



UNIVERSITY OF MICHIGAN 

NERS/BIOE 481

Lecture 11 B
Computed Tomography (CT)

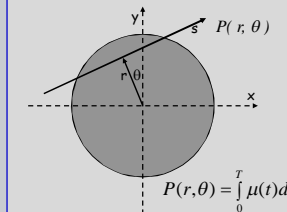
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mikef@rad.hfh.edu



Henry Ford
Health System

RADIOLOGY RESEARCH

VII.B.1 - X-ray projection measurements



For an object with a variable attenuation coefficient $\mu(x,y)$, the transmitted x-ray intensity is given by the projection;

$$I(r, \theta) = I_o \exp[-P(r, \theta)]$$

Thus the projection can be deduced by measuring the transmission;

$$P(r, \theta) = -\text{Log}_{nat}[I(r, \theta)/I_o]$$

- CT scanner devices are periodically calibrated using a phantom to determine the reference signal I_o .
- The projection, $P(r, \theta)$, is determined using correction factors for x-ray spectral hardening and scattered radiation.

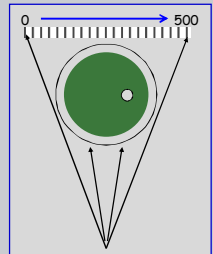
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VII - Computed Tomography

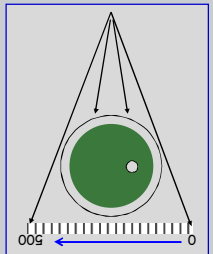
A) X-ray Computed Tomography ... (L11)
B) CT Reconstruction Methods ... (L11/L12)

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VII.B.1 - Fan beam projection views - 0 & 180 degrees

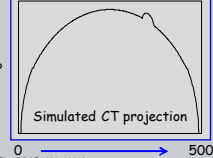


0 500

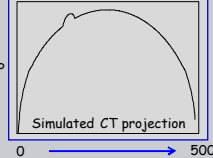


005 0

As a CT gantry rotates, the projection of a small target is recorded on the detector at positions that shift from one side to the other.



0 500



0 500

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VII.B - CT Reconstruction

B) CT Reconstruction (L12)

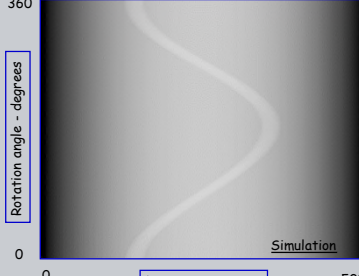
- 1) [Projection geometry \(5 slides\)](#)
- 2) Fourier Domain Solution
- 3) Convolution / Backprojection
- 4) Cone beam reconstruction
- 5) Iterative Reconstruction

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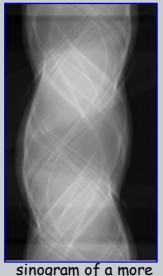
VII.B.1 - Projection views: 0° to 360°

Sinogram:

- An image with the projection values organized as rotation angle versus detector position is referred to as the sinogram.
- The sinogram depicts all of the transmission data used to perform a reconstruction of the object attenuation values.



360
0
0 500



sinogram of a more complex object

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VII.B.1 - Inverse solution (computed tomogram)

$\mu = .022$
 $\Delta\mu_{rel} = .1$
 $H^\# = 100$

$\mu = .020$
 $\Delta\mu_{rel} = 0$
 $H^\# = 0$

Simulation

$\mu_{H2O} = .020$

- Attenuation values:**
Image reconstruction results in the value for the material attenuation coefficient.
- Hounsfield Units (HU):**
Medical standards define the Hounsfield number as the reconstructed attenuation coefficient relative to water,

$$\Delta\mu_{rel}(x,y) = (\mu(x,y) - \mu_{H2O}) / \mu_{H2O}$$

$$H^\# = 1000 \Delta\mu_{rel}(x,y)$$

$H^\# \text{ water} = 0$
 $H^\# \text{ air} = -1000$

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VII.B.2 - Central Slice Theorem Theorem first presented in L07

The central slice theorem from Fourier analysis provides a method to easily demonstrate that an object can be reconstructed from projections.

The values of the 1D transform of an object projection are equal to the values of the 2D transform of the object along a line through the (0,0) coordinate that is perpendicular to the projection direction.

NERS/BIOE 481 - 2019 Barrett & Swindell, 1981, Pg 384 10

VII.B.1 - CT tissue values

CT numbers for Medical CT images

- For soft tissues, the Hounsfield numbers are between 0 and 100.
- This corresponds to a 1% range of attenuation coefficient values.
- Air (~-1000) and bone (> 1000) provide high contrast.

CT - number = $\frac{\mu - \mu_{\text{water}}}{\mu_{\text{water}}} \cdot 1000$

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VII.B.2 - Central Slice Theorem - proof Theorem first presented in L07

The central slice theorem is easily proven by considering the values of the Fourier transform of an object, $O(x,y)$, along the $\omega_y = 0$ axis,

$$\mathfrak{Z}(\omega_x, \omega_y) = \iint O(x, y) \cdot e^{-2\pi i(\omega_x x + \omega_y y)} dx dy$$

$$\mathfrak{Z}(\omega_x, 0) = \int \left(\int O(x, y) dy \right) \cdot e^{-2\pi i(\omega_x x)} dx$$

$$\mathfrak{Z}(\omega_x, 0) = \int (P(r, 0)) e^{-2\pi i(\omega_x x)} dx$$

The inner integration reduces to the projection in a direction parallel to the y axis ($\theta=0$). Other directions can be considered by a simple rotation of the object.

NERS/BIOE 481 - 2019 Barrett & Swindell, 1981, Pg 384 11

VII.B - CT Reconstruction

B) CT Reconstruction

- 1) Projection geometry
- 2) **Fourier Domain Solution (9 slides)**
- 3) Convolution / Backprojection
- 4) Cone beam reconstruction
- 5) Iterative Reconstruction

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VII.B.2 - Fourier reconstruction method.

Projections measured from many directions are transformed to describe the 2D Fourier transform of the object. The Object material properties are estimated using the 2D inverse Fourier transform

The Fourier coefficients are interpolated from (r, θ) to (x, y) coordinates

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VII.B.2 - Angular sampling requirement

Full sampling of the Fourier domain requires that the radial frequency coefficients be closely spaced in the high frequency portion of the domain.

$\Delta\theta = 2/N$ radians
 $N_\theta = \pi/\Delta\theta = N\pi/2$

$\Delta\omega$ is determined by the detector sampling pitch, Δu .
 $\omega_{\min} = 1/2\Delta u$
 $\Delta\omega = 2 \omega_{\min} / N = 1/\Delta u$

$N \times N/2$ frequency coefficients to reconstruct an $N \times N$ image.

Angular sampling may be doubled to overlap the detectors element for each projection sample

Views required to reconstruct a 512x512 image

800 views	180°	
1600 views	360°	quarter offset geometry
3200 views	360°	1/4 offset + double sampling

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VII.B.2 - Cone beams, circular orbits

A fan beam of radiation used to acquire $P(u)$ with angular sampling produces frequency samples in the 2D Fourier domain in arcs through the 0,0 axis.

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VII.B.2 - quarter-quarter offset

- Angular sampling over 180 degrees is sufficient to describe an object in the Fourier domain.
- However, 360 degree sampling is commonly done with the rotation center offset by $(\frac{1}{4}, \frac{1}{4})$ of the sample increment, $\Delta\mu$.

$\frac{1}{4}, \frac{1}{4}$ offset sampling improves resolution by decreasing the effective sampling increment, $\Delta\mu$, by a factor of two.

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VII.B.2 - Cone beams, circular orbits

A cone beam of radiation used to acquire $P(u,v)$ with angular sampling DOES NOT FULLY SAMPLE the 3D Fourier domain in the region of the axis.

Each projection is associated with a dish shaped surface of Fourier coefficients going through the 3D frequency domain. When rotated, there is a void of coefficients along the axis of rotation.

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VII.B.2 - Parallel beams, circular orbits

A parallel beam of radiation used to acquire $P(u,v)$ using circular rotational sampling completely samples the 3D Fourier domain.

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VII.B.2 - cone beam, circle plus line

- The Radon values of all planes intersecting the object have to be known in order to perform an exact reconstruction. The Tuy sufficiency condition (Tuy 1983) states that exact reconstruction is possible if all planes intersecting the object also intersect the source trajectory at least once.
- The circular trajectory does not satisfy the Tuy-Smith condition as illustrated. It is therefore necessary to extend the trajectory with an extra circle or line if exact reconstruction is required.

Tuy, H. (1983). An inversion formula for cone-beam reconstruction. *SIAM Journal of Applied Mathematics* 43, 546-552.

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VII.B - CT Reconstruction

B) CT Reconstruction

- 1) Projection geometry
- 2) Fourier Domain Solution
- 3) Convolution / Backprojection (11 slides)
- 4) Cone beam reconstruction
- 5) Iterative Reconstruction

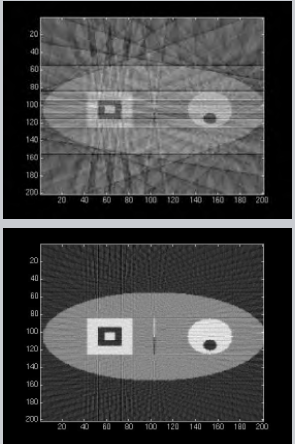
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VII.B.3 - Girod example

Filtered - Backprojection

1. Measure projections.
2. Filter projections.
3. Backproject.

For every point in the reconstruction image, the value for each filtered projection is interpolated and added to the the image.

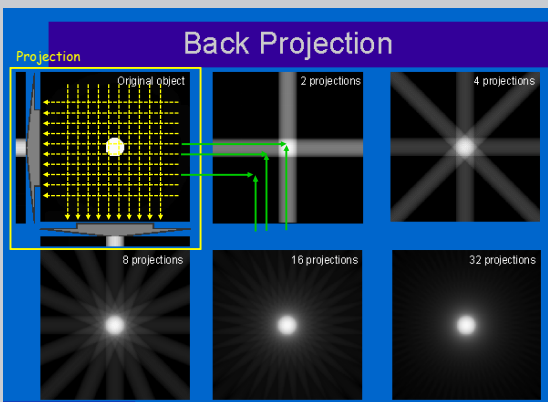


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VII.B.3 - Back Projection Method

From: impactscan.org

Back Projection

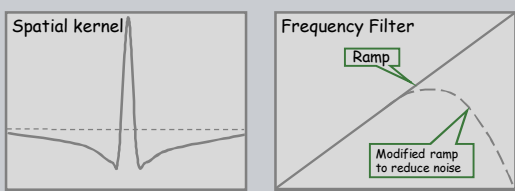


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VII.B.3 - Filter shape

Projections are filtered either by

- convolution with a spatial kernel or
- Fourier transformations with a filter function



Equivalent:

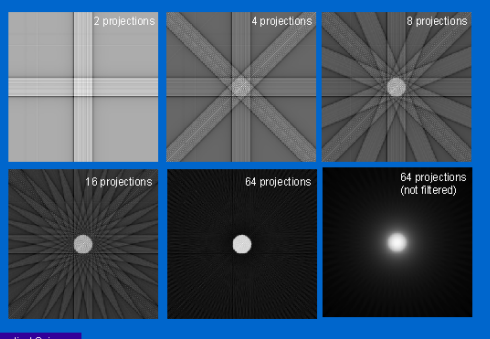
- Convolution Backprojection
- Filtered Backprojection

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VII.B.3 - Filtered projections

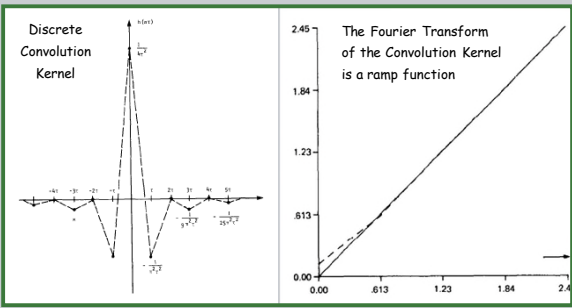
From: impactscan.org

Filtered back projection **FPB**



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VII.B.3 - Discrete kernels/filters



The Fourier Transform of the Convolution Kernel is a ramp function

- A. C. Kak and Malcolm Slaney, Principles of Computerized Tomographic Imaging, IEEE Press, 1988.
- <http://www.slaney.org/pct/pct-toc.html>

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VII.B.3 - Modified filters

The ideal filter (ramp) is usually modified to smooth noise or sharpen edges.

Smooth Sharp

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VII.B.3 - FBP reconstruction, 2D parallel, discrete notation

- A useful solution for the noise in a reconstruction can be found for the special case of a cylindrical homogenous object.
- We then consider only the noise of the reconstruction in the center which is influenced by the noise in the central ray projection, P_0 .
- The central ray projections are rotationally similar with a noise of σ_p .

Note: The fan beam solution (central cone) is the same as the parallel beam for the central ray (see VII.B.4).

- If the projection noise does not vary with angle, σ_p , then projection variance, σ_p^2 , can be taken out of the summations.
- The angular summation is now trivial and results in an N_θ term that cancels one in the denominator.

$$\sigma_{\mu(x,y)}^2 = \frac{(\pi \Delta u)^2}{N_\theta} \alpha^2 \sigma_p^2$$

$$\alpha^2 = \sum_{l=-\infty}^{+\infty} \{K(l\Delta u - u)\}^2$$

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VII.B.3 - FBP reconstruction, 2D parallel, integral notation

- Consider (x, y) positions in a plane of an object rotated about the z axis thru θ degrees.
- The projection thru this point to the position u on the detector is,

$$P(u, \theta) = \int \mu(s) ds$$

$$u(x, y, \theta) = x \sin(\theta) + y \cos(\theta)$$

For each point (x, y) , the value of μ is equal to the integral of the convolved projection over all angles where:

- $u = u(x, y, \theta)$
- The convolution Kernel is the inverse Fourier transform of the ramp function, $|\omega|$, and any additional smoothing filter, $F(\omega)$.

$$\mu(x, y) = \int P^*(u, \theta) d\theta$$

$$P^*(u, \theta) = P(u, \theta) * K(u)$$

$$= \int K(u' - u) P(u', \theta) du'$$

$$K(u) = \int F(\omega) |\omega| e^{i\omega u} d\omega$$

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VII.B.3 - FBP reconstruction, 2D parallel, discrete notation

- A common smoothing function used to modify the ramp filter is the sinc function, $\text{sinc}(\omega)/\omega$.
- For this the α^2 term is a function of the limiting spatial frequency,

$$\omega_{lim} = 1/(2\Delta u)$$

Note: See the lecture notes on CT noise propagation for the derivation of α^2 .

$$\alpha^2 = \frac{\omega_{lim}^2}{2\pi^2} \text{ sinc filter}$$

$$\sigma_\mu^2 = \frac{\omega_{lim}^2}{2N_\theta} \sigma_p^2 = \frac{1}{2^3 N_\theta (\Delta u)^2} \sigma_p^2$$

We recall now that the Projection is proportional to the natural log of the detected signal and thus the projection noise is equal to the relative noise of the detector signal.

$$P = -\ln(S/S_0) = \ln(S_0) - \ln(S), \quad \sigma_p = \sigma_S/S$$

In terms of the noise equivalent quanta, Q_{eq} , the projection noise is thus,

$$\sigma_p^2 = 1/SNR^2 = 1/(Q_{eq} A_d) = 1/(Q_{eq} \Delta u S_w)$$

where A_d = detector area, S_w = slice width.

$$\sigma_\mu^2 = \frac{1}{2^3 N_\theta (\Delta u)^2} \frac{1}{Q_{eq} \Delta u S_w} \sigma_p^2 = \frac{1}{2^3 N_\theta (\Delta u)^3 S_w} \left(\frac{1}{Q_{eq}} \right)$$

Note:

- For cone beam CT, $S_w = \Delta u$, and the CT noise is inversely proportional to the pixel area.
- Noise is inversely proportional to $(mas)^{1/2}$ due to Q_{eq} .
- CT SNR = μ / σ_μ

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VII.B.3 - FBP reconstruction, 2D parallel, discrete notation

- For a detector with discrete elements spaced at a distance of Δu , the convolution can be written as a sum over the discrete kernel.
- While this sum is written with infinite limits, it is bounded by the object beyond which P is zero.
- The reconstruction can be similarly written as a discrete sum with a constant corresponding to the angular range.

$$P^*(u, \theta_i) = \sum_{l=-\infty}^{+\infty} K(l\Delta u - u) P(l\Delta u, \theta_i) \Delta u$$

$$\mu(x, y) = \frac{\pi}{N_\theta} \sum_{k=1}^{N_\theta} P^*(u, \theta_k)$$

If we write the solution as a double sum for the convolution and the backprojection, we can see that the noise of the results will be determined by the noise in the projection values at each position and angle.

$$\mu(x, y) = \frac{\pi}{N_\theta} \Delta u \sum_{k=1}^{N_\theta} \sum_{l=-\infty}^{+\infty} K(l\Delta u - u) P(l\Delta u, \theta_k)$$

$$\sigma_{\mu(x,y)}^2 = \left\{ \frac{\pi}{N_\theta} \Delta u \right\}^2 \sum_{k=1}^{N_\theta} \sum_{l=-\infty}^{+\infty} \{K(l\Delta u - u)\}^2 \sigma_{p(l\Delta u, \theta_k)}^2$$

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- The Noise Power Spectrum (NPS) for CT reflects the modified ramp function used to filter the projections.
- As expected, the variance of reconstructed image values will be proportional to the area under the NPS.

Ghetti2013, JACMP
NPS measured for a SOMATOM Definition Flash CT scanner (Siemens).

- B40s is a standard FBP filter.
- I40s S[1-5] are filters used with the SAFIRE reconstruction algorithm.

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VII.B - CT Reconstruction

B) CT Reconstruction

- 1) Projection geometry
- 2) Fourier Domain Solution
- 3) Convolution / Backprojection
- 4) Cone beam reconstruction (11 slides)
- 5) Iterative Reconstruction

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VII.B.4 - 3D Solution, parallel beam

- Resample detector data for alignment with rotation axis

$$P_{\theta}(u, v)$$

- Convolve the Projection Data in the direction of rotation

$$P_{\theta}^{*}(u, v) = P_{\theta}(u, v) * h(u)$$

- Backproject the convolved Projection Data

$$\mu(x, y, z) = \int_0^{2\pi} P_{\theta}^{*}(x', z') d\theta$$

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VII.B.4 - FDK - Feldkamp, Davis, Kress


FDK

Practical cone-beam algorithm

L. A. Feldkamp, L. C. Davis, and J. W. Kress
Research Staff, Ford Motor Company, Dearborn, Michigan 48121
Received November 11, 1983; accepted February 28, 1984

The cone-beam reconstruction algorithm developed at FMC is still widely used for both laboratory and clinical systems. The original paper has been cited over 5000 times (google scholar, 2017)

The original mCT system from Ford Motor Co.
Fein Focus source
Image intensifier (relocated to HFHS)



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VII.B.4 - 3D Solution, cone beam

The Feldkamp solution weights the projection data and scales the backprojection

- Weight Projection Data

$$P_{\theta}'(u, v) = \frac{d_s}{\sqrt{d_s^2 + v^2 + u^2}} P_{\theta}(u, v)$$

- Convolve Weighted Projection Data

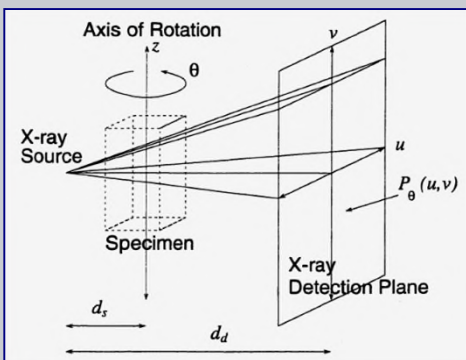
$$P_{\theta}^{*}(u, v) = P_{\theta}'(u, v) * h(u)$$

- Backproject the convolved weighted Projection Data

$$\mu(x, y, z) = \int_0^{2\pi} \frac{d_s^2}{(d_s - y)^2} P_{\theta}^{*} \left(\frac{d_s x'}{d_s - y'}, \frac{d_s z'}{d_s - y'} \right) d\theta$$

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VII.B.4 - Geometry nomenclature



Reimann, WSU thesis, 1998

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VII.B.4 - FDK pseudocode A - process the projection views

```

1 for each vertical detector position v (N_v positions):
2   for each horizontal detector position u (N_u positions):
3     precompute weights w(u, v) = d_s / sqrt(d_s^2 + u^2 + v^2)
4   end u loop
5 end v loop
6
7 for each theta (N_theta views):
8   Weight Projection
9   for each vertical detector position v (N_v positions):
10    for each horizontal detector position u (N_u positions):
11      p'_theta(u, v) = p_theta(u, v) * w(u, v)
12    end u loop
13  end v loop
14
15 Fourier Filter Zero Padded Projections 2 rows at a time
16 for every other vertical detector position v (N_v/2 positions):
17   P(u) = FFT{p'_theta(u, v) and p'_theta(u, v + 1)}
18   for each zero padded horizontal detector position u (2N_u positions):
19     P*(u) = P(u) * h(u)
20   end u loop
21   p''_theta(u, v) and p''_theta(u, v + 1) = FFT^-1{P*(u)}
22 end v loop
23
  
```

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VII.B.4 - FKD pseudocode B - Backproject

```

24 for each y position (Ny positions):
25   for each x position (Nx positions):
26     compute x' = x cos θ + y sin θ
27     compute y' = y cos θ - x sin θ
28     compute M = dx / (y' - dx)
29     compute u = x'M + u0
30     compute M2
31     compute M2c2 = M2([u])
32     compute M2c1 = M2(1 - [u])
33   For each z position (Nz positions):
34     compute v = zM + v0
35     compute c4 = [v]
36     compute c3 = 1 - [v]
37     interpolate M2p02(u, v) = c4(c2P([u + 1], [v + 1])
38       + c1P([u], [v + 1])) + c3(c2P([u + 1], [v]) + c1P([u], [v]))
39     increment I(x, y, z) by M2p02(u, v)
40   end z loop
41 end x loop
42 end y loop

```

Backproject each view column by column.

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VII.B.4 - FKD pseudocode C computation overhead

N	64	128	256	512	1024
Trad.	1 · 10 ⁸	2 · 10 ⁹	3 · 10 ¹⁰	5 · 10 ¹¹	9 · 10 ¹²
Fast	2 · 10 ⁸	2 · 10 ⁹	2 · 10 ¹⁰	2 · 10 ¹¹	1 · 10 ¹²

Table 3.4 Number of FLOPs on projection data in traditional and fast backprojection as a function of $N = N_x = N_y = 2N_z = N_t = N_q = N_\theta/2$. The estimate for fast backprojection when $N = 1024$ is extrapolated.

- Henrik Turbell, Cone-Beam Reconstruction Using Filtered Backprojection, PhD Dissertation no. 672, Linköping University, Sweden, February, 2001
- <http://www.cvl.isy.liu.se/ScOut/Theses/>

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VII.B.4 - FKD pseudocode C computation overhead

Line Number	Floating Point Operations Required per Step	Total Times Step is Executed	Total Floating Point Operations Required
3	7	$N_\theta N_u$	$7N_\theta N_u$
11	1	$N_\theta N_u N_u$	$N_\theta N_u N_u$
17	$9(2N_u \log_2(2N_u))$	$N_\theta(N_u/2)$	$9N_\theta N_u N_u \log_2(2N_u)$
19	2	$N_\theta(N_u/2)(2N_u)$	$2N_\theta N_u N_u$
21	$9(2N_u \log_2(2N_u))$	$N_\theta(N_u/2)$	$9N_\theta N_u N_u \log_2(2N_u)$
26	3	$N_\theta N_y N_z$	$3N_\theta N_y N_z$
27	3	$N_\theta N_y N_z$	$3N_\theta N_y N_z$
28	2	$N_\theta N_y N_z$	$2N_\theta N_y N_z$
29	2	$N_\theta N_y N_z$	$2N_\theta N_y N_z$
30	1	$N_\theta N_y N_z$	$N_\theta N_y N_z$
31	1	$N_\theta N_y N_z$	$N_\theta N_y N_z$
32	2	$N_\theta N_y N_z$	$2N_\theta N_y N_z$
34	2	$N_\theta N_z N_y N_z$	$2N_\theta N_z N_y N_z$
36	1	$N_\theta N_z N_y N_z$	$N_\theta N_z N_y N_z$
37	8	$N_\theta N_z N_y N_z$	$8N_\theta N_z N_y N_z$
38	1	$N_\theta N_z N_y N_z$	$N_\theta N_z N_y N_z$

The heavy lifting is in the column backprojection

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VII.B.4 - HFHS animal femur mCT 1500 × 1500 × 1250

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VII.B.4 - FKD pseudocode C computation overhead

To reconstruct an $N \times N \times N$ volume,

$$N_\theta = N\pi/2$$

$$FLOPS = N^4 6\pi + \delta$$

Thus,

$$512^3 \rightarrow 1300 \times 10^9 \text{ FLOPS}$$

FLOPS - Floating Point Operations Per Second
 3 GHz Xeon processors are rated by Intel at 50 GFLOPS.
 However, for reconstruction problems speed is often limited by memory i/o rates.

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VII.B.4 - HFHS femur mCT ZOOM

- 1536 × 1920 Acq. PaxScan 2520
- Cubic spline resample Thevenaz, 'interpol'
- Compress proj data JPEG2000 8-1 Kakadu
- Filtered Backproj. MPI cluster (6/12)
- DICOM VCT data JPEG2000 8-1 DCMTK (OFFIS)

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VII.B - CT Reconstruction

B) CT Reconstruction

- 1) Projection geometry
- 2) Fourier Domain Solution
- 3) Convolution / Backprojection
- 4) Cone beam reconstruction
- 5) Iterative Reconstruction (9 slides)

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VII.B.5 - Iterative Reconstruction

• We then take the difference between the original projection, P2, and the estimate, P2e, and backproject it to get a second estimate.

• This is then reprojected in the P3 direction.

2.3	2.7	1.0
7.3	7.7	6.0
6.3	6.7	5.0

$1/3 \quad 2/3 \quad -3/3$
(P2 - P2e)

16	17	12
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P2

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VII.B.5 - Iterative Reconstruction

In general, iterative reconstructions make an initial guess as to the tomographic solution, then

- reproject in a particular direction
- examine the difference between the reprojected estimate and an actual measurement.
- distribute the difference back to the solution estimate

Consider the simple example from the Webb reading assignment where a 3 x 3 tomograph is considered.

- The actual object has values from 1 to 9.
- 4 projection measurements are made.

1	2	3
8	9	4
7	6	5

16	17	12
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P2

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VII.B.5 - Iterative Reconstruction

• Similarly, the difference between the original projection, P3, and the estimate, P3e, and the estimate, P3e is backprojected, and the result is reprojected in the P4 direction.

2.3	2.0	2.3
7.3	9.0	4.7
7.7	5.3	5.0

$12.7 \quad 16.3 \quad 7.3$
(P3 - P3e)

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VII.B.5 - Iterative Reconstruction

• We now start by distributing the P1 values horizontally across a 3 x 3 solution matrix.

• These are then reprojected in the P2 direction

2	2	2
7	7	7
6	6	6

15	15	15
----	----	----

P2e

16	17	12
----	----	----

P2

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VII.B.5 - Iterative Reconstruction

• Finally, the difference between the original projection, P4, and the estimate, P4e, and the estimate, P4e is backprojected.

• the result after 1 iteration closely approximates the original values.

1.9	2.1	2.3
8.0	8.6	4.0
7.7	6.0	4.6

$13.7 \quad 13.3 \quad 13.7$
(P4 - P4e)

Original

1	2	3
8	9	4
7	6	5

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VII.B.5 - Maximum Likelihood CT reconstruction

- Maximum Likelihood (ML) reconstruction methods offers the possibility to include the Poisson statistics of the photons in the reconstruction. Since the projections, i , are independent, the log-likelihood, L , can be written as,

$$L = \sum_i \left(-d_i e^{-\sum_j A_{ij} \mu_j} - Y_i \sum_j A_{ij} \mu_j \right) + c_1$$
 Equation 5
Ziegler 2007
- where
 - d_i is the expected number of photons leaving the source along the i th projection,
 - Y_i are the observed photon counts along projection i ,
 - μ_j is the absorption coefficient of the j th supporting grid point,
 - A_{ij} are the elements of the system matrix, and c_1 is an irrelevant constant
- An approximate solution of maximizing L leads to an iterative step n to $n+1$ of,

$$\mu_j^{n+1} = \mu_j^n + \mu_j^n \frac{\sum_i A_{ij} [d_i e^{-\sum_j A_{ij} \mu_j^n} - Y_i]}{\sum_i A_{ij} A_{ij} \mu_j^n d_i e^{-\sum_j A_{ij} \mu_j^n}}$$
 Equation 6
Ziegler 2007
- Using an ordered subset method, Ziegler demonstrated that ML reconstruction can result in a signal to noise improvement of about 3 for equal resolution relative to filtered backprojection methods (FBP).

Ziegler, Medical Physics, 2007

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VII.B.5 - ASIR Hara 2009

Adaptive Statistical Iterative Reconstruction,
GE Medical Systems.

Dose Index = 8
ASIR
Dose Index = 22
Traditional Reconstruction

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VII.B.5 - Maximum Likelihood CT reconstruction

Center
midway
edge

Ziegler, Medical Physics, 2007

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VII.B.5 - IR techniques Geyer et al., Iterative CT Reconstruction Techniques (review), Radiology, Aug. 2015

GE Medical Systems.

- 2008 - ASIR : Adaptive Statistical Iterative Reconstruction
- 2010 - VEO : Model based (computationally intensive)
- 2013 - ASIR-V : Hybrid ASIR-VEO

Philips Medical Systems.

- 2010 - iDose4 : Adaptive Statistical Iterative Reconstruction.
- 2013 - IMR : Model based reconstruction.

Siemens Medical Systems.

- 2008 - IRIS : Iterative reconstruction in image space.
- 2010 - SAFIRE : Sinogram affirmed iterative reconstruction
- 2015 - ADMIRE : Advanced modeled iterative reconstruction.

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VII.B.5 - Maximum Likelihood CT reconstruction Ziegler, Medical Physics, 2007

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VII.B.5 - Model based IR

Differential system models

Model based IR:

- Rather than treating all measurements with equal weighting, a statistical model allows differing degrees of credibility among data.
- Three dimensional models describe the data acquisition process (source, gantry geometry, active detector) including the radiation interactions in a 3D model of the subject.
- Most IR methods specify a parameter that influences the amount of noise reduction.

De Marco, J. Appl. Clin. Med. Phys., Jan. 2018 54

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