called the reduced matrix element.

Measuring the lifetime (decay probability) of a nuclear state thus gives a value for the \( B(\lambda L : J_i \rightarrow J_f) \).

For lifetimes, \( \tau \) in units of seconds where the transition probability per unit second, \( T = \frac{1}{\tau} \), \( (E_\gamma \text{ in MeV}) \),

\[ T(E1) = 1.587 \times 10^{15} E_\gamma^3 B(E1) \quad (5.0.4) \]
\[ T(E2) = 1.223 \times 10^9 E_\gamma^5 B(E2) \quad (5.0.5) \]
\[ T(E3) = 5.698 \times 10^2 E_\gamma^7 B(E3) \quad (5.0.6) \]
\[ T(M1) = 1.779 \times 10^{13} E_\gamma^3 B(M1) \quad (5.0.7) \]
\[ T(M2) = 1.371 \times 10^7 E_\gamma^5 B(M2) \quad (5.0.8) \]
\[ T(M3) = 6.387 \times E_\gamma^7 B(M3) \quad (5.0.9) \]

The units of \( B(E\lambda) \) are \( e^2 \text{fm}^{2\lambda} \) and the units of \( B(M\lambda) \) are \( (e\hbar 2Mc)^2 \text{fm}^{2\lambda-2} \).
The EM transition rate depends on $E_\gamma^{2\lambda+1}$, the highest energy transitions for the lowest $\lambda$ are (generally) favoured. This results in the preferential population of yrast and near-yrast states.
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\[ = \text{gamma-ray between yrast states} \]
The EM transition rate depends on $E_\gamma^{2\lambda+1}$, (for E2 decays $E_\gamma^5$) 
Thus, the highest energy transitions for the lowest $\lambda$ are usually favoured. 
Non-yrast states decay to yrast ones (unless very different $\phi$, K-isomers)
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Schematic for $(f_{7/2})^2$ configuration.
4 degenerate states if there are no residual interactions.

Residual interactions between two valence nucleons give additional binding, lowering the (mass) energy of the state.
The effective interaction between nucleons deduced from nuclear spectra

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William W. True
University of California, Davis, California 95616

B. General features in the experimental matrix elements

Several comparisons of the experimental matrix elements seem worthwhile before embarking on detailed analyses. To gain a qualitative impression of spectra we adopt plots of matrix elements as a function of $\theta_{12}$, the classical angle of orientation between the two angular momentum vectors $j_1$ and $j_2$ coupled to $J$ which gives a measure of the overlap of their orbital wavefunctions. $\theta_{12}$ is defined by

$$\cos \theta_{12} = \frac{J(J + 1) - j_1(j_1 + 1) - j_2(j_2 + 1)}{2 \sqrt{j_1 j_2 (j_1 + 1)(j_2 + 1)}}.$$  (II.2)
\[ \Delta E(j^2 J) \approx -\frac{V_0 F_R}{\pi} \tan \frac{\theta}{2} \quad (T = 1, J \text{ even}) \]

\[ \Delta E\left(j_{\rho}j_{\sigma}J\right)_{j_{\rho}=j_{\sigma}} = -\frac{V_0 F_R}{\pi} \left(\cot \frac{\theta}{2}\right) \left[1 + \frac{1}{\cos^2\left(\frac{\theta}{2}\right)}\right] \quad (T = 0, J \text{ odd}) \]

GEOmetrical Interpretation

EQUIVALENT ORBITS

**Fig. 4.9.** Angular dependence of the \(\delta\)-function residual interaction strength (lower values correspond to more attractive residual interactions) for two particles in equivalent orbits. (Left) The \(T = 1\) (\(J\) even) states. (Right) The \(T = 0\) (\(J\) odd) states. The analytic expressions are indicated above their respective plots.
FIG. 4.10. Empirical proton–neutron multiplets for two particle equivalent orbit configurations for comparison with the behavior shown in Fig. 4.9. The curves are drawn through the data (Schiffer, 1971).
J. P. Schiffer and W. W. True: Effective interaction between nucleons
FIG. 4. Plots of the various \( j_1 = j_2 \) multiplets. The data (crosses) are compared to calculations (circles for \( T = 0 \) and squares for \( T = 1 \)) with the 12-parameter interaction of Table XVII \( (r_2 = 2.0 \text{ fm}) \). The solid points for the \((1g_{9/2})^2\) multiplet represent calculations with the purely central five-parameter interaction of Table XIII.
Fig. 4.12. A geometrical analysis of the $|h_9/2g_9/2J\rangle$ p–n multiplet in $^{210}\text{Bi}$. The empirical levels are shown on the left along with the semiclassical angle between the orbits of the two nucleons. The right side shows that the levels split into two families, according to $J$ even or $J$ odd. The solid lines are drawn to connect the points:
Ground state
Configuration.
Spin/parity $I^\pi=0^+$; $E_x = 0$ keV

Excitation energy (keV)
4+/2+ energy ratio: mirrors 2\(^+\) systematics.

Ground state
Configuration.
Spin/parity I\(^\pi\)=0\(^+\);
E\(_x\) = 0 keV
Evolution of nuclear structure
(as a function of nucleon number)

B(E2; 2^+ → 0^+)

Magic

(sph. vib.)

Mid-shell (ellipsoidal)

(sph. vib.)

Magic

\[ R_{4/2} < 2 \]

\[ R_{4/2} \approx 2.0 \]

\[ R_{4/2} \approx 3.33 \]

\[ R_{4/2} < 2 \]
What about both valence neutrons and protons? In cases of a few valence nucleons there is a lowering of energies, development of multiplets. \[ R_{4/2} \rightarrow \sim 2-2.4 \]
Quadrupole Vibrations in Nuclei?

• Low-energy quadrupole vibrations in nuclei?
  - Evidence?
  - Signatures?
  - Coupling schemes?
$\beta_2$

$E_n$

$n=0$

$n=1$

$n=2$

$n=3$

http://npl.kyy.nitech.ac.jp/~arita/vib
\( \lambda = 1 \) is equivalent to motion of center of mass.

Lowest physical mode is \( \lambda = 2 \)

Quadrupole

\[ Y_{2\mu} \]

\( 2^+ \)

\( 0^+ \)
We can use the m-scheme to see what states we can make when we couple together 2 quadrupole phonon excitations of order $J = 2\hbar$.

(Note phonons are **bosons**, so we can couple identical ‘particles’ together).

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\[ E = 2E_{ph} \rightarrow 4^+ \rightarrow 2^+ \rightarrow 0^+ \quad \text{Superpose 2 Quad vibes} \]

\[ E = E_{ph} \rightarrow 2^+ \quad \text{Quad. vib. rel. to g.s.} \]

\[ E = 0 \rightarrow 0^+ \]
Table 6.2 $m$ scheme for three-quadrupole phonon states

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$J = 6, 4, 3, 2, 0$

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From, *Nuclear Structure From a Simple Perspective*, by R.F. Casten, Oxford University Press.
For an idealised quantum quadrupole vibrator, the (quadrupole) phonon (={"d-boson"}) selection rule is $\Delta n=1$, where $n=$ phonon number.

**Fig. 6.2.** Low-lying levels of the harmonic vibrator phonon model.
For an idealised quantum quadrupole vibrator, the phonon (='d-boson') selection rule is $\Delta n=1$.
For an idealised quantum quadrupole vibrator, the phonon (=\textit{d-boson}) selection rule is $\Delta n_p = 1$.
Collective (Quadrupole) Nuclear Rotations and Vibrations

• What are the (idealised) excitation energy signatures for quadrupole collective motion (in even-even nuclei)?
  - (extreme) theoretical limits

Perfect, quadrupole (ellipsoidal), axially symmetric quantum rotor with a constant moment of inertia \( I \) has rotational energies given by (from \( E_{\text{class(rotor)}} = \frac{L^2}{2I} \))

\[
E_J = \frac{\hbar^2}{2\ell} J(J+1), \quad \frac{E(4^+)}{E(2^+)} = \frac{4(5)}{2(3)} = \frac{20}{6} = 3.33
\]
Collective (Quadrupole) Nuclear Rotations and Vibrations

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Perfect, quadrupole vibrator has energies given by the solution to the harmonic oscillator potential ($E_{\text{classical}} = \frac{1}{2} k\Delta x^2 + \frac{p^2}{2m}$).

$$E_N = \hbar \omega N \quad \frac{E(4^+)}{E(2^+)} = \frac{2}{1} = 2.00$$
Other Signatures of (perfect) vibrators and rotors

Decay lifetimes give $B(E2)$ values.
Also selection rules important
(eg. $\Delta n=1$).

$E_\gamma = \hbar \omega$ ; $\Delta E_\gamma (J\rightarrow J-2)=0$

For ('real') examples, see
Other Signatures of (perfect) vibrators and rotors

Decay lifetimes give $B(E2)$ values. Also selection rules important (e.g., $\Delta n = 1$).

\[ E_x = \left(\frac{\hbar^2}{2I}\right)J(J+1), \text{ i.e., } E_\gamma (J \rightarrow J-2) = \left(\frac{\hbar^2}{2I}\right)[J(J+1) - (J-2)(J-3)] = \left(\frac{\hbar^2}{2I}\right)(6J-6); \quad \Delta E_\gamma = \left(\frac{\hbar^2}{2I}\right) \times 12 = \text{const.} \]
Other Signatures of (perfect) vibrators and rotors

Decay lifetimes give $B(E2)$ values. Also selection rules important (eg. $\Delta n=1$).

For ('real') examples, see J. Kern et al., Nucl. Phys. A593 (1995) 21

$E_\gamma = \hbar \omega ; \Delta E_\gamma (J \rightarrow J-2)=0$

$E_x = (\hbar^2/2I)J(J+1)$, i.e., $E_\gamma (J \rightarrow J-2) = (\hbar^2/2I)[J(J+1) - (J-2)(J-3)] = (\hbar^2/2I)(6J-6); \Delta E_\gamma = (\hbar^2/2I)*12 = \text{const.}$
So, what about ‘real’ nuclei?
Many nuclei with $R(4/2)\sim2.0$ also show $I^\pi=4^+,2^+,0^+$ triplet states at $\sim2E(2^+)$. 
First Observation of a Near-Harmonic Vibrational Nucleus

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(Received 4 May 1987)

Evidence is presented for five closely spaced states in $^{118}$Cd near $E_{\text{ex}} \approx 2$ MeV, which are interpreted as a near-harmonic three-phonon quintuplet. Candidates for even higher-lying multiphonon states are also found. The experimental spectrum is compared with an anharmonic vibrator [or U(5) spectrum] and an interacting-boson-approximation calculation incorporating a two-particle, four-hole intruder configuration.
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<tr>
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<tr>
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<td>4^+ 2^+ 0^+</td>
<td>nd</td>
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<tr>
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<td>2^+</td>
<td>2^+</td>
<td>nd</td>
</tr>
<tr>
<td>0</td>
<td>0^+</td>
<td>0^+</td>
<td>nd</td>
</tr>
</tbody>
</table>

*G.S. BAND*  **γ-BAND**  **β_1-BAND**  **β_2-BAND**

**HARMONIC VIBRATOR**  **BAND GROUPING**  **DEGENERATE IBM - 1**
Note on ‘near-yrast feeding’ for vibrational states in nuclei.

If ‘vibrational’ states are populated in very high-spin reactions (such as heavy ion induced fusion evaporation reactions), only the decays between the (near)-YRAST states are likely to be observed.

The effect is to (only?) see the ‘stretched’ E2 cascade from $J_{\text{max}} \rightarrow J_{\text{max}}-2$ for each phonon multiplet.

= the ‘yrast’ stretched E2 cascade.
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Nuclear Rotations and Static Quadrupole Deformation
\[ T(E2) = 1.223 \times 10^9 E_\gamma^5 B(E2) \]

\( T(E2) \) = transition probability = \( 1/\tau \) (secs);

\( E_\gamma \) = transition energy in MeV

\[ B(E2: 0^+_1 \rightarrow 2^+_1) \propto \langle 2^+_1 \mid E2 \mid 0^+_1 \rangle^2 \]
Rotational model, $B(E2: I \rightarrow I-2)$ gives:

$$B(E2) = \frac{5}{16\pi} Q_o^2 \frac{3(I - K)(I - K - 1)(I + K)(I + K - 1)}{(2J - 2)(2J - 1)J(2J + 1)}$$

$T(E2) = 1.223 \times 10^9 E_\gamma^5 B(E2)$

$T(E2)$ = transition probability = $1/\tau$ (secs);
$E_\gamma$ = transition energy in MeV

$Q_o =$ INTRINSIC (TRANSITION) ELECTRIC QUADRUPOLE MOMENT.

This is intimately linked to the electrical charge (i.e. proton) distribution within the nucleus.

Non-zero $Q_o$ means some deviation from spherical symmetry and thus some quadrupole ‘deformation'.

$B(E2: 0^+_1 \rightarrow 2^+_1) \propto \left\langle 2^+_1 \mid E2 \mid 0^+_1 \right\rangle^2$
Bohr and Mottelson, Phys. Rev. 90, 717 (1953)

Isomer spin in $^{180}$Hf, $I^\pi > 11$ shown later to be $I^\pi = K^\pi = 8^-$ by Korner et al. Phys. Rev. Letts. 27, 1593 (1971).

K-value very important in understanding isomers.

$$E_x = \left( \frac{\hbar^2}{2I} \right) \times J(J+1)$$

$I = \text{moment of inertia.}$

This depends on nuclear deformation and $I \sim kM R^2$

Thus, $I \sim kA^{5/3}$

(since $r_{nuc} = 1.2A^{1/3}$fm)

---

**Fig. 2.** Suggested decay schemes of $^{71}$Lu$^{176}$ and $^{72}$Hf$^{180}$. The gamma-ray energies as well as the $\beta$-decay data of Lu$^{176}$ are taken from reference 2. The excitation energies listed in parentheses are obtained from Eq. (1) adjusted to give the energy of the first excited state. All energies are given in kev.
Therefore, plotting the moment of inertia, divided by $A^{5/3}$ should give a comparison of nuclear deformations across chains of nuclei and mass regions....
Nuclear static moment of inertia for \( E(2+) \) states divided by \( A^{5/3} \) trivial mass dependence.
Should show regions of quadrupole deformation.
Lots of valence nucleons of both types: emergence of deformation and therefore rotation

\[ R_{4/2} \rightarrow \sim 3.33 = \frac{[4(4+1)]}{[2(2+1)]} \]
Perfect rotor limit $R(4/2) = 3.33 = \frac{4(4+1)}{2(2+1)}$
Best nuclear ‘rotors’ have largest values of $N_{\pi}N_{\nu}$

This is the product of the number of ‘valence’ protons, $N_{\pi}$, and the number of valence neutrons $N_{\nu}$.
Alignments and rotational motion in ‘vibrational’ $^{106}$Cd (Z=48, N=58),
Some useful nuclear rotational, ‘pseudo-observables’...

\[ E_{\text{rot}}(I) = \frac{\hbar^2}{2I^{(0)}(I)} I(I + 1) \]

The kinematic moment of inertia is given by

\[ I^{(1)}(I) = \frac{I}{\omega} \]

while the dynamic moment is given by

\[ I^{(2)} = \frac{dI}{d\omega} \approx \frac{4\hbar}{\Delta E_\gamma} \]
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\]

Rotational ‘frequency’, \(\omega\) given by,

\[
\omega = \frac{dE(I)}{dI_x(I)} \approx \frac{E(I + 1) - E(I - 1)}{I_x(I + 1) - I_x(I - 1)}
\]

\[
I_x(I) = \sqrt{I(I + 1) - K^2} \approx \sqrt{\left(I + \frac{1}{2}\right)^2} - K^2
\]

\[
\omega \approx \frac{E_{\gamma}}{\sqrt{(I + \frac{3}{2})^2 - K^2 - \sqrt{(I - \frac{1}{2})^2 - K^2}}}
\]
8qp states, $E_x \sim 8\Delta$

6qp states, $E_x \sim 6\Delta$

4qp states, $E_x \sim 4\Delta$

2qp states, $E_x \sim 2\Delta$

Figure 2.21: Partial decay scheme for $^{178}W$ showing 0, 2, 4, 6 and 8 quasi-particle structures [9].

Fig. 1 — continued.
Fig. 1 — continued.
Fig. 14. Experimental and calculated bandhead energies.
Fig. 17. Comparison of experimental and calculated moments of inertia.
Transitions from Vibrator to Rotor?
Vibrator: \( E_n = n\hbar\omega = \frac{J}{2}\hbar\omega, \quad E_\gamma = \hbar\omega \)

Rotor: \( E_J = \frac{\hbar^2}{2\ell} J(J + 1), \quad E_\gamma = \frac{\hbar^2}{2\ell} (4J - 2) \)

\[
R = \frac{E_\gamma (J \rightarrow J - 2)}{J}
\]

Vibrator: \( R = \frac{\hbar\omega}{J} \xrightarrow{J \rightarrow \infty} 0 \)

Rotor: \( R = \frac{\hbar^2}{2\ell} \left(4 - \frac{2}{J}\right) \xrightarrow{J \rightarrow \infty} 4\left(\frac{\hbar^2}{2\ell}\right) \)
$^9\text{Be} + ^{96}\text{Zr} \rightarrow 3n + ^{102}\text{Ru}$

$\gamma^3$ data from YRASBALL

Counts/0.5 keV

$E_\gamma$ (keV/ℏℏ) of $^{102}\text{Ru}$

Vibrational regime

Rotational regime

Spin (J)

$E_\gamma$ (keV/ℏℏ) of $^{102}\text{Ru}$
Vibrator-Rotator phase change is a feature of near stable (green) A~100.

‘Rotational alignment’ can be a crossing between quasi-vibrational GSB & deformed rotational sequence.
(stiffening of potential by population of high-j, equatorial ($h_{11/2}$) orbitals).