UNIVERSITY OF MICHIGAN — Michigan Engineering

NERS/BIOE 481

Lecture 07
Metrics of Image Quality

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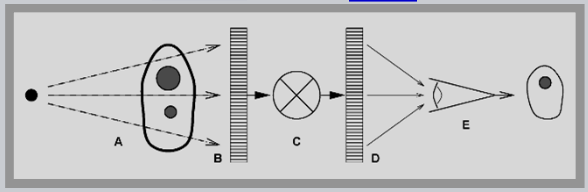
Henry Ford Health System
RADIOLOGY RESEARCH

I.A.1 - General Model – xray imaging

Xrays are used to examine the interior content of objects by recording and displaying transmitted radiation from a point source.

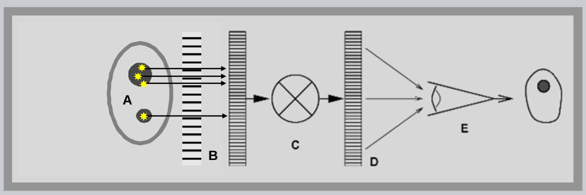
DETECTION DISPLAY



(A) Subject contrast from radiation transmission is
(B) recorded by the detector and
(C) transformed to display values that are
(D) sent to a display device for
(E) presentation to the human visual system.

I - General Model – radioisotope imaging

Radioisotope imaging differs from xray imaging only with repect to the source of radiation and the manner in which radiation reaches the detector.



Pharmaceuticals tagged with radioisotopes accumulate in target regions. The detector records the radioactivity distribution by using a multi-hole collimator.

V.A – Linear Systems Analysis

V) Fidelity of images
- A. Linear Systems Analysis (21)
- B. Resolution
- C. Noise
- D. Detective efficiency

V.A.1 – Linear System Properties

- Linearity
  - For many inputs to a system, the output corresponds to a the sum of the outputs that would occur if each input was separately applied.
  - Multiplication of the input by a constant multiplies the output by the same constant.
- Spatial invariance

  The image resulting from a point input is the same for all input positions. For some systems, the response may be large scale invariant with respect to the response of adjacent detector elements, but small scale variant with respect to input positions within one detector element.
- Isotropic response

  Imaging systems for which the point response function is the same in all directions can be described by one dimensional response functions.

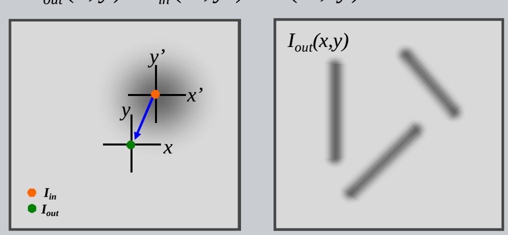
See Rossman, Radiology1969

V.A.2 – Convolution

- For a linear imaging system, the output image can be computed as the 2D input signal convolved with the system point spread function.

$$I_{out}(x,y) = \iint I_{in}(x',y')P(x-x',y-y')dx'dy'$$

$$I_{out}(x,y) = I_{in}(x',y') \otimes P(\Delta x, \Delta y)$$



V.A.2 – Convolution

A special case of convolution is the auto-correlation function, ACF, which is the convolution of a function with the same function.

$$ACF(\chi,\eta)=\iint f(x,y)\,f(x+\chi,y+\eta)\,dxdy$$

We will see shortly where the ACF is useful for describing noise based on the deviation of the signal from the mean, $\Delta S(x,y)$.

In this case, $ACF(0,0)$ is just σ_S^2.

V.A.3 – Fourier Analysis

- The Fourier transform can be used to evaluate the frequency composition of a signal. For imaging systems the signal is often a function of a two spatial variables. For simplicity, consider only the variation of the image signal in one dimension, $S(x)$.
- The Fourier transform operates on $S(x)$ to produce a function which varies with spatial frequency, $S(\omega_x)$.
- When the variable x has units of mm the variable ω_x will have units of cycles/mm.
- The Fourier transform may be mathematically expressed as an integral transformation involving an imaginary trigonometric operator.

$$S(\omega_x)=\int_{-\infty}^{\infty} S(x)e^{-i2\pi x\omega_x}dx$$

$$e^{ix}=\cos(x)+i\sin(x)$$

V.A.3 – Fourier Analysis

- Symmetric signals

  If the signal is symmetric about $x=0$, then the Fourier transform is real. For example, if $S(x)$ is a unit impulse function at $x=0$, then the tranform is simply $S(\omega_x)$ = 1 for all values of ω_x.

$$S(\omega_x)=\int_{-\infty}^{\infty} S(x)\cos(2\pi x\omega_x)dx$$

- Inverse Transform

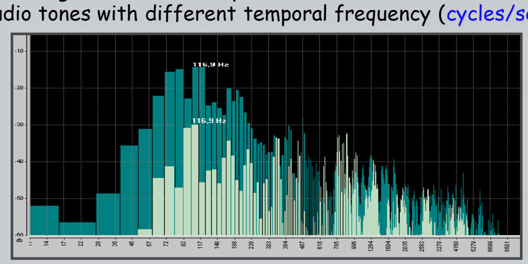
  A similar inverse Fourier transform operates on a frequency domain function to produce the corresponding spatial domain function.

$$S(x)=\int_{-\infty}^{\infty} S(\omega_x)e^{i2\pi x\omega_x}d\omega_x$$

- If $S(w_x)$ is obtained from the Fourier transform of $S(x)$, then the inverse Fourier transform of $S(w_x)$ results in the original function $S(x)$.
- $S(x)$ and $S(w_x)$ are thus often referred to as the spatial domain and frequency domain representations of the same signal.

V.A.3 – Spatial Frequencies

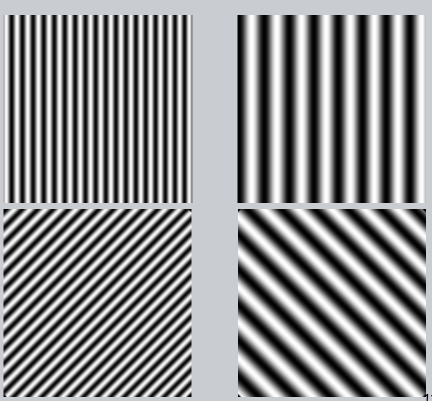
Audio signals are commonly described as a distribution of audio tones with different temporal frequency (cycles/sec).



Signal that vary with position, x, are described by the amplitude of different spatial frequencies (cycles/mm).

V.A.3 – Spatial Frequencies

Similarly, Images can be described as a set of signals with varying spatial frequency and direction.

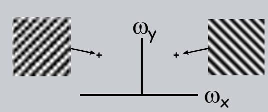


V.A.3 – Fourier Analysis

- The two dimensional Fourier transform can be used to evaluate the spatial frequency composition of an image. The inverse 2D transform is defined similarly.

$$S(\omega_x,\omega_y)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} S(x,y)e^{-i2\pi(x\omega_x+y\omega_y)}dxdy$$

- $S(w_x, w_x)$ for a particular value of (w_x, w_x) , corresponds to a 2D image with directionally oriented sinusoidal signal variation.



- For numeric computation, the Discrete Fourier transform (DFT) is used.

$$S\left(n/_{N\Delta x}\right)=\sum_{k=0}^{N-1} S\left(k\Delta x\right)e^{-i2\pi nk/N}$$

- Two dimensional transforms are computed by first transforming all rows with a 1D transform, and then all columns with a 1D transform.

$$S\left(k\Delta x\right)=\frac{1}{N}\sum_{n=0}^{N-1} S\left(n/_{N\Delta x}\right)e^{i2\pi nk/N}$$

- The numeric steps in the DFT may be factored to achieve significant improvement in computational speed. Most computer algorithms use some form of a Fast Fourier Transform (FFT).
  - Cooley JW and Tukey JW, "An algorithm for machine calculation of complex Fourier series", Math. Computation (April 1965), Vol 19, pp 297-301.
  - Public domain optimized FFT: http://www.fftw.org

FFT Demonstrations: A. fourier.jar B. imageJ FFT

Convolution Theorem

If $f(x)$ has the Fourier transform $F(\omega)$ and $g(x)$ has the Fourier transform $G(\omega)$, then $f(x)\otimes g(x)$ has the Fourier transform $F(\omega)G(\omega)$; that is, convolution of two functions means multiplication of their transforms,

$$\int\left(f(x)*g(x)\right)e^{-i2\pi xs}dx=F(\omega)G(\omega)$$

- Proof:

$$\begin{aligned}\int\left(f(x)*g(x)\right)e^{-i2\pi xs}dx &= \int\left(\int f(x')g(x-x')dx'\right)e^{-i2\pi xs}dx\\ &=\int f(x')\left(\int g(x-x')e^{-i2\pi xs}dx\right)dx'\\ &=\int f(x')\left(\int g(\chi)e^{-i2\pi(\chi+x')s}d\chi\right)dx'\\ &=\int f(x')e^{-i2\pi x's}G(\omega)dx'\\ &=F(\omega)G(\omega)\end{aligned}$$

Note: * is used here rather than the classical ⊗ as is commonly seen.

From Bracewell

Description of Blur
spatial and the frequency domain

$$I_{in}(x,y)\ \otimes\ P(\Delta x,\Delta y)\ =\ I_{out}(x,y)$$

$$\downarrow\qquad\qquad\downarrow\qquad\qquad\uparrow$$

$$\mathcal{F}\qquad\qquad\mathcal{F}\qquad\qquad\mathcal{F}^{-}$$

$$\downarrow\qquad\qquad\downarrow\qquad\qquad\uparrow$$

$$I_{in}(\omega_x,\omega_y)\ \times\ \mathrm{MTF}(\omega_x,\omega_y)\ =\ I_{out}(\omega_x,\omega_y)$$

Autocorrelation Theorem (Wiener-Khinchin theorem)

If $f(x)$ has the Fourier transform $F(\omega)$, then it's auto-correlation function has the fourier transform $|F(\omega)|^2$.

$$\int_{-\infty}^{\infty}|F(\omega)|^2e^{i2\pi x\omega}d\varpi=\int_{-\infty}^{\infty}f^*(u)f(u+x)du$$

- Proof:

$$\begin{aligned}\int_{-\infty}^{\infty}|F(\omega)|^2e^{i2\pi x\omega}d\omega &= \int_{-\infty}^{\infty}F(\omega)F^*(\omega)e^{i2\pi x\omega}d\omega\\ &= f(x)\otimes f^*(-x)\\ &=\int_{-\infty}^{\infty}f(u)f^*(u-x)du\\ &=\int_{-\infty}^{\infty}f^*(u)f(u+x)du\end{aligned}$$

f^* is the complex conjugate of f.

From Bracewell

As a special case of the autocorrelation theorem, the autocorrelation function of signal deviations is the Fourier transform of its noise power spectrum.

$$ACF(\chi,\eta)=\iint NPS(\omega_x,\omega_y)e^{+2\pi i(\chi\omega_x+\eta\omega_y)}d\omega_x d\omega_y$$

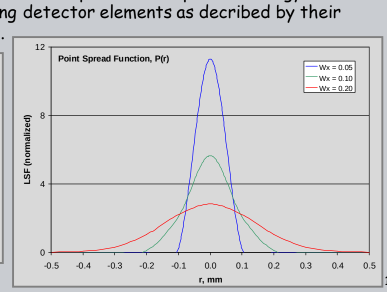
$$NPS(\omega_x,\omega_y)=\iint ACF(\chi,\eta)e^{-2\pi i(\chi\omega_x+\eta\omega_y)}d\chi d\eta$$

Where the NPS is the Fourier transform of signal deviations from the mean, ΔS,

$$NPS(\omega_x,\omega_y)=\left\langle\frac{1}{2X\,2Y}\left|\int_{-X}^{X}\int_{-Y}^{Y}\Delta S(x,y)e^{-2\pi i(x\omega_x+y\omega_y)}dxdy\right|^2\right\rangle$$

- To illustrate convolution, autocorrelation, and noise power, consider a radiograph taken with a uniform beam of 10^6 photons/mm^2.
- For a detector with elements of 0.1 mm x 0.1 mm size, an ideal detector would record an average of 10^4 photons with quantum noise having a standard deviation of 100.
- Consider three detectors that spread the deposited energy amongst the neighboring detector elements as decribed by their point spread functions.

- These 2D spread functions are normalized to have an integral value of 1.0.
- The widths have Wx values in mm that are the standard deviation value of a gaussian distribution.
- These spread function might describe;
  - high resolution
  - standard resolution
  - low resolution

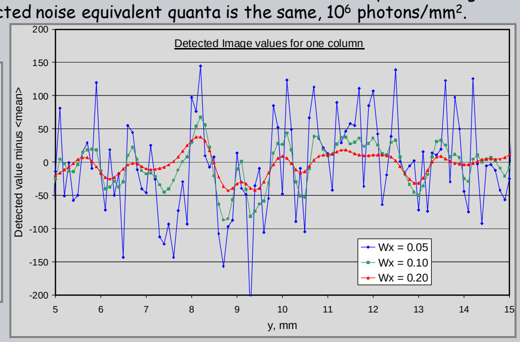


V.A.3 – Fourier Analysis Theorems

As expected the image values for the low resolution detector have noise deviations that are highly correlated from one element to the next. The standard deviation of the detected values vary even though the detected noise equivalent quanta is the same, 10^6 photons/mm^2.

Std. Dev.

- high resolution 63.9
- standard resolution 27.9
- low resolution 13.8

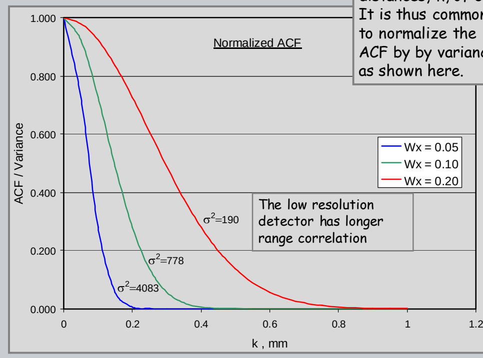


The detected values reflect the convolution of the ideal response and the point spread function.

V.A.3 – Fourier Analysis Theorems

The ACF describes the correlation with distance of the noise texture.

The ACF is equal to the variance, σ^2, for displacement distances, k, of 0. It is thus common to normalize the ACF by by variance as shown here.

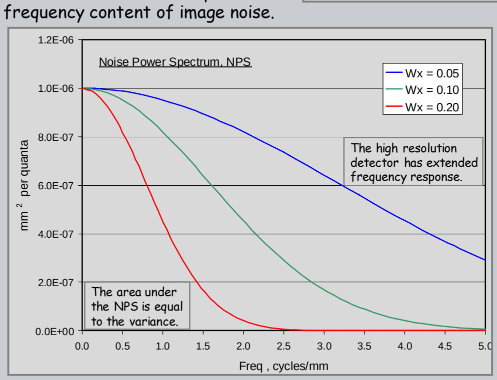


The low resolution detector has longer range correlation

V.A.3 – Fourier Analysis Theorems

The NPS describes the spatial frequency content of image noise.

The units of the NPS are the inverse of noise equivalent quanta units, 1/(quanta per mm^2)



The high resolution detector has extended frequency response.

The area under the NPS is equal to the variance.

V.A.3 – Fourier Analysis Theorems

- The area under the Noise Power Spectrum, NPS, is equal to the variance associated with the signal deviations from the mean, σ^2, as noted in the previous slide.
- This property is explained by Parseval's theorem.

Parseval's theorem

Loosely stated, Parseval's theorem says that the sum (or integral) of the square of the spatial function is equal to the sum (or integral) of the square of it's Fourier transform.

$$\int_{-\infty}^{\infty} |f(x)|^2 dx = \int_{-\infty}^{\infty} |F(\omega)|^2 d\omega$$

$$\sum_{n=0}^{N-1} |f_n|^2 = \frac{1}{N} \sum_{k=0}^{N-1} |F_k|^2$$

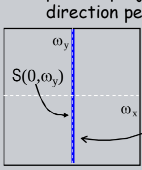
Parseval presented this theorem without proof in 1799, http://en.wikipedia.org/wiki/Parseval's_theorem

V.A.3 – Fourier Analysis Theorems

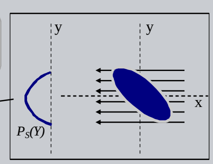
Barrett & Swindell, 1981, Pg 384

Central slice theorem

The values of the 2D Fourier transform along a line (slice) through the origin are equal to the 1D tranform of the parallel projection (line integral) of the spatial function in a direction perpendicular to the slice.



$$S(0,\omega_y) = \int_{-\infty}^{\infty} P_S(y) e^{-i2\pi(y\omega_y)} dy$$



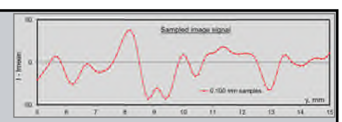
The proof can be easily understood by considering the 2D Fourier transform values along the ω_y axis.

$$S(\omega_x,\omega_y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} S(x,y) e^{-i2\pi(x\omega_x + y\omega_y)} dxdy$$

$$S(0,\omega_y) = \int_{-\infty}^{\infty} \left(\int_{-\infty}^{\infty} S(x,y) dx \right) e^{-i2\pi(y\omega_y)} dy$$

$$S(0,\omega_y) = \int_{-\infty}^{\infty} P_S(y) e^{-i2\pi(y\omega_y)} dy$$

V.A.3 – Fourier Analysis Theorems



The Sampling theorem

- The Nyquist-Shannon sampling theorem establishes a sufficient condition for a sampling interval that permits a discrete sequence of samples to capture all the information from a continuous band limited signal.
- The Fourier transform of a band limited signal will be zero above some limiting frequency
- The reconstruction is accurate if the signal is sampled at intervals of $\Delta x = 1/(2\ \omega_{lim})$.

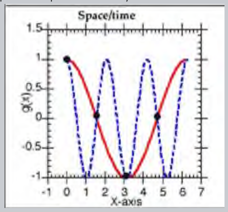
$\omega_{lim} = 1/(2\ \Delta x)$, is called the Nyquist frequency.

https://en.wikipedia.org/wiki/Nyquist-Shannon_sampling_theorem

In the 19th century, Helmholz related visual resolution to the spacing of receptors in the retina. In the 1930s this was used by Nyquist for commications signal sampling

V.A.3 – Fourier Analysis Theorems

http://research.opt.indiana.edu/Library/FourierBook/ch07.html#H2



The Sampling theorem and Aliasing

When the sampling theorem conditions are not met and signal frequencies extend beyond the Nyquist frequency, the signal is falsely sampled and the high frequency components appear as false low frequency signals.

A poorly sampled image of a brick building illustrates aliasing artifacts sometimes referred to as a Moire pattern.

http://www.svi.nl/AntiAliasing

V.B – Resolution

V) Fidelity of images

A. Linear Systems Analysis

B. Resolution (22 slides)

C. Noise

D. Detective efficiency

V.B.3 – Image detail

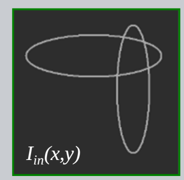
Good detector resolution permits the recording of detailed object structures.

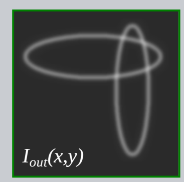
V.B.1 – Spatial Spread Functions - $P(x,y)$ and $P(r)$

- In the previous section, the output image was shown to equal the 2D input signal convolved with the system point spread function.

$$I_{out}(x,y) = I_{in}(x,y) \otimes P(x,y)$$

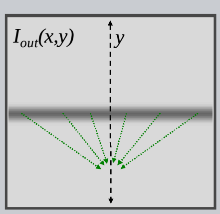
- The point spread function (PSF) can be written in polar coordinates, $P(r,\theta)$.
- For systems with an isotropic response, the PSF is then one dimensional, $P(r)$.



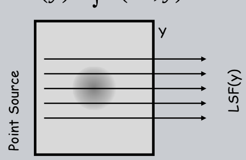


V.B.1 – Spatial Spread Functions - $L(y)$

- The response to a line source is known as the Line Spread Function (LSF), $L(y)$, which is the response normal to the line.
- For a linear system, the LSF can be computed as the integration of the PSF for point sources distributed in a line.
- Written in this form, the LSF can be seen to be the line integral of the PSF in the x direction for all values of y.
- This is often referred to as the projection of the PSF.

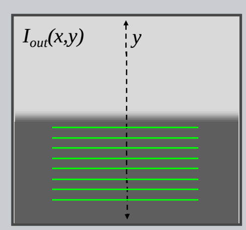


$$L(y) = \int P(-x, y)dx$$



V.B.1 – Spatial Spread Functions - $E(y)$

- The response normal to the edge of a broad source is known as the Edge Spread Function (ESF), $E(y)$.
- For a linear system, the ESF can be computed as the integration of the LSF for distributed line sources.
- Written in this form, the LSF can be seen to be the derivative of the ESF.

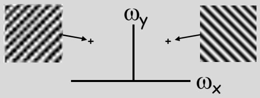


$$E(y) = \int_{y}^{\infty} L(y')dy'$$

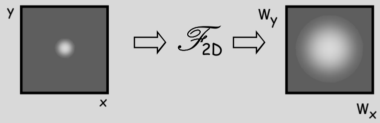
$$L(y') = \frac{d[E(y)]}{dy}$$

V.B.2 – Modulation Transfer Function (MTF)

- The 2D MTF is defined as the modulation transfer for 2D sinusoidal patterns of varying spatial frequency and orientation.



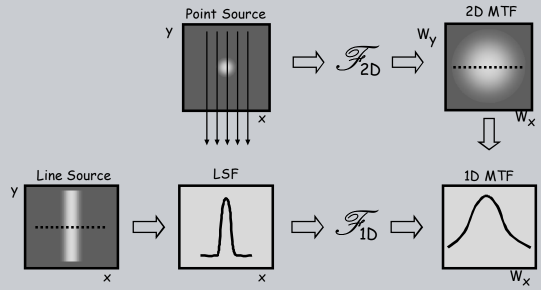
- The 2D MTF may be computed as the magnitude of the 2D Fourier transform of the systems point spread function, PSF.



- The PSF describes the systems response to x-rays normally incident to the detector at a specific point.

V.B.2 – MTF: Central Slice Theorem

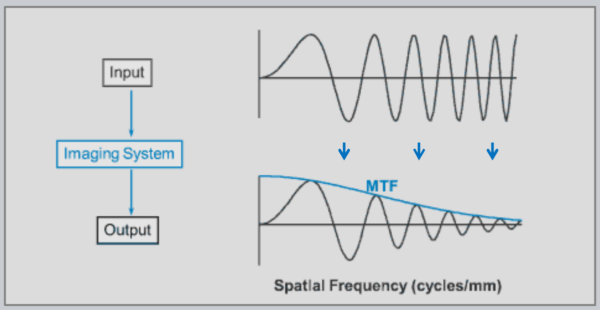
- The projection of a PSF is equal to the LSF for a line source oriented parallel to the projection direction.
- The 1D MTF is equal to the values of the 2D MTF along a line through the origin and perpendicular to the line source



K. Rossman, Radiology, Vol 93, 1964

V.B.3 – MTF

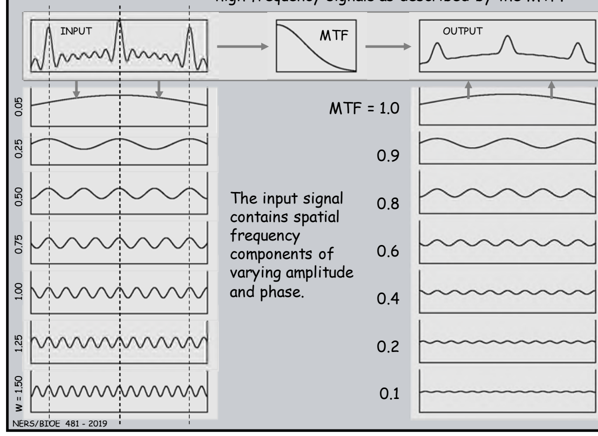
The modulation transfer function (MTF) quantifies the ability of the imaging system to transfer any given spatial frequency from an input to an output.



Rossman, 'PSF, LSF, & MTF, Tools for .. imaging systems', Radiology 93:257, 1969

V.B.3 – Image blur

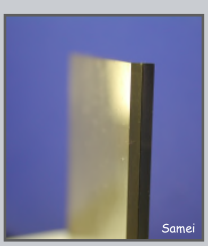
The imaging system reduces the strength of high frequency signals as described by the MTF.



V.B.3 – MTF: Edge Phantom

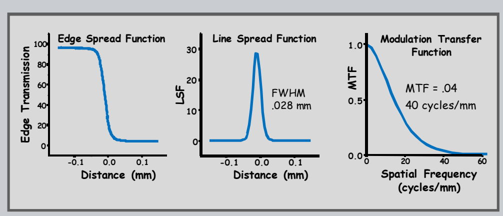
IEC 62220-1-2 ED. 1.0 B:2007

- MTF is commonly measured using an Edge Test Device.
- This edge test device has been fabricated by milling and then lapping the sides of a high Z material laminated between lucite slabs.



- Edge Test Device
  - 5 x 5 cm, 0.1-mm-thick polished Platinum-Iridium foil laminated between two 1-mm slabs of Lucite
- Image Acquisition
  - Device placement at the center of the field of view
  - Acquisition at ~ 4-6 mR (w/o grid)

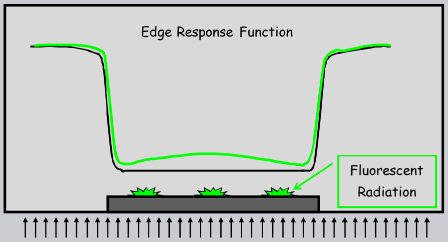
V.B.2 – MTF: Edge Spread Function



- The edge spread function, ESF, is generally the easiest reponse function to measure experimentally.
- The LSF can then be computed as the ESF derivative.
- Then the MTF can be computed from the LSF

•Samei, Med. Phys., V 25, N 1, 1998

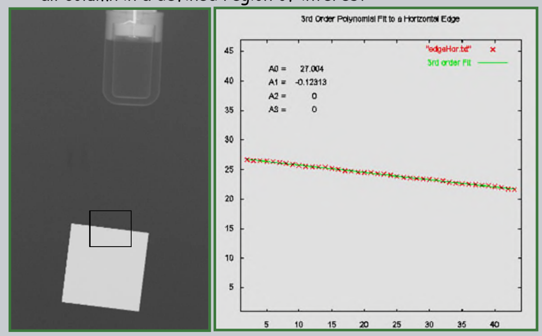
V.B.3 – MTF: Edge Phantom Material



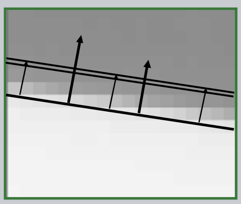
- If the kVp of the x-ray beam is above the binding energy of the edge material, fluorescent radiation can cause low spatial frequency distortion of the edge response.
- Edges are be made of very high Z material to minimize this effect;
  Lead, Platinum-Iridium, depleted Uranium
- For low energies, a Brass-Aluminum laminate would be appropriate

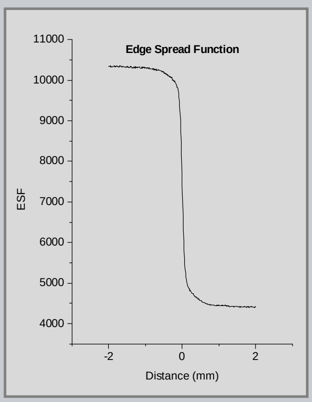
V.B.3 – Edge Fit

- Edge positions are estimated for each column using a numerical derivative of the column data.
- A polynomial fit is performed on the edge estimates for all column in a defined region of interest



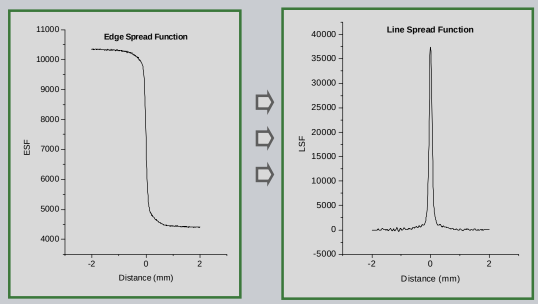
V.B.3 – MTF: Distance Binning





- For each pixel in the ROI;
  - The closest distance to the edge is determined and converted to an integer index.
  - The pixel value is added to a 1D array element.
- The average value of the ESF is then computed from the accumulated values.

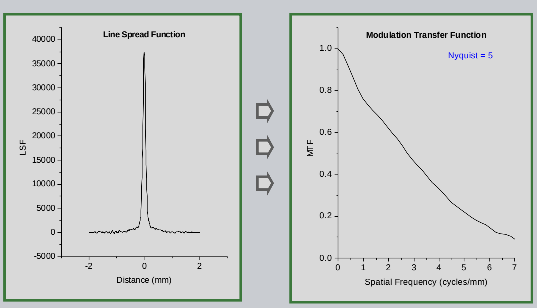
V.B.3 – MTF - ESF to LSF



ESF to LSF transformation

- The ESF is smoothed in regions of low slope using a moving polynomial fit
- The ESF is numerically differentiated to deduce the LSF
- The baseline of the LSF is corrected using the ends of the data

V.B.3 – MTF: LSF to MTF

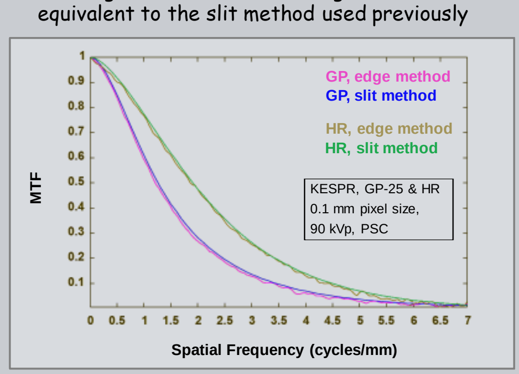


LSF to MTF transformation

- The LSF is multiplied by a Hamming window function.
- The magnitude of the Fourier transform of the LSF is computed.
- The MTF is determined by normalizedto 1.0 at a spatial frequency of 0.0.

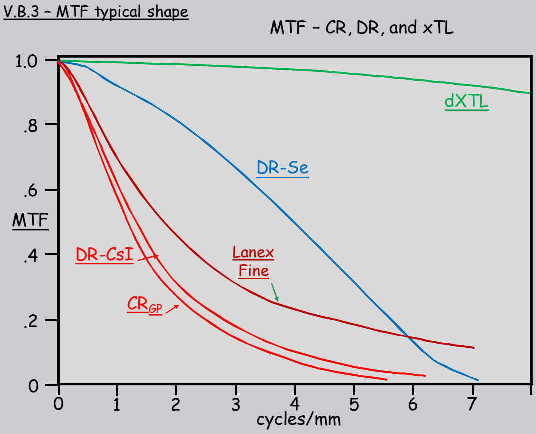
V.B.3 – MTF: Edge vs Slit

The edge method for measuring the MTF is equivalent to the slit method used previously

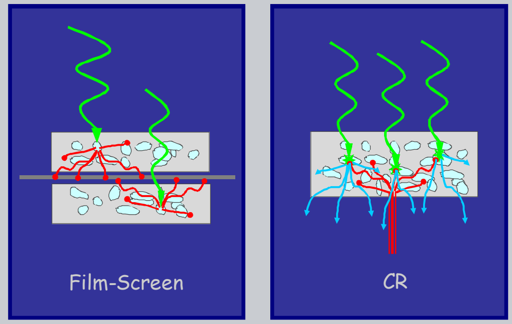


Samei, Med. Phys, 1998

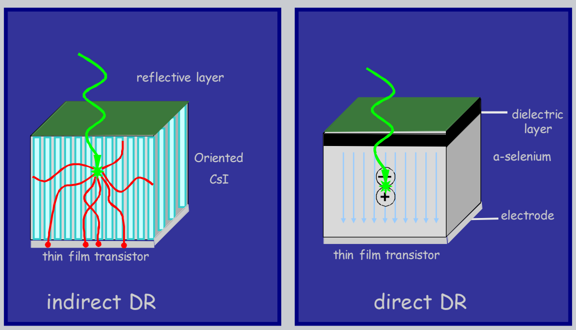
V.B.3 – MTF typical shape



V.B.3 – Image Blur from Light Diffusion

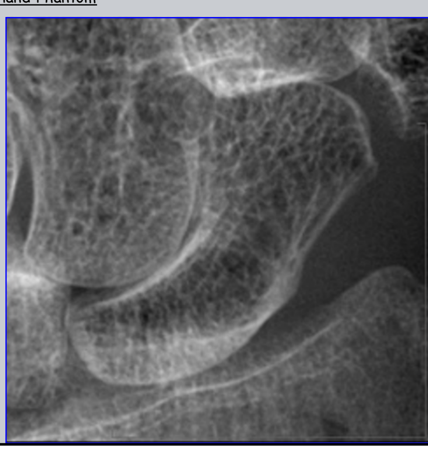


V.B.3 – Blur in Digital Radiography

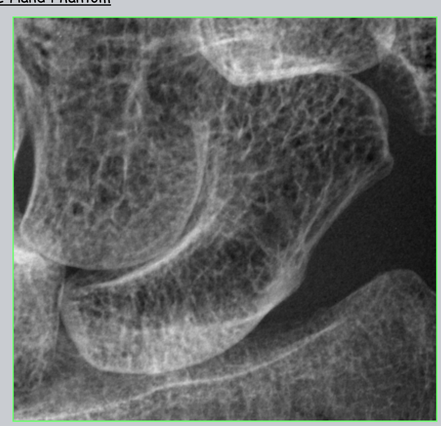


V.B.3 – CR Hand Phantom

V.B.3 – DR Hand Phantom



V.B.3 – XTL Hand Phantom

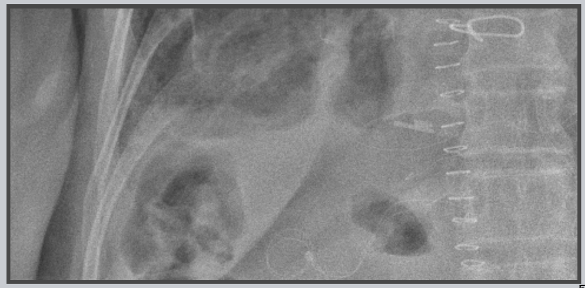


V.C - Noise

V) Fidelity of images
- A. Linear Systems Analysis
- B. Resolution
- C. Noise (22 slides)
- D. Detective efficiency

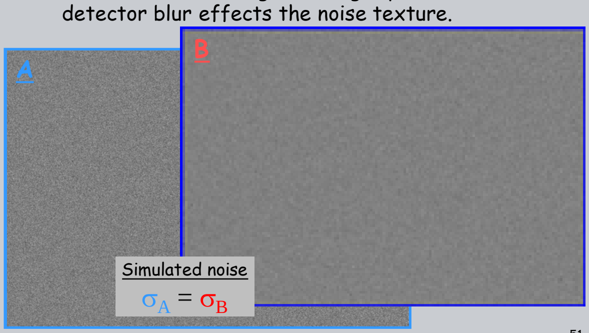
V.C.1 - Noise

- Image noise associated with quantum mottle limits the detection of large low contrast features.
- When evaluating the performance of CR/DR systems, the noise amplitude and texture should be consistent with the incident exposure and the detector type.



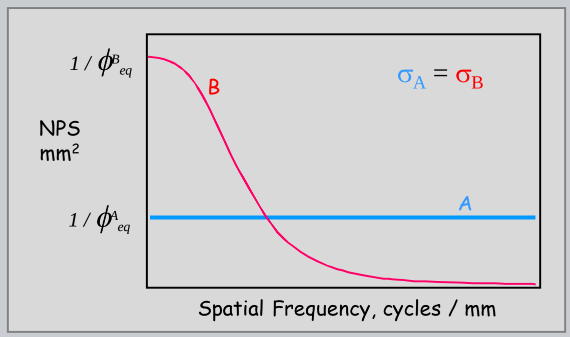
V.C.1 - Noise texture

- Statistical fluctuations in the number of x-rays detected in each pixel cause image noise.
- Correlation of the signal amongst pixels from detector blur effects the noise texture.



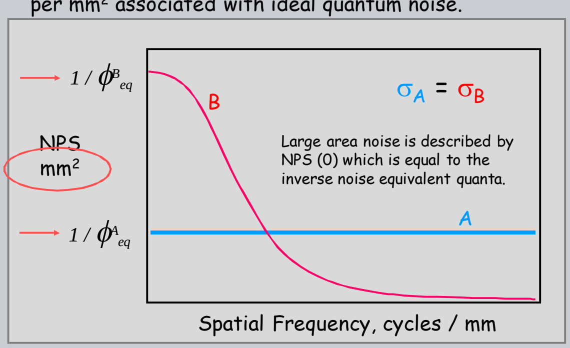
V.C.2 - Noise Power Spectrum, NPS

The relative strength of various spatial frequencies in a noise image is referred to as the noise power spectrum, NPS.



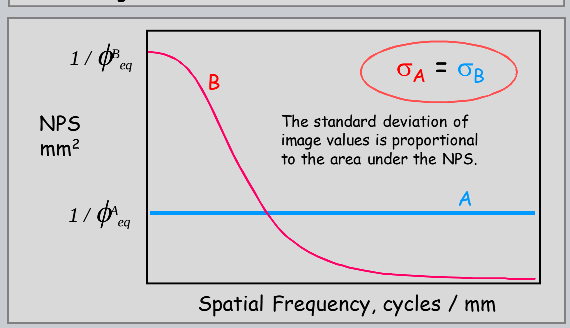
V.C.2 - NPS and Noise Equivalent Quanta

The units of the NPS are mm²/quanta. This is the inverse of ϕ_{eq} , the equivalent number of quanta per mm² associated with ideal quantum noise.



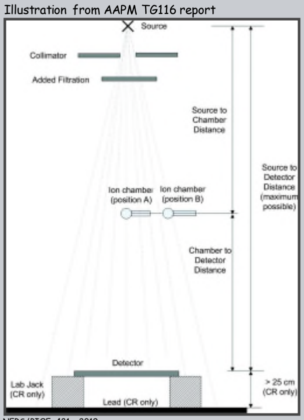
V.C.2 - NPS and Spatial Variations

- Smoothing operations reduce STD but not NPS(0).
- STD is NOT a good measure of low contrast detection when image texture and the NPS are different



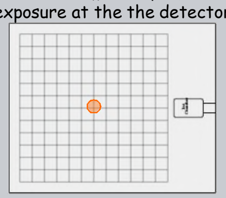
V.C.3 – NPS measurement conditions

Illustration from AAPM TG116 report



In the absence of instrument noise, the NPS is inversely proportional to the radiation exposure.

- The chamber is placed midway between the source and the detector and exposure measure at A and B.
- The measurements are corrected for distance and A to B offset.
- Measures at B are used to deduce estimates of the exposure at the the detector.



V.C.3 – NPS measurement - Radiographic Technique

IEC and AAPM have published documents on Exposure Indices that include a standard beam condition similar to RQA5.

	HVL	kV	Added Cu	Added Al
IEC 62494	6.8 +/- .3	70 +/- 4	0.5 mm	2 mm
AAPM TG116	6.8 +/- .25	70 +/- 4	0.5 mm	0-4 mm

Most previously published work has used IEC RQA beam conditions.

	Added Al	HVL mm Al	Nominal kVp	Typical kVp
RQA5	21 mm	7.1	70	72-77
RQA9	40 mm	11.5	120	120-124

It is now common to measure NPS over a range of exposure values

Suggested values:

0.1	0.3	0.6	1.0	3.0	6.0	10.0	mR
-	0.3	-	1.0	-	6.0	-	mR

V.C.3 – Linear 'For Processing' image data

- Ideally, we would like to obtain image data that is linear with exposure but includes
  - Defective pixel corrections
  - Gain and offset corrections (flat field)
- Systems often export 'For Processing' images data that include these corrections but are proportional to the log of the detector input exposure, E. If the pixel value relationship is known, a linear approximation for small signal deviations can be used.

$$\bar{I}_p = A + B\ln(\bar{E})$$

$$\bar{I}_p + \Delta I = A + B\ln(\bar{E} + \Delta E)$$

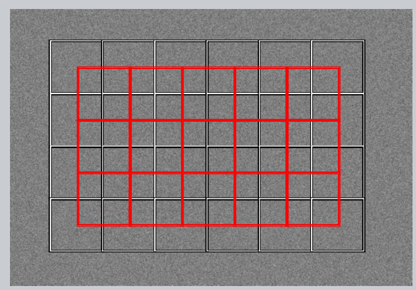
$$\Delta I = B\ln\left(1 + \frac{\Delta E}{\bar{E}}\right) \cong B\left(\frac{\Delta E}{\bar{E}}\right)$$

AAPM TG116 recommends that exported 'For Processing' images have pixel values with $I_p = 1000\log_{10}(K_{std})$ where K_{std} is the standard beam input air kerma in nanoGray units

V.C.3 – NPS: 2D Block Average

To reduce noise in the estimate of the NPS at the expense of spectral resolution, the 2D NPS is computed for many small blocks using a 2D FFT and the results are averaged.

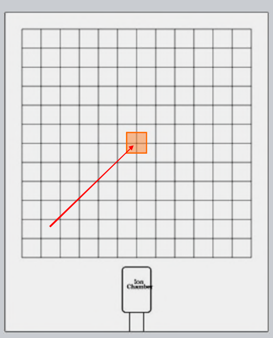
Additional improvement in the noise of the NPS estimate can be achieved by computing the 2D FFT from overlapping blocks.



Flynn, Med. Phys., V 26, N 8, 1999

V.C.3 – NPS adjustment for non-uniformity

- The linear signal mean in each block is used to correct the NPS of each block to the exposure at the center of the detector.
- This accounts for variations in the input signal due to the x-ray tube heel effect.



V.C.3 – FFT estimate of the NPS

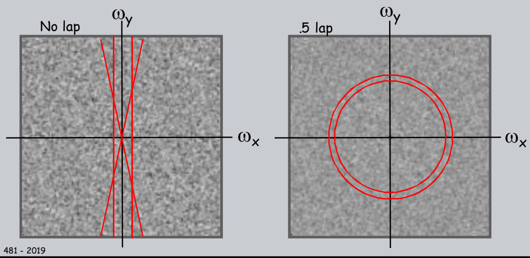
The estimate of the NPS for each block is done using a Fast Fourier Transform, FFT, as described in Flynn1999.

1. A bi-quadratic surface is fit to the block values to obtain the mean value and low frequency trend (see Zhou, MedPhys 2011)
2. Relative noise deviations are computed based on whether the data is linear or logarithmic.
3. Values are adjusted for image pixel area.
4. Block values are modified by a spectral window function (Hamming).
5. The NPS is computed as the magnitude squared of the Fourier transform.

Flynn, Med. Phys., V 26, N 8, 1999

V.C.3 – NPS: 1D Results from 2D NPS

- The 2D NPS can be displayed as an image with values proportional to the log of the NPS.
- An 1D estimate can be derived from the 2D NPS by averaging all values within;
  - NPS(y) : A horizontal band about the w_x axis
  - NPS(x) : A vertical band about the w_y axis
  - NPS(r) : A circular band centered on the origin



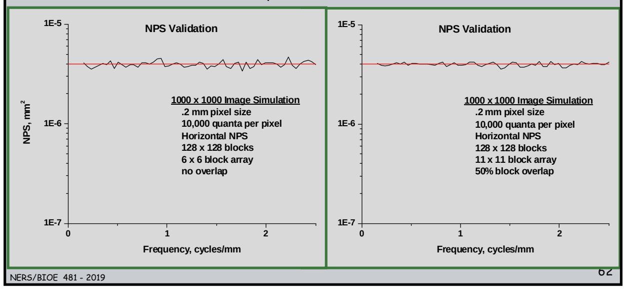
V.C.3 – NPS: 2D Block Average

NPS for a simulated image

uncorrelated gaussian noise

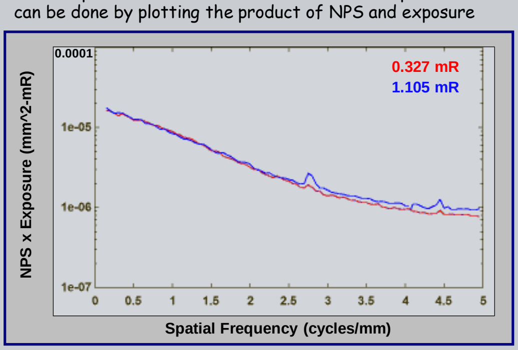
10,000 #/pixel x 25 pixels/mm^2 = 250,000 #/mm2

1/250,000 = 4E-6 mm^2



V.C.3 – NPS: NPS exposure product

The comparison of results made at different exposures can be done by plotting the product of NPS and exposure

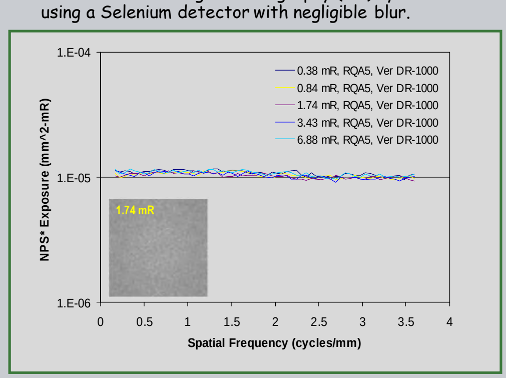


NPS for a Computed Radiography (CR) system

Samei & Flynn, SPIE, 1997

V.C.3 – NPS: Se DR detector

NPS for a direct digital radiography (DR) system using a Selenium detector with negligible blur.



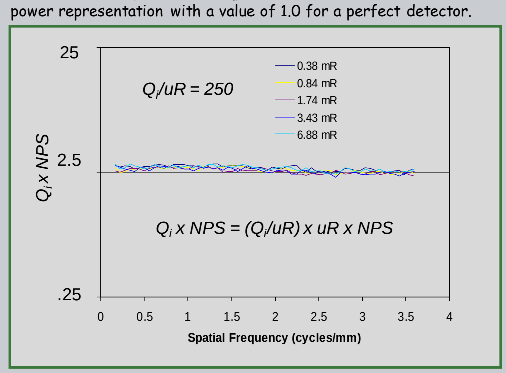
V.C.3 – NPS for an ideal detector

- Thicker detectors with more efficient absorption produce less noise and lower NPS for the same exposure.
- It is informative to compare the measured NPS to that expected from an ideal detector.
- The NPS for an ideal detector will be constant in relation to frequency with a value equal to the inverse of the noise equivalent quanta for an ideal integrating detector, $1/\Phi_{eq}$.
- From lecture 05;

$$\Phi_{eq}^{ideal} = \frac{\left(\int_0^{kVp} E\Phi(E)dE\right)^2}{\int_0^{kVp} E^2\Phi(E)dE} = Q_i$$

V.C.3 – NPS: Se DR detector

Multiplying the NPS (mm2/quanta units) by the quanta/mm2 incident on the detector (i.e. ideal NEQ) results in a dimensionless noise power representation with a value of 1.0 for a perfect detector.



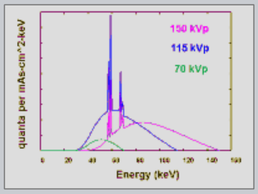
Samei & Flynn, Med.Phys., 2003

V.C.3 – NPS for an ideal detector

For experimental use, values of Q_i and exposure, X, are computed using a model of the spectral shape and expressed as Q_i/X in relation to kVp.

From L05

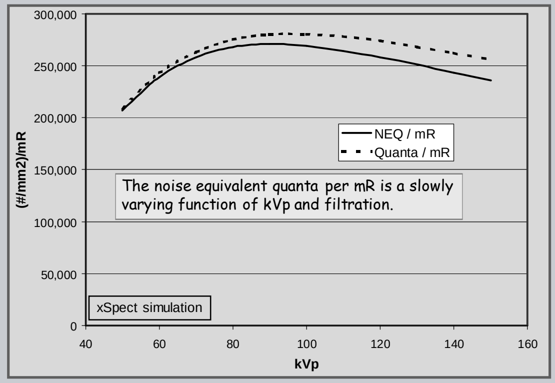
$$Q_i = \frac{\left(\int_0^{kVp} E\Phi(E)dE\right)^2}{\int_0^{kVp} E^2\Phi(E)dE}$$



From L03

- Values for X_f are obtained by computing the energy absorbed in air for the spectrum $\Phi(E)$ using mass enery-absorption coefficient data obtained from the National Institute of Standards and Technology.
- The energy absorbed in air is then converted to charge using a W value of 33.97 J/C (i.e. eV/ion pair) (Boutillon 1987).

V.C.3 – Q,quanta per mR



Ideal Noise Equivalent Quanta

[ ] denotes converted values

Reference	Beam Conditions	Added Filtration	Method	Q, #/mm2 per µR	Q, #/mm2 per nG
Dobbins ... MedPhys 1992	70 kVp -	0.5 mm Cu	Provided by Manf.	270	[30.8]
Kengylelics ... MedPhys 1998	75 kVp -	1.5 mm Cu	Photon fluence	[310]	35.4
Stierstorfer .. MedPhys 1999	70 kVp 7.1 mm HVL	21 mm Al	NEQ (energy intgr. detector)	[258]	29.4
Flynn & Samei MedPhys 1999	70 kVp 6.3 mm HVL	19 mm Al	NEQ (energy intgr. detector)	262	[29.9]
Samei & Flynn MedPhys 2002	70 kVp -	19 mm Al	NEQ (energy intgr. detector, HVL adj.)	246-249	[28.1-28.4]
Granfors ... MedPhys 2003	75 kVp 7.0 mm HVL	20 mm Al	Photon fluence	[261]	29.8
IEC 62220-1 1st ed. 2003	77 kVp 7.1 mm HVL	21 mm Al (RQA 5)	Photon fluence	[264]	30.2
Samei & Flynn MedPhys 2003	74-78 kVp 7.1 mm HVL	21 mm Al	NEQ (energy intgr. detector, HVL adj.)	256-259	[29.2-29.6]
Samei MedPhys 2003	70 kVp 6.4,6.5 mmHVL	19 mm Al	NEQ (energy intgr. detector, HVL adj.)	255-258	[29.1-29.5]
Siewerdsen ... MedPhys 2005	60, 80 kVp -	4 mm Al 0 .6 mm Cu	Photon fluence	259, 283	[29.6,32.3]

Q per µR for TG116/IEC beam conditions

Noise equivalent quanta computed from a spectral model.

- Qi/uR E integr: Ideal energy integrating detector
- Qi/uR Fluence: Ideal counting detector
- Qi/uR dDR: Direct DR detector (.5 mm Se)

kV	Added Cu Filtration	Added Al Filtration	HVL	Qi/µR E integr.	Qi/µR Fluence	Qd/µR dDR
70	0.5 mm	0.0 mm	6.6	251	(258)	145
70	0.5 mm	2.0 mm	6.8	255	(262)	144
70	0.5 mm	4.0 mm	7.0	258	(265)	144

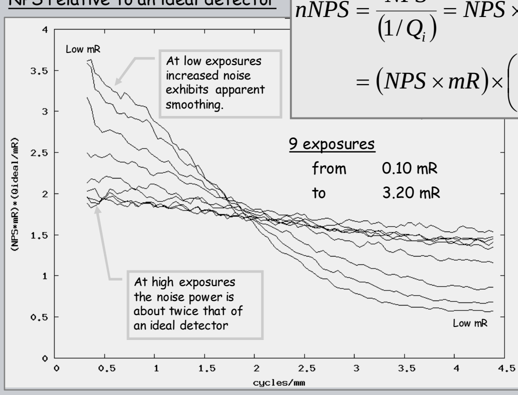
- Within the range of added filtration and HVL acceptable for AAPM EI beam conditions, the Qi per mR varies by about +/- 1.5%
- The Qi per mR for an ideal counting detector is about 2.7% higher that that for an ideal energy integrating detector.
- The noise equivalent quanta, and therefor NPS(0) for a DR detector varies little with beam conditions.

V.C.3 – Normalized NPS

NPS relative to an ideal detector

$$nNPS = \frac{NPS}{(1/Q_i)} = NPS \times Q_i$$

$$= (NPS \times mR) \times \left(\frac{Q_i}{mR}\right)$$



V.D – Detective Efficiency

V) Fidelity of images
A. Linear Systems Analysis
B. Resolution
C. Noise
D. Detective efficiency (6 slides)

V.D.1 – Frequency dependant SNR

- Since the *NPS* is normalized to the inverse of ϕ_{eq} at zero frequency,
- and ϕ_{eq} is understood to be the square of the signal to noise ratio,
- the $NPS(\omega)$ can be understood as the square of the frequency dependant noise relative to the overall signal,

$$NPS(\omega) = \left(\frac{\sigma(\omega)}{S}\right)^2$$

- Since the frequency dependant signal is $S * MTF(\omega)$, the frequency dependant signal to noise ratio can be defined as,

$$\{SNR(\omega)\}^2 = \frac{(S \times MTF(\omega))^2}{(\sigma(\omega))^2} = \frac{(MTF(\omega))^2}{NPS(\omega)}$$

Note: as written, this is the actual frequency dependant SNR that can be computed from experimental measures of the MTF and NPS

V.D.1 – Frequency dependent NEQ

The frequency dependent SNR, $SNRmeas(\omega)$, has been referred to as the frequency dependent NEQ, $NEQ(\omega)$ or $\phi_{eq}(\omega)$.

$$\{SNR_{meas}(\omega)\}^2 = \frac{(MTF(\omega))^2}{NPS(\omega)} = NEQ(\omega)$$

Since $NPS(\omega)$ is equal to $1/\phi_{eq}$ for $f=0$,
and $MTF(\omega) = 1.0$ for $f=0$,
$SNRmeas(\omega) = \phi_{eq}$ for $f=0$

The frequency dependent $NEQ(\omega)$ is thus consistent with the NEQ and ϕ_{eq} that we have previously considered.

- Shaw, R., "Thé Equivalent Quantum Efficiency of the Photographic Process," J. Photogr. Sci., Vol. 11, pp. 199 -204. 1963.
- J. C. Dainty and R. Shaw, Image Science Academic Press, London, 1974. (a) Ch. 5. (b) Ch. 8.

V.D.2 – DQE: Detective Quantum Efficiency

- An ideal detector was previously defined as one that detects all of the energy of all of the incident radiation. If it also has no blur, the ideal $SNR(\omega)^2$ will be the ideal Φ_{eq} , i.e. the Q_i defined in the prior section, and will be constant for all spatial frequencies.
- A popular figure of merit (FOM) is to compare the measured $SNR(\omega)$ to the ideal $SNR(\omega)$. This frequency dependant FOM is known as the Detective Quantum Efficiency;

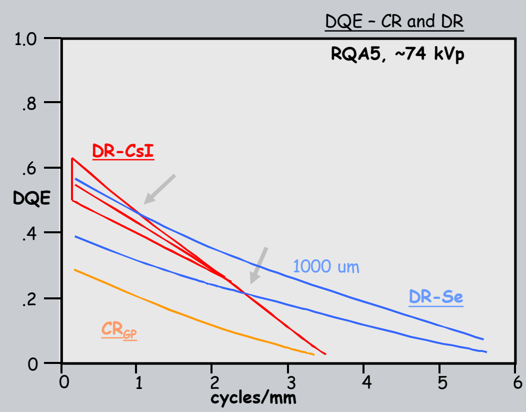
$$DQE(\omega) = SNR^2_{meas}(\omega) / SNR^2_{ideal}$$

$$DQE(\omega) = \frac{(MTF^2(\omega)/NPS(\omega))}{Q_i}$$

$$DQE(\omega) = \frac{MTF^2(\omega)}{Q_i \times NPS(\omega)} = \frac{MTF^2(\omega)}{nNPS(\omega)}$$

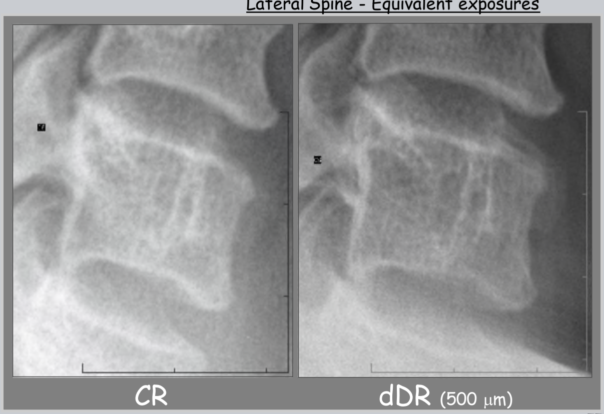
Note: DQE(ω) is seen above to be just the ratio of the MTF(ω) squared to the normalized NPS obtained by using the ideal quanta per mR.
It is usually more informative to report the MTF and normalized NPS separately rather than combining them into one FOM.

V.D.3 – experimental DQE examples



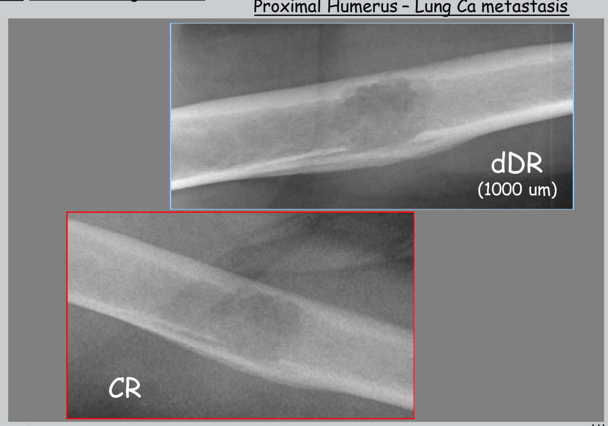
V.D.3 – Spine signal/noise

Lateral Spine - Equivalent exposures



V.D.3 – Arm signal/noise

Proximal Humerus – Lung Ca metastasis



Is the diagnostic value of reduced noise (dose) in mediastinal regions more important than the value of improved detail in lung regions where quantum noise is minimal

?

EKC CR 900