Consider a chain of radioactive decays 1 → 2 → 3 → 4 where nuclei of type 4 are stable.

1. Write down the 4 differential rate equations that describe the growth and/or decay for the 4 types of nuclei.
2. Integrate each differential equation directly, with the initial conditions: $N_i(0) \neq 0$.
3. Show, independently, from your results in part 1.0 and, independently, from your results in part 2.) above, that: $N_1(t) + N_2(t) + N_3(t) + N_4(t) = N_1(0) + N_2(0) + N_3(0) + N_4(0)$.
4. Verify your results for all nuclei of types 1, 2 and 3, with the predictions provided by the Bateman equations, (6.37) and (6.38) from Krane’s book.
5. Now consider the small $t$ behavior of the system under consideration. Use a Taylor expansion to extract the smallest non-zero time-dependent term term for each type of nucleus, and interpret. For example, for type 1, the Taylor expansion would yield, $N_1(t) \rightarrow N_0(1 - \lambda_1 t)$.
6. Take the $t \rightarrow \infty$ limit for all nuclei, and interpret.
7. Consider the case of the near-stable parent, and rapidly decaying daughters, namely, $\lambda_1 \ll \lambda_{i(\geq 2)}$. For times such that:
   \[
   \frac{1}{\lambda_{i(\geq 2)}} \ll t \ll \frac{1}{\lambda_1},
   \]
   identify any conditions suggestive of secular equilibrium.

Consider the following set of radioactive decays:
1 → 2, via rate constant $\lambda_{12}$
1 → 3, via rate constant $\lambda_{13}$
2 → 3, via rate constant $\lambda_{23}$
Nuclei of type 3 are stable.

1. Write down the 3 differential rate equations that describe the growth and/or decay for the 3 types of nuclei.
2. Draw a graphical representation of the decay scheme.
3. Solve the differential equations, with the initial conditions: $N_1(0) = N_0$, $N_2(0) = N_3(0) = 0$.
4. What is the solution in the special case: $\lambda_{12} + \lambda_{13} = \lambda_{23}$?
5. What is the solution in the special case: $\lambda_{23} \rightarrow \infty$?

Krane problem 5, page 189.