NERS/BIOE 481

Lecture 02
Radiation Physics

Michael Flynn, Adjunct Prof
Nuclear Engr & Rad. Science
mikef@umich.edu
mikef@rad.hfh.edu

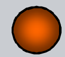
Henry Ford Health System
RADIOLOGY RESEARCH

A) Properties of Materials

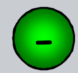
1) Atoms
2) Condensed media
3) Gases

- The primary components of the nucleus are paired protons and neutrons.
- Because of the coulomb force from the densely packed protons, the most stable configuration often includes unpaired neutrons

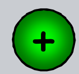
Neutrons
neutral charge
1.008665 AMU



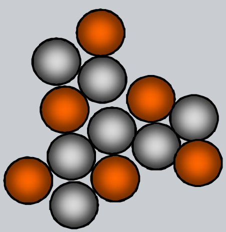
Protons
+ charge
1.007276 AMU



Electrons
- charge
0.0005486 AMU



Positrons
+ charge
0.0005486 AMU



$$^{13}_{6}C$$

Carbon 13
13 nucleons
6 protons
7 neutrons

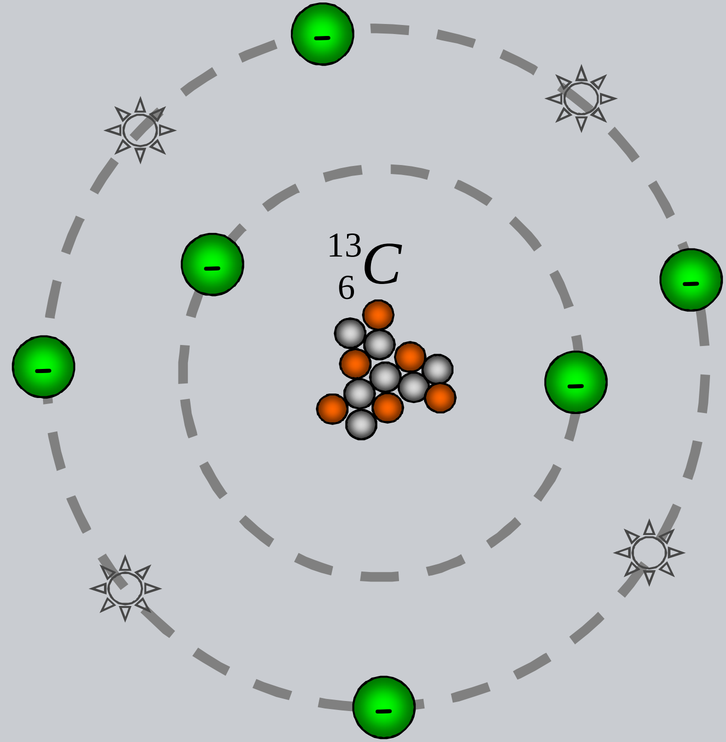
Terminology

$$^{A}_{Z}X$$

A = no. nucleons

Z = no. protons

II.A.1 – Atoms – the Bohr model



- The Bohr model of the atom explains most radiation imaging phenomena.
- Electrons are described as being in orbiting shells:
    - K shell: up to 2 e-, n=1
    - L shell: up to 8 e-, n=2
    - M shell: up to 18 e-, n=3
    - N shell: up to 32 e-, n=4
- The first, or K, shell is the most tightly bound with the smallest radius. The binding energy (Ionization energy in eV) neglecting screening is ……. → $I = I_0 (Z^2 / n^2)$ $\quad I_0 = 13.60\ eV$

Niels Bohr (1913). "On the Constitution of Atoms and Molecules, Part I", Philosophical Magazine 26 (151): 1–24.
Niels Bohr (1913). "On the Constitution of Atoms and Molecules, Part II", Philosophical Magazine 26 (153): 476-502.
Niels Bohr (1913). "On the Constitution of Atoms and Molecules, Part III", Philosophical Magazine 26 (155): 857-875.

- For condensed material, the molecules per cubic cm can be predicted from Avogadro's number (atoms/cc)

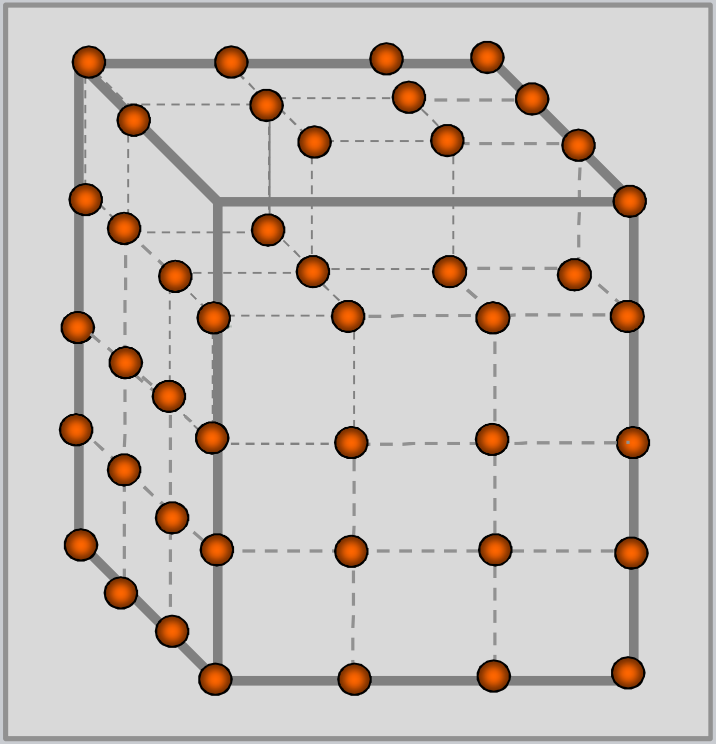
$$N_m = N_a \frac{\rho}{A}$$

- Consider copper with one atom per molecule,

$$A = 63.50$$

$$\rho = 8.94\ gms/cc$$

$$N_{cu} = 8.47 \times 10^{22}\ \#/cc$$



- If we assume that the copper atoms are arranged in a regular array, we can determine the approximate distance between copper atoms (cm);

$$l_{Cu} = 1 \Big/ N_{Cu}^{1/3} = 2.3\times10^{-8}\ cm$$

- The Angstrom originated as a unit appropriate for describing processes associated with atomic spacing dimensions. 1.0 Angstroms is equal to 10^{-8} cm. Thus the approximate spacing of Cu atoms is 2.3 Angstroms.

- In relation, the radius of the outer shell electrons (M shell) for copper can be deduced from the unscreened Bohr relationship (Angstroms);

$$r_m = \alpha_H \left(n^2 / Z\right)$$
$$r_{Cu} = .52917\left(3^2 / 29\right)$$
$$r_{Cu} = .16, Angstroms$$

Thus for this model of copper, the atoms constitute a small fraction of the space,

$$V_{Cu} = (4/3)\pi 0.16^3 = .017\ A^3$$
$$V_{Cu} / (2.3^3) = .001$$

α_H is the 'Bohr radius', the radius of the ground state electron for Z = 1

- An ideal gas is defined as one in which all collisions between atoms or molecules are perfectly elastic and in which there are no intermolecular attractive forces. One can visualize it as a collection of perfectly hard spheres which collide but which otherwise do not interact with each other.
- An ideal gas can be characterized by three state variables: absolute pressure (P), volume (V), and absolute temperature (T). The relationship between them may be deduced from kinetic theory and is called the “ideal gas law”.

$$PV = nRT = NkT$$

P = pressure, pascals(N/m^2)
V = volume, m^3

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n = number of moles
T = temperature, Kelvin
R = universal gas constant
= 8.3145 J/mol·K(N·m/mol·K)

N = number of molecules
k = Boltzmann constant
= 1.38066 x 10-23 J/K
= 8.617385 x 10-5 eV/K
$k = R/N_A$
N_A = Avogadro's number
= 6.0221×10^{23} /mol

- The density of a gas can be determined by dividing both sides of the gas equation by the mass of gas contained in the volume V.

$$\rho = (P/T)/R_g$$

- The gas constant for a specific gas, R_g, is the universal gas constant divided by the grams per mole, m/n.

$$R_g = R\,/\,(m/n)$$

- m/n is the atomic weight.

$$PV = nRT$$

$$P\left(\frac{V}{m}\right) = \left(\frac{n}{m}R\right)T$$

$$\frac{P}{\rho} = R_g T \qquad \left(\frac{n}{m}R\right) = R_g$$

R_g = specific gas constant

Dry Air example

Molar weight of dry air = 28.9645 g/mol

R_{air} = (8.3145/28.9645) =.287 J/g·K

Pressure = 101325 Pa (1 torr, 760 mmHg)

Temperature = 293.15 K (20 C)

Density = 1204 (g/m^3) = .001204 g/cm^3

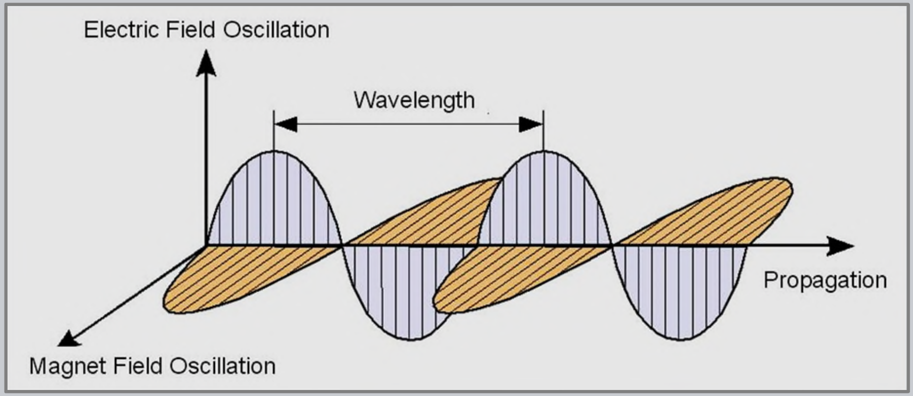
Note: these are standard temperature and pressure, STP, conditions.

B) Properties of Radiation

1) EM Radiation
2) Electrons

II.B.1 – EM radiation

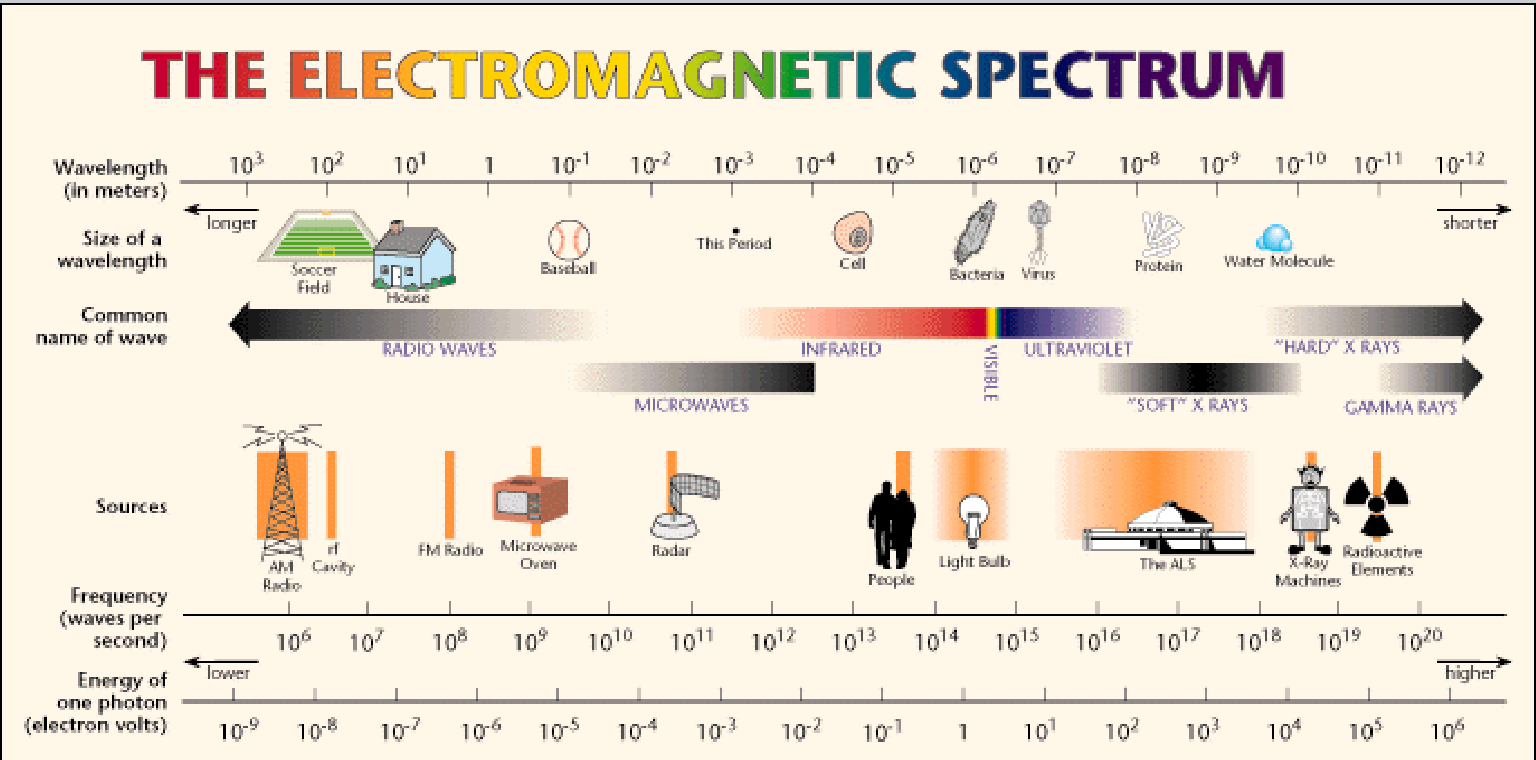
Electromagnetic radiation involves electric and magnetic fields oscillating with a characteristic frequency (cycles/sec) and propagating in space with the speed of light.



- The electric and magnetic fields are perpendicular to each other and to the direction of propagation.
- X-ray and gamma rays are both EM waves (photons)
  - Xrays – produced by atomic processes
  - Gamma rays – produced by nuclear processes
- The energy of an EM radiation wave packet (photon) is related to the oscillation frequency and thus the wavelength;
  - $E = 12.4/\lambda$, for E in keV & λ in Angstrom
  - $E = 1.24/\lambda$, for E in keV & λ in nm
  - $E = 1240/\lambda$, for E in eV & λ in nm

II.B.1 – EM radiation

The electromagnetic spectrum covers a wide range of wavelengths and photon energies. Radiation used to "see" an object must have a wavelength about the same size as or smaller than the object.



Lawrence Berkeley Lab: www.lbl.gov/MicroWorlds/ALSTool/EMSpec/EMSpec2.html

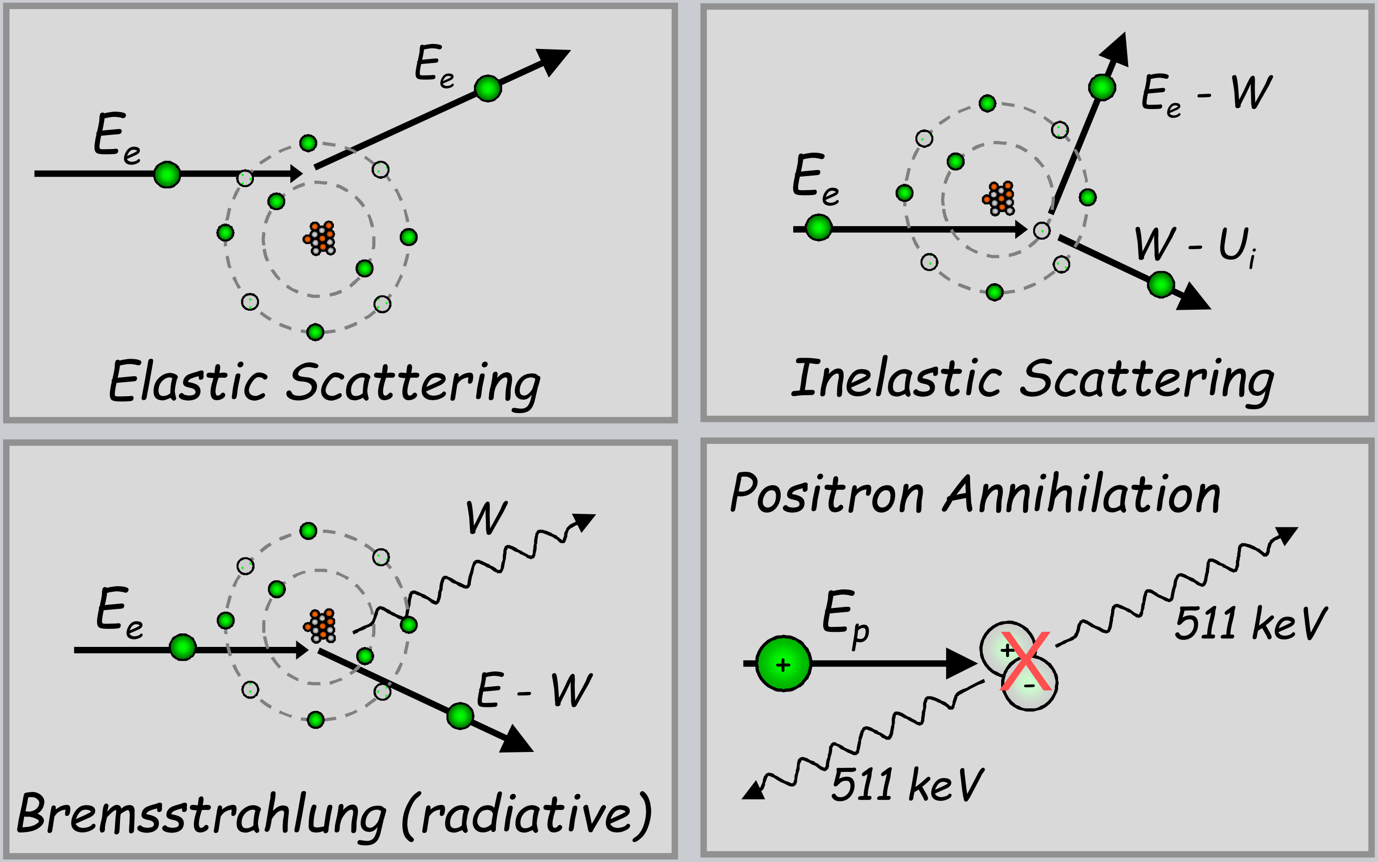
- The electron is one of a class of subatomic particles called leptons which are believed to be “elementary particles”. The word "particle" is somewhat misleading however, because quantum mechanics shows that electrons also behave like a wave.
- The antiparticle of an electron is the positron, which has the same mass but positive rather than negative charge.

  - Mass-energy equivalence = 511 keV
  - Molar mass = 5.486×10^{-4} g/mol
  - Charge = 1.602×10^{-19} coulombs

http://en.wikipedia.org/wiki/Electron

C) Radiation Interactions
   1) Electrons
   2) Photons
      a. Interaction cross sections
      b. Photoelectric interactions
      c. Compton scattering (incoherent)
      d. Rayleigh scattering (coherent)

Basic interactions of electrons and positrons with matter.



II.C.1 – Electron multiple scattering

Numerous elastic and inelastic deflections cause the electron to travel in a tortuous path.

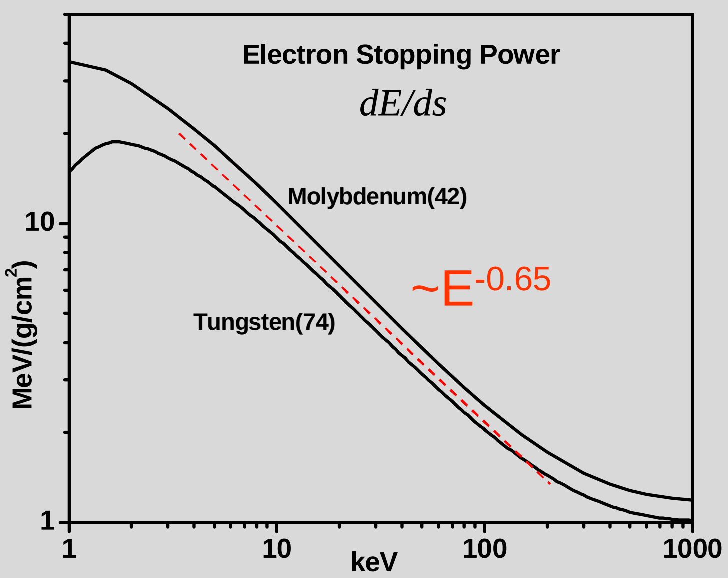
PENELOPE

- Tungsten
- 10μm x 10μm
- 100 keV

For take-off angles of 17.5°-22.5° 0.0006 of the electrons produce an emitted x-ray of some energy.

II.C.1 – Electron path

- A very large number of interactions with typically small energy transfer cause gradual energy loss as the electron travels along the path of travel.
- The Continuous Slowing Down Approximation (CSDA) describes the average loss of energy over small path segments.
  - ICRU reports 37 (1984) and 49 (1993).
  - Berger & Seltzer, NBS 82-2550A, 1983.
  - Bethe, Ann. Phys., 1930

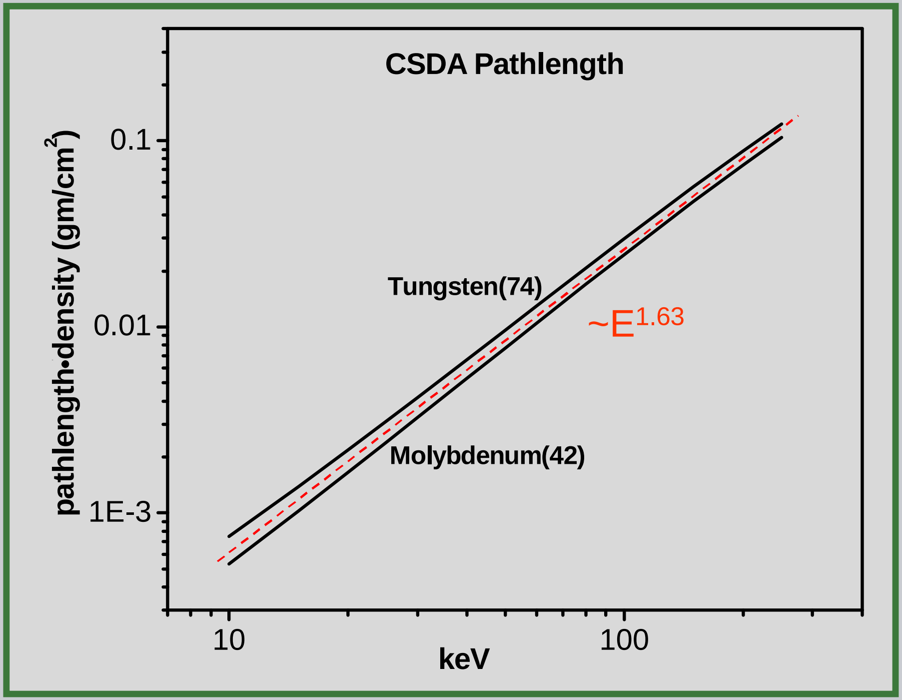


MeV/(g/cm²) - For radiation interaction data, units of distance are often scaled using the material density, distance * density, to obtain units of g/cm².

http://physics.nist.gov/PhysRefData/Star/Text/ESTAR.html

The total pathlength traveled by the electron along the path of travel is obtained by integrating the inverse of the stopping power, i.e. $1/(dE/ds)$,

$$R_{CSDA} = \int_T^0 \frac{1}{dE/ds} dE$$



gm/cm²

Pathlength is often normalized as the product of the length in cm and the material density in gm/cm³ to obtain gm/cm².

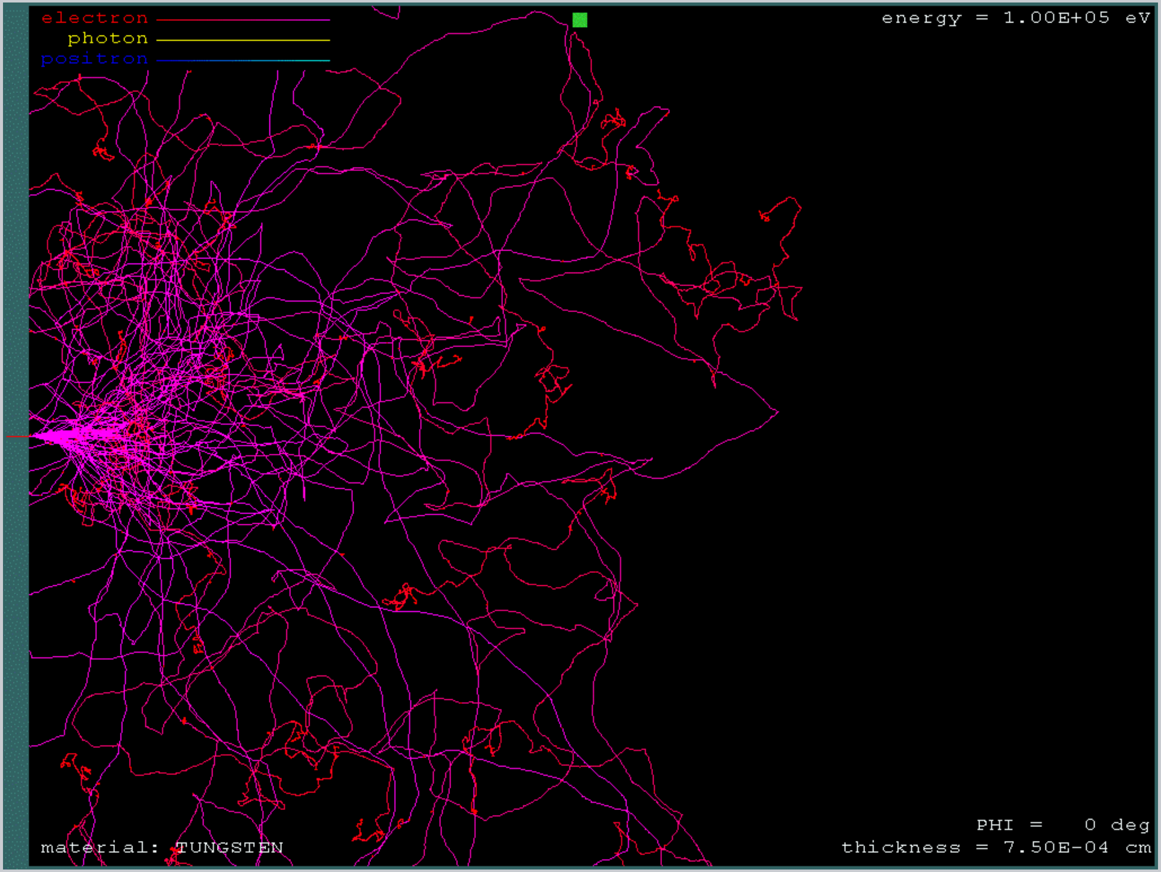
CSDA –

Continuous Slowing Down Approximation.

- 100 keV, Tungsten, 15.4 μm
- 30 keV, Molybdemun, 3.2 μm

- A beam of many electrons striking a target will diffuse into various regions of the material.

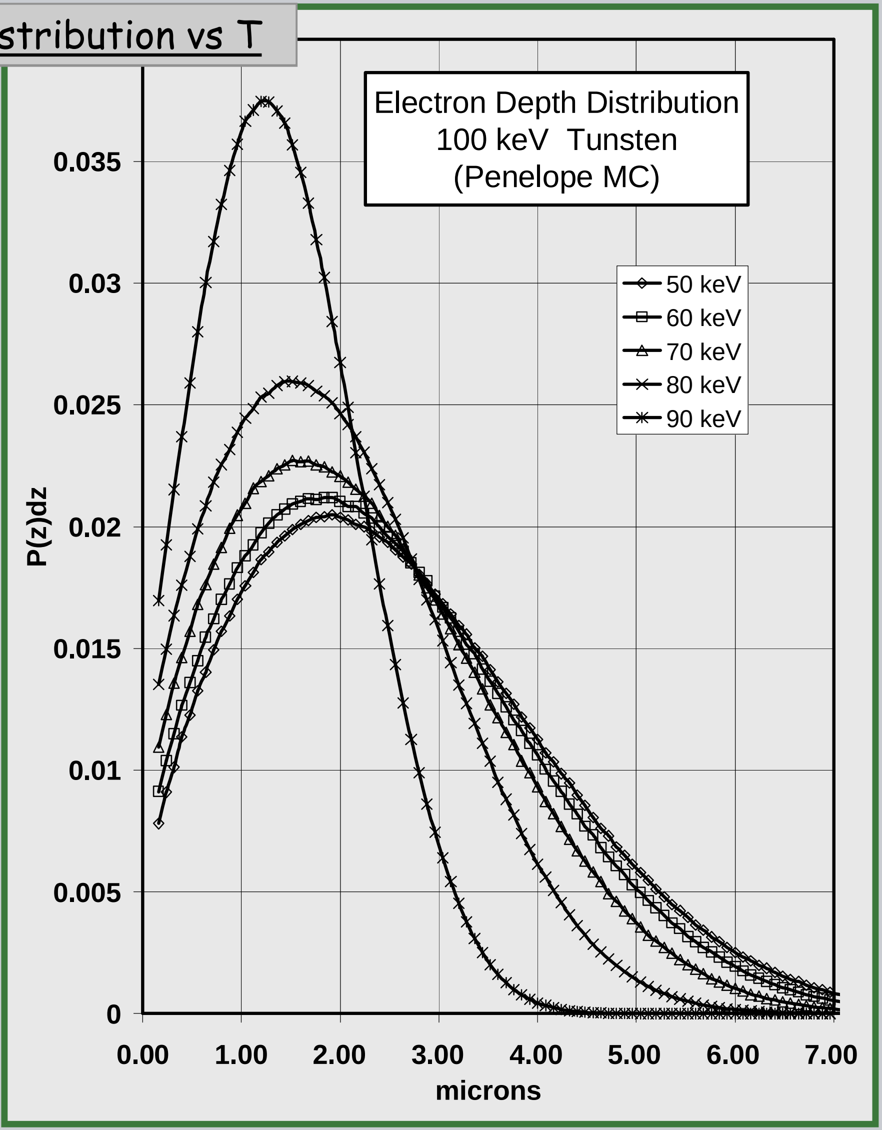
50 e



II.C.1 – Electron depth distribution vs T

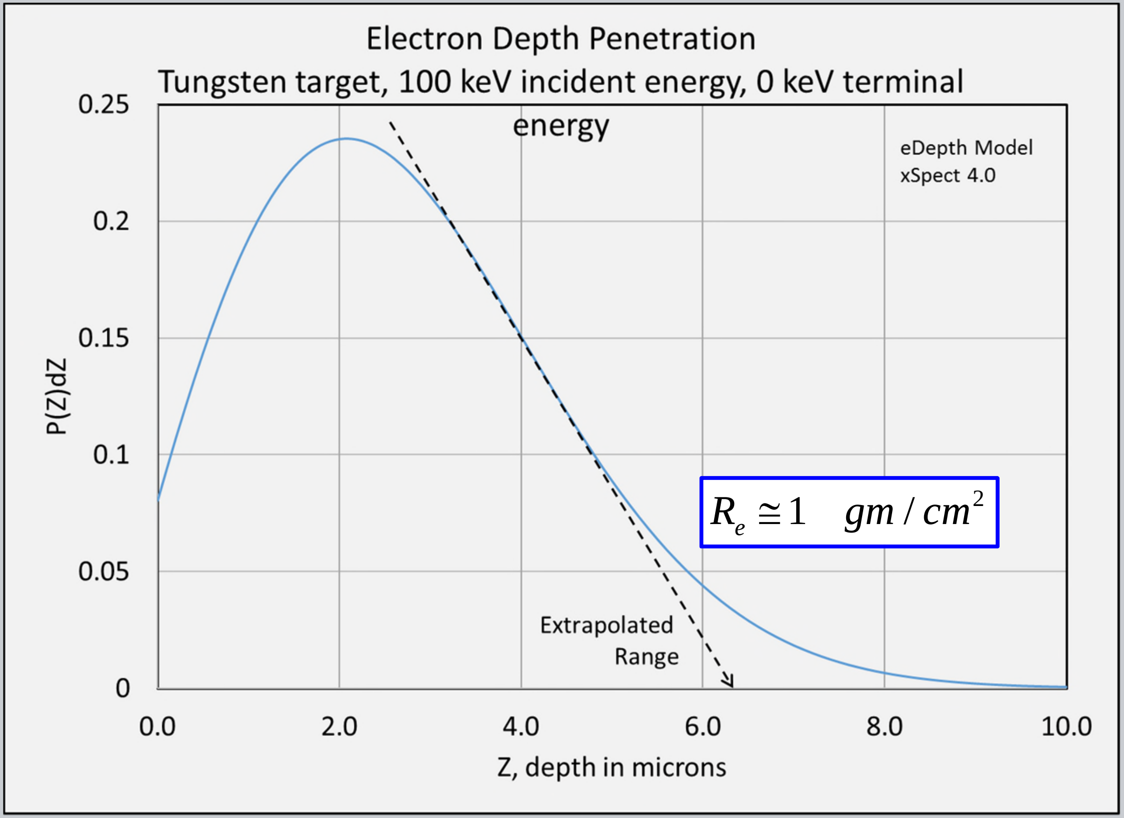
- Electrons are broadly distributed in depth, Z, as they slow down due to extensive scattering.
- Most electrons tend to rapidly travel to the mean depth and diffuse from that depth.

$Pz(T,Z)$ is the differential probability (1/cm) of that an electron within the target is at depth Z



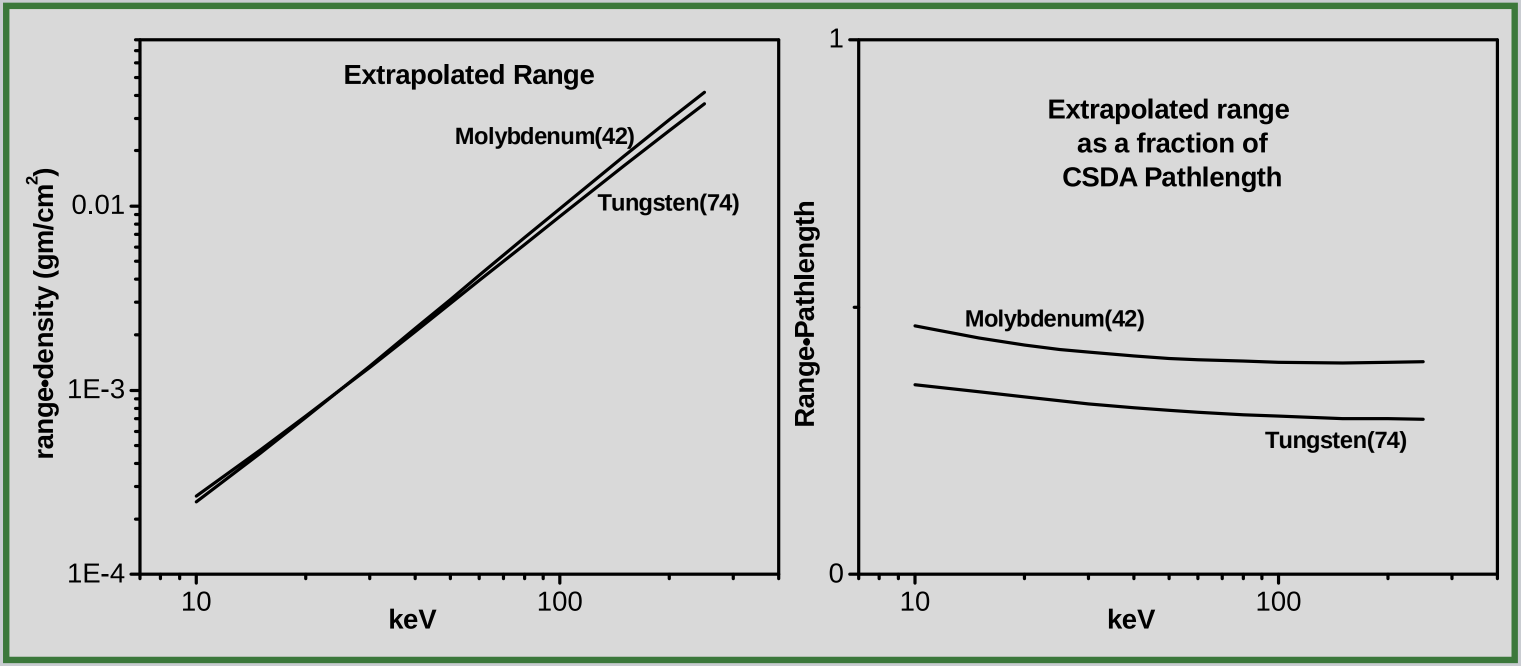
II.C.1 – Electron extrapolated range

The extrapolated range is commonly measured from electron transmission data measured with foils of varying thickness. It is defined as the point where the tangent at the steepest point on the almost straight descending portion of the penetration curve.



II.C.1 – Electron extrapolated range

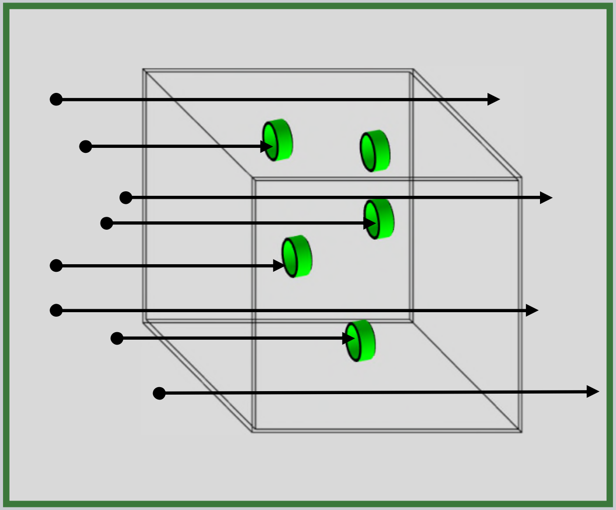
- The extrapolated range in units of gm/cm2 is nearly independent of atomic number.
- The extrapolated range is about 30-40% of the CSDA pathlength.



Tabata, NIM Phys. Res. B, 1996

C) Radiation Interactions
   1) Electrons
   2) Photons
      a. Interaction cross sections
      b. Photoelectric interactions
      c. Compton scattering (incoherent)
      d. Rayleigh scattering (coherent)

- The probability of a specific interaction per atom is known as the cross section, σ.
- The probability is expressed as an effective area per atom.
- The 'barn' is a unit of area equal to 10^{-24} cm^2 (non SI unit).



- The probability per unit thickness that an interaction will occur is the product of the cross section and the number of atoms per unit volume.

$$\mu = \sigma N \text{, } (\text{cm}^2/\text{atom})\cdot(\text{atoms/cm}^3) = \text{cm}^{-1}$$

$$N = N_a\,(\rho/A)$$

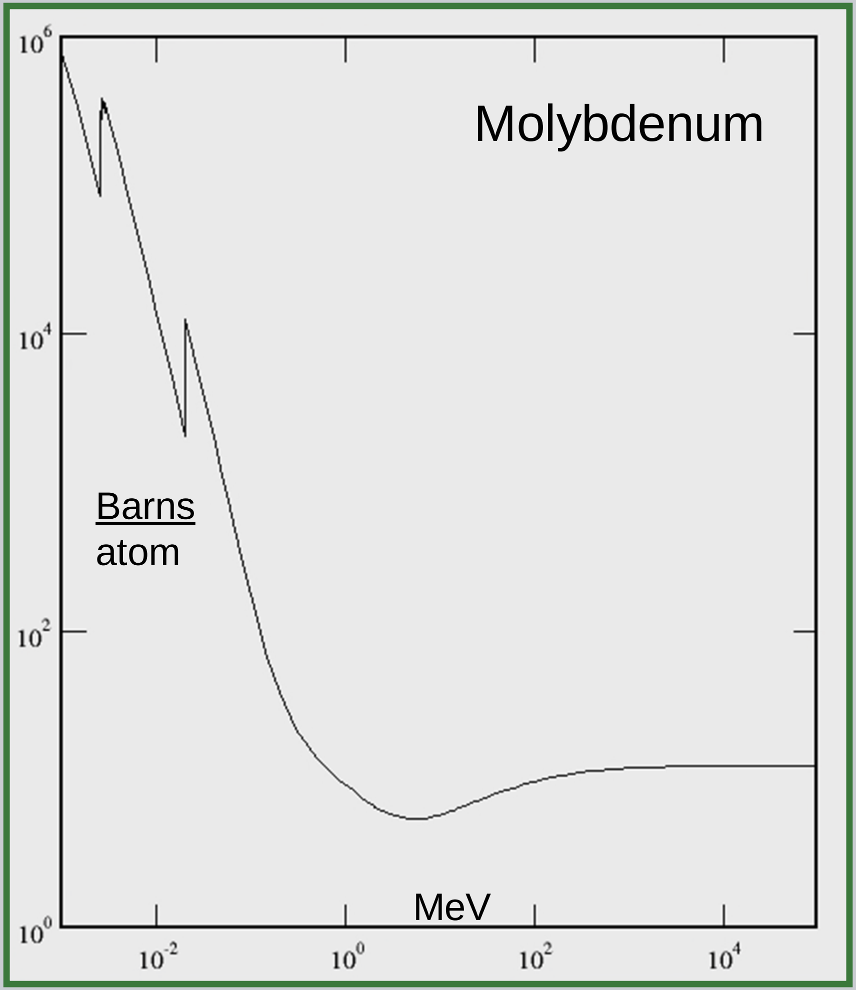
N_a - Avogadro's # (6.022×10^{23})
ρ - density
A - Atomic Weight

- Cross section values are tabulated for the elements and many common materials.
- Values range from 10^1-10^4 barns (10 to 100 keV) depending on Z.
- Cross sections are typically small relative to the area of the atom.

Cu, 30 keV

$\sigma = 1.15 \times 10^3$ b/atom

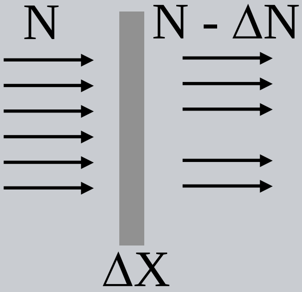
$A_{Cu} = 8.04 \times 10^6$ b/atom



http://physics.nist.gov/PhysRefData/Xcom/Text/XCOM.html

II.C.2.a – Linear attenuation coefficients

- The probability per unit thickness that an x-ray will interact when traveling a small distance called the 'linear attenuation coefficient'.
- For a beam of x-rays, the relative change in the number of x-rays is proportional to the incident number.
- For a thick object of dimension x, the solution to this differential equation is an exponential expression known as Beer's law.



$$\frac{\Delta N}{\Delta X} = \frac{dN}{dX} = -\mu N$$

$$N(x) = N(0) e^{-\mu x}$$

$$Transmission = N(x) / N(0) = e^{-\mu x}$$

II.C.2.a – Xray Interaction types

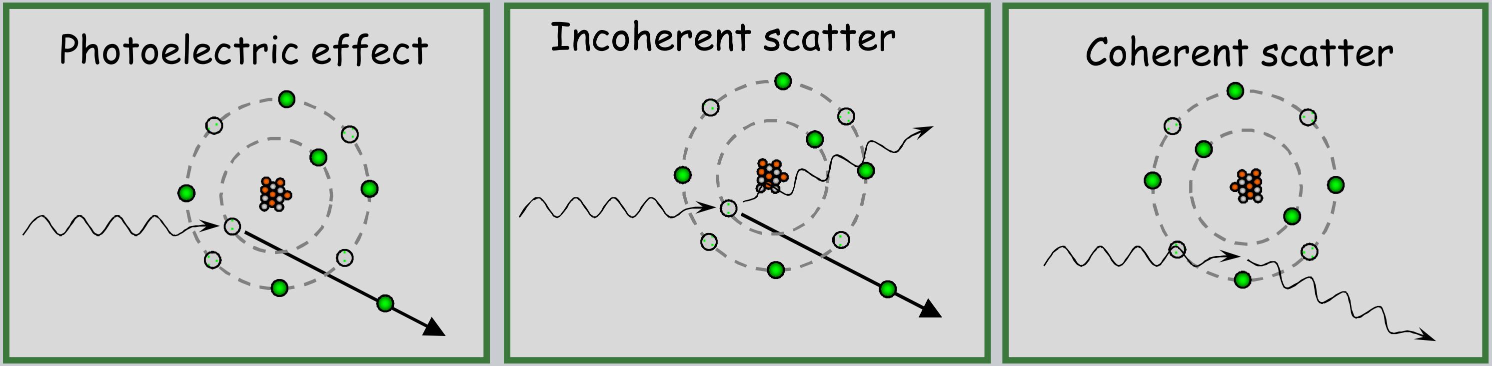
Attenuation is computed from Beer's law using the linear attenuation coefficient, μ, computed from cross sections and material composition.

$$\mu_{PE} = N_1\sigma^1{}_{PE} + N_2\sigma^2{}_{PE} + N_3\sigma^3{}_{PE}$$

$$N = N_o e^{-\mu x}$$

Where N_i is the number of atoms of type i and σ^i is the cross section for type i *atoms*.

In the energy range of interest for diagnostic xray imaging, 10 – 250 keV, there are three interaction processes that contribute to attenuation.



$$\mu = \mu_{PE} + \mu_{INC} + \mu_{COH}$$

Photoelectric Absorption

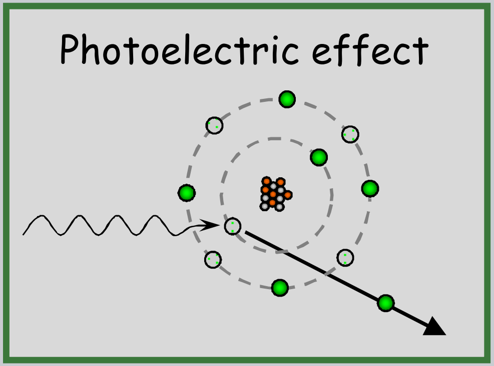
- The incident photon transfers all of it's energy to an orbital electron and disappears.
- The photon energy must be greater than the binding energy, I, for interaction with a particular shell. This cause discontinuities in absorption at the various I values.
- An energetic electron emerges from the atom;

$$E_e = E_\gamma - I$$

- Interaction is most probable for the most tightly bound electron. A K shell interaction is 4-5 times more likely than an L shell interaction.
- Very strong dependance on Z and E.

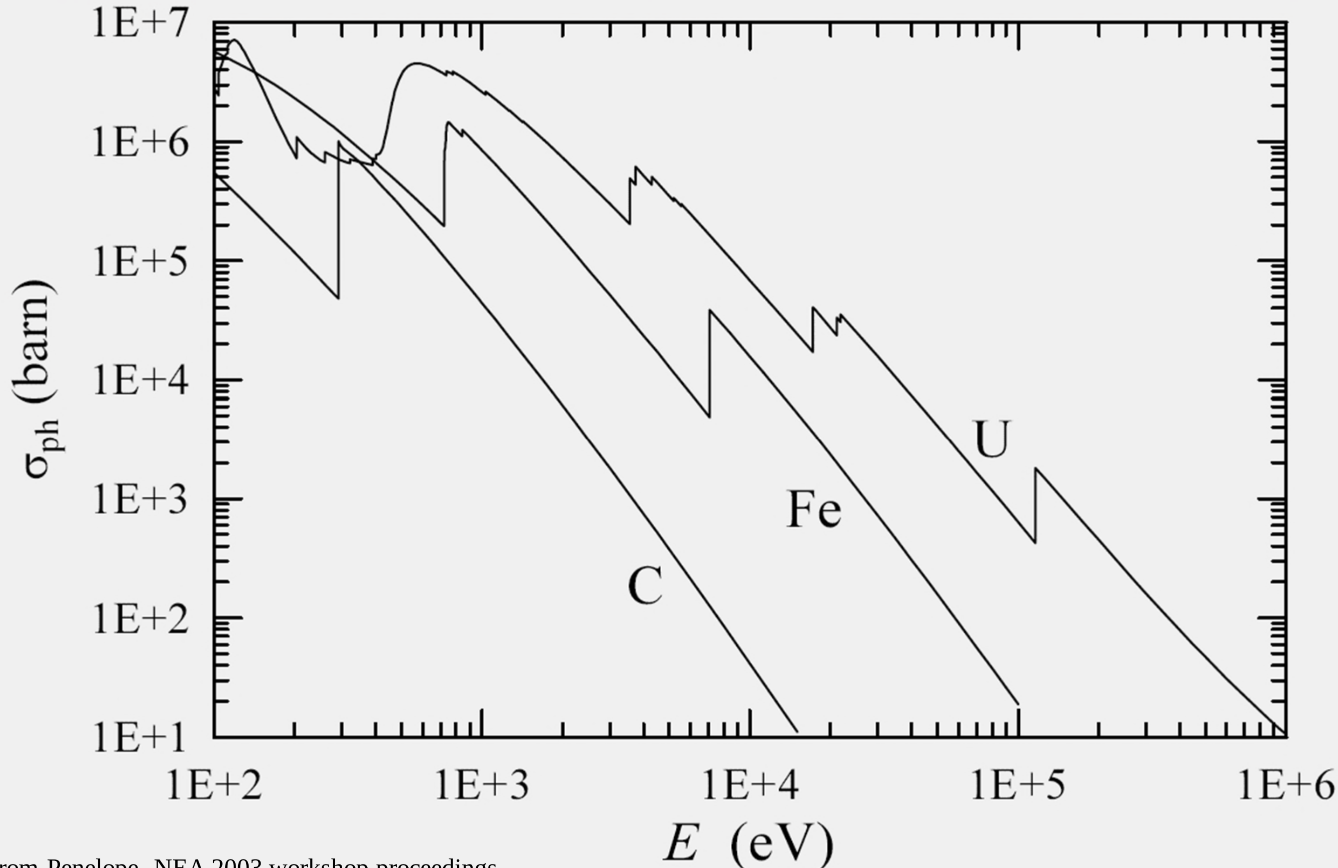
$$I \cong I_0 \left(Z^2 / n^2 \right)$$

$$I_0 = 13.60 \quad eV$$

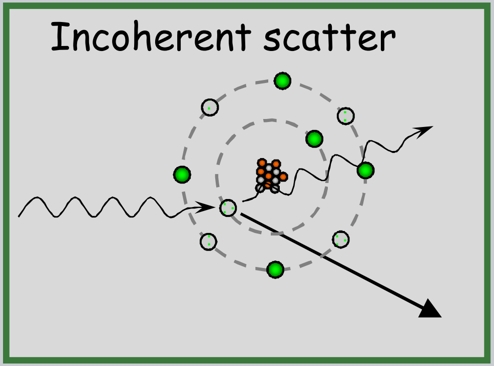


$$\mu^{pe} \cong k \frac{\rho}{A} \frac{Z^4}{E^3}$$

Atomic photoelectric cross sections for carbon, iron and uranium as functions of the photon energy E.



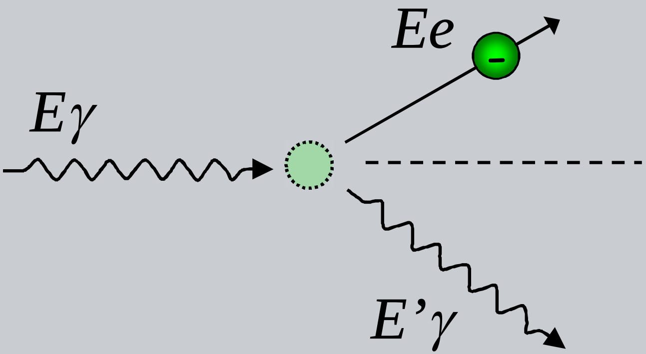
from Penelope, NEA 2003 workshop proceedings



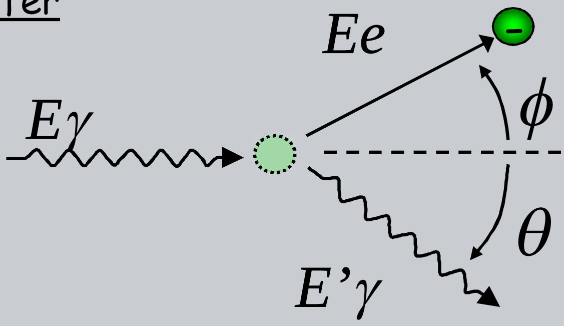
Compton Scattering

- An incident photon of energy E_γ interacts with an electron with a reduction in energy, E'γ, and change in direction. (i.e. incoherent scattering)

- The electron recoils from the interaction in an opposite direction with energy E_e.



II.C.2.c – Compton Scattering – energy transfer



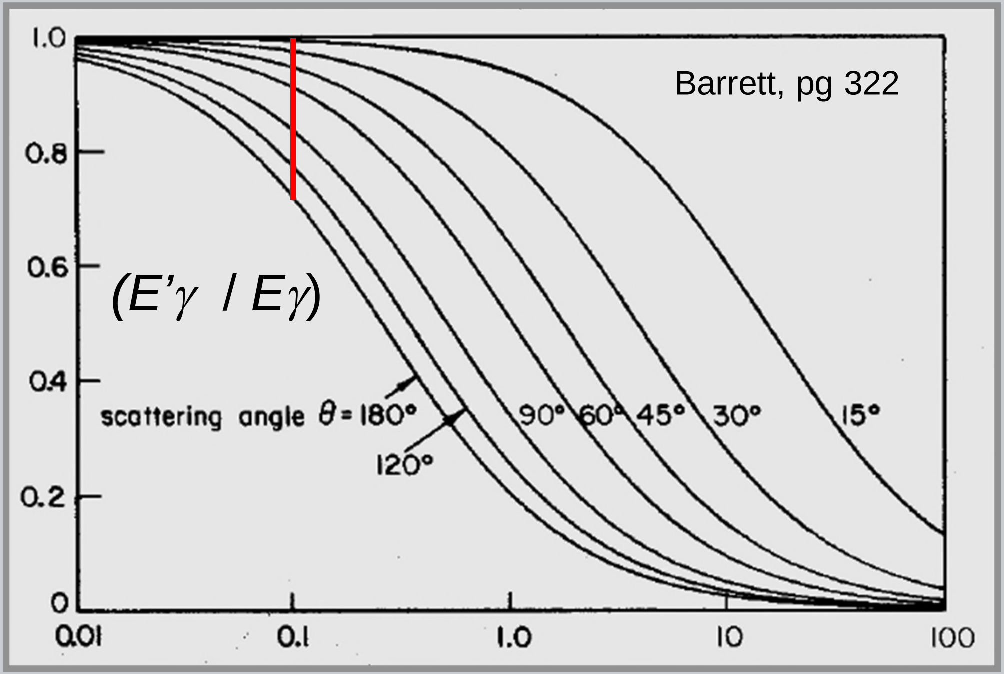
$$\frac{1}{E'_\gamma} - \frac{1}{E_\gamma} = \frac{1}{E_\gamma}\alpha\left(1-\cos\theta\right)$$

$$\alpha = \frac{E_\gamma}{m_o c^2}$$

$$m_o c^2 = 511(keV)$$

The scattered energy as a fraction of incident energy depends on the angle and scaled energy, α.

$$\frac{E'_\gamma}{E_\gamma} = \frac{1}{\left(1+\alpha\left(1-\cos\theta\right)\right)}$$



Incident Energy, $E\gamma$, MeV

II.C.2.c – Compton Scattering – cross section

- The cross section for compton scattering is expressed as the probability per electron such that the attenutation coefficient for removal of photons from the primary beam is;

$$\mu^c = n_e \sigma^c = \left(N_o \rho \frac{Z}{A} \right) \sigma^c$$

Since Z/A is about 0.5 for all but very low Z elements, the mass attenuation coeffient, μ^c/ρ , is essentially the same for all materials !

- The Klein-Nishina equation describes the compton scattering cross section in relation to a classical cross section for photon scattering that is independent of energy (σ_o , Thomson free electron cross section).

$$\sigma^c = \sigma_o f_{KN}(\alpha)$$

$\sigma_o = (8\pi/3)\, r_e^2$

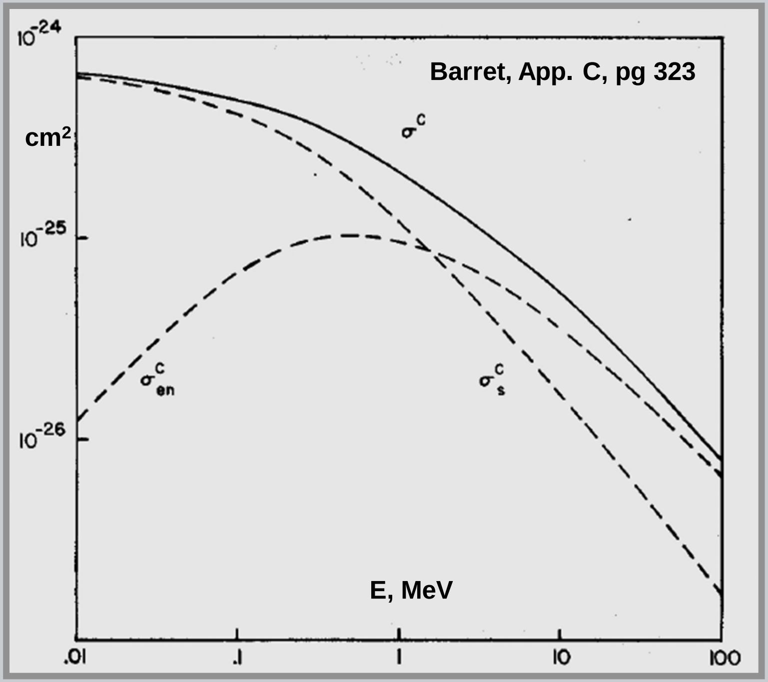
$r_e = e^2/(m_o c^2)$

$\sigma_o = 0.6652$ barn

$$f_{KN}(\alpha) = \frac{3}{4}\left[\frac{2(1+\alpha)^2}{\alpha^2(1+2\alpha)} + \frac{\ln(1+2\alpha)}{\alpha}\left(\frac{1}{2} - \frac{1+\alpha}{\alpha^2} \right) - \frac{1+3\alpha}{(1+2\alpha)^2} \right]$$

Barret, App. C, pg 323

- The free electron Compton scattering cross section is slowly varying with energy for low photon energies.
- At high energy, $\alpha > 1$, the cross section is proportional to $1/E$.

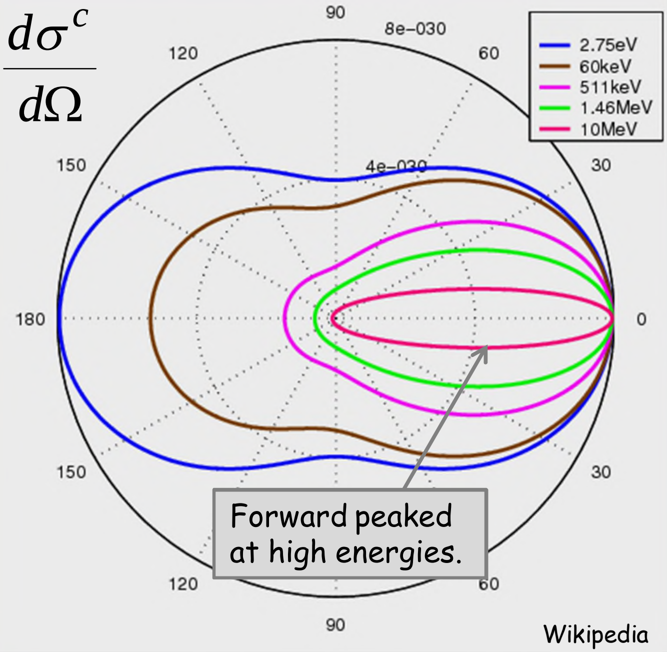


- The total scattering cross section is made of two component probabilities, σ_{en} and σ_s, describing how much of the photon energy is absorbed by recoil electrons and how much by the scattered photon. This difference is important in computations of dose and exposure.

II.C.2.c – Compton Scattering – cross section vs E

The differential scattering cross section describes the probability that the scattered photon will be deflected into a the differential solid angle, $d\Omega$, in the direction (θ,ϕ).

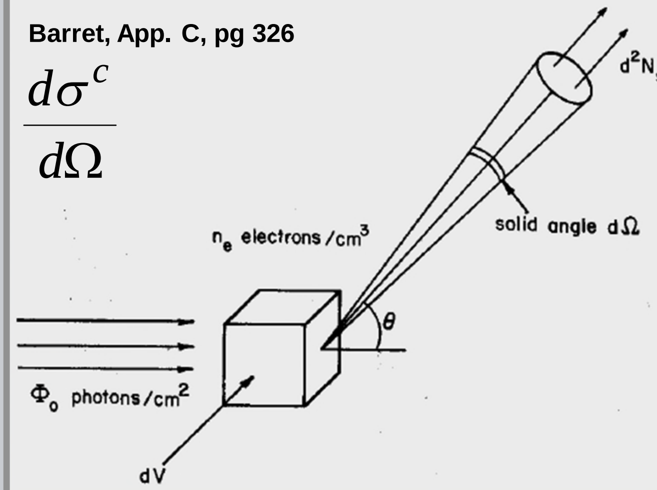
$$\frac{d\sigma^c}{d\Omega}$$



Wikipedia

Barret, App. C, pg 326

$$\frac{d\sigma^c}{d\Omega}$$



In the next lecture, we will learn more about solid angle integrals. Since the differential cross for unpolarized photons depends only on θ, we will find,

$$\sigma^c = \int_0^{\pi} \left(\frac{d\sigma^c}{d\Omega}\right)_{\theta} 2\pi \sin\theta d\theta$$

II.C.2.c – Compton vs Photoelectric

Photoelectric

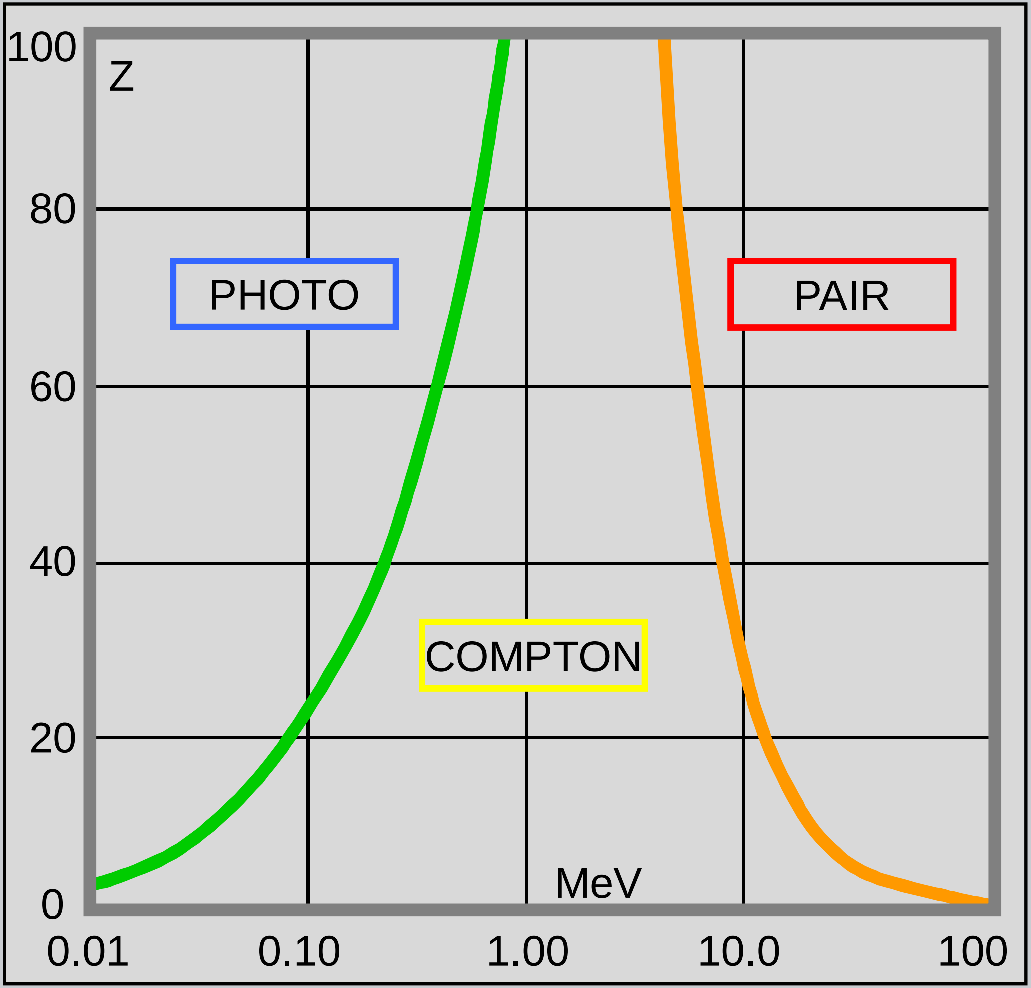
Dominant for high Z at low energy.

Compton

Dominant for low Z at medium energy.

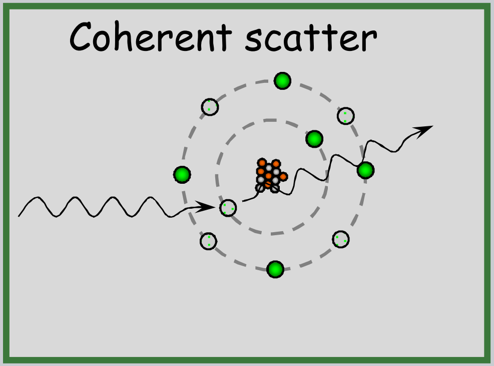
Pair production

Dominant for high Z at very high energy.



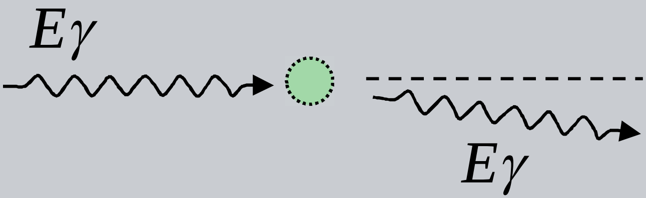
In the field of a nucleus, a photon may annihilate with the creation of an electron and a positron.

$E\gamma > 2 \times 511$ keV



Rayleigh Scattering

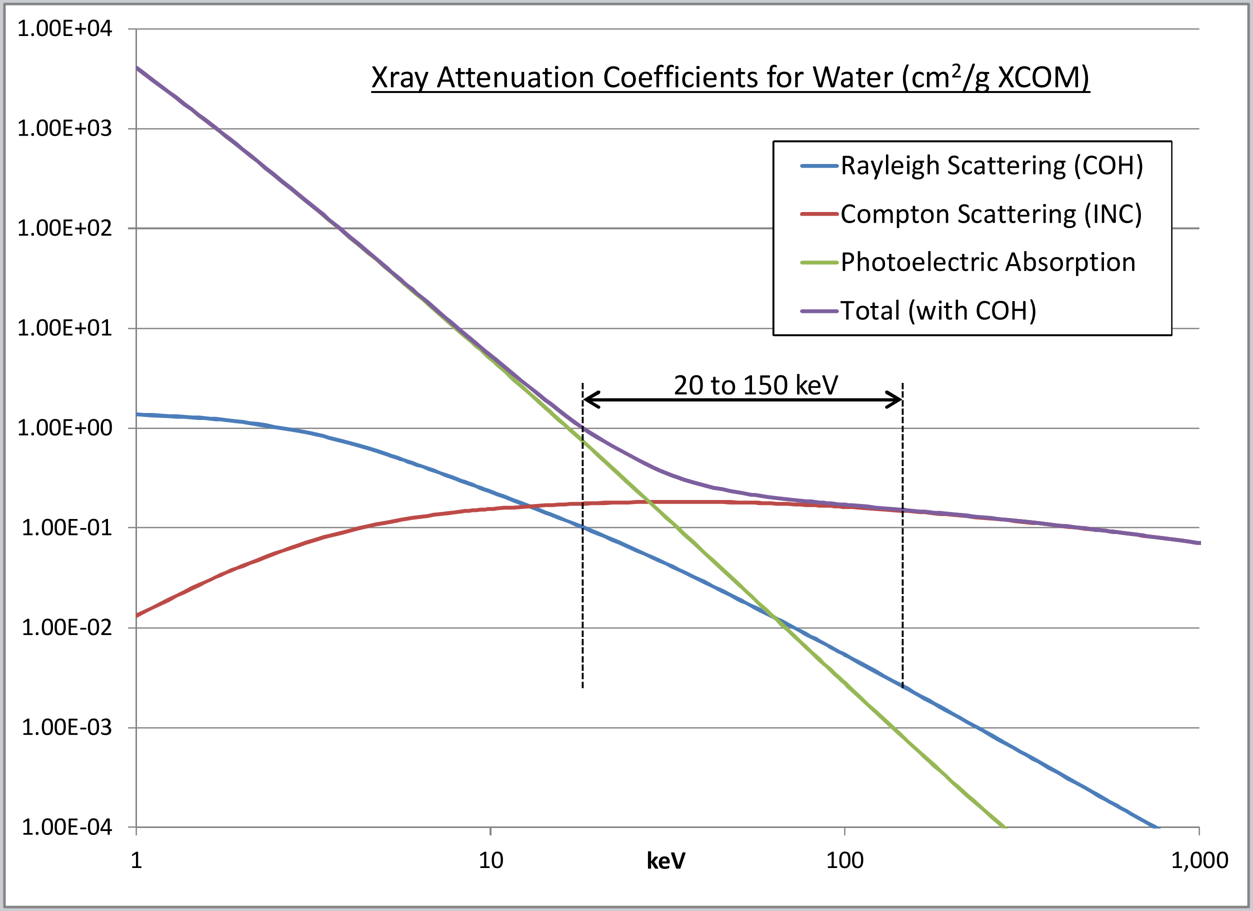
- An incident photon of energy E_γ interacts with atomic electrons with a change in direction but no reduction in energy. (i.e. coherent scattering)



The Thomson cross section can be understood as a dipole interaction of an electromagnetic wave with a stationary free electron. The electric field of the incident wave (photon) accelerates the charged particle which emits radiation at the same frequency as the incident wave. Thus the photon is scattered.

II.C.2d – Rayleigh Scattering

In the energy range from 20 – 150 keV, coherent scattering contributes 10% - 2% to the water total attenuation coefficient.



II.C.2d – Rayleigh Scattering

Even though there are other components to the total coherent scattering, such as nuclear Thomson, Delbruck, and resonant nuclear scattering, Rayleigh scattering is the only significant coherent event for keV photons.

Rayleigh Scattering

- Rayleigh scattering is the coherent interaction of photons with the bound electrons in an atom. The angular dependence is described by the differential cross section, $d\sigma_R/d\Omega$.
- It is common to consider coherent scattering as a modification, with an atomic form factor, $F(\chi, Z)$, of the Thomson cross section,

$$\frac{d\sigma_R}{d\Omega} = |F(\chi, Z)|^2 \frac{d\sigma_{Th}}{d\Omega}$$

$$\chi = \sin\left(\frac{1}{2}\theta\right)/\lambda$$

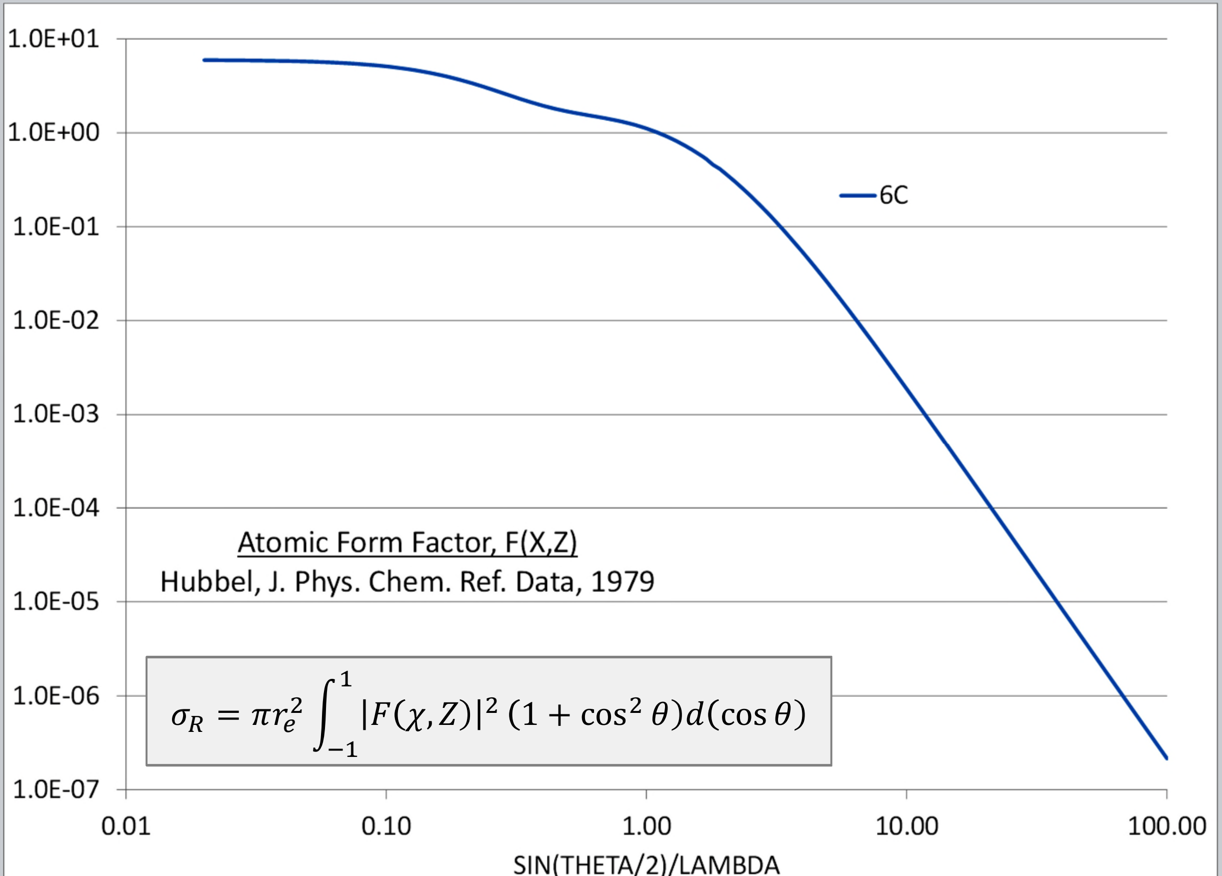
$$\frac{d\sigma_{Th}}{d\Omega} = \frac{1}{2} r_e^2 (1 + \cos^2\theta)$$

- For the differential Thomson cross section per electron,
  - It depends on the classical electron radius, r_e^2 = 0.079b (slide 30).
  - When integrated, the total cross section is the same as σ_o (slide 30).
  - The angular distribution is the same as for low energy Compton scattering (slide 32).
- The total cross section is then given by,

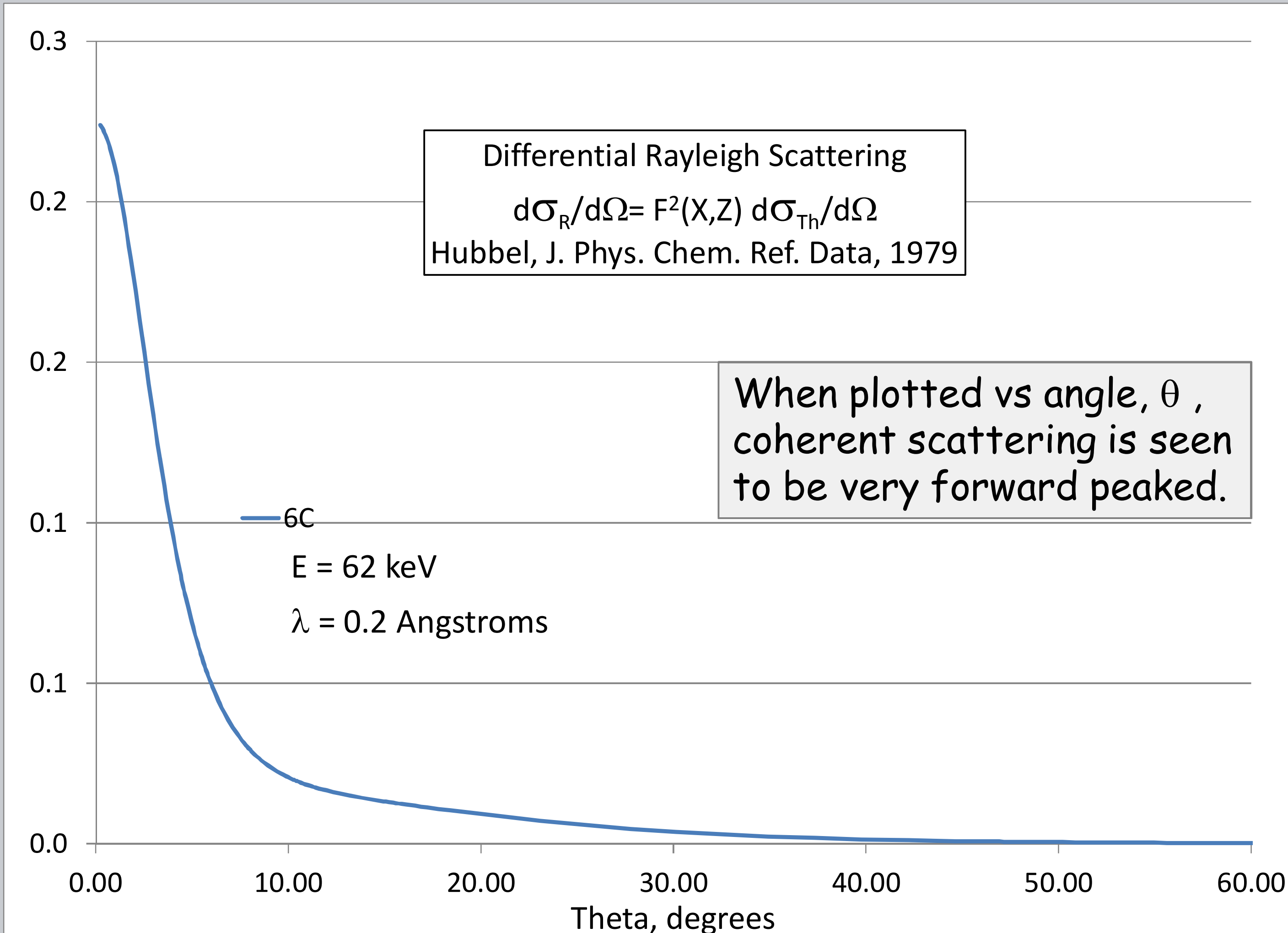
$$\sigma_R = \pi r_e^2 \int_{-1}^{1} |F(\chi, Z)|^2 \, (1 + \cos^2\theta) d(\cos\theta)$$

II.C.2d – Rayleigh Scattering

At low values of χ (small angle, low energy) the form factor is equal to the number of electrons (i.e 6 for Carbon in this example).

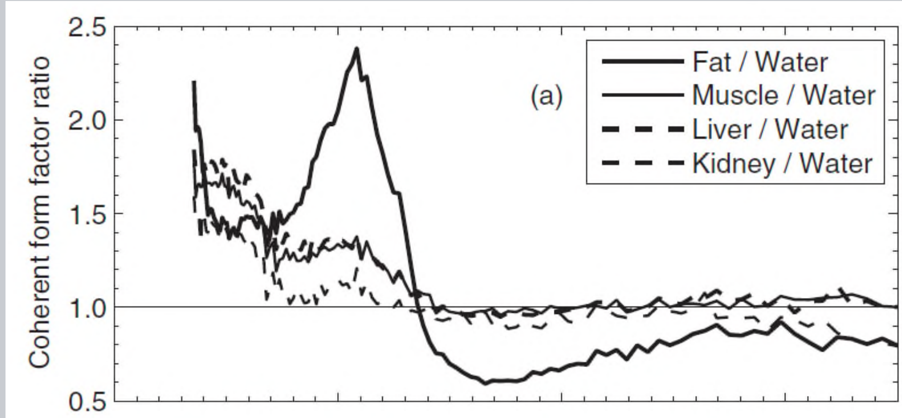


$$\sigma_R = \pi r_e^2 \int_{-1}^{1} |F(\chi, Z)|^2 \, (1 + \cos^2\theta) d(\cos\theta)$$

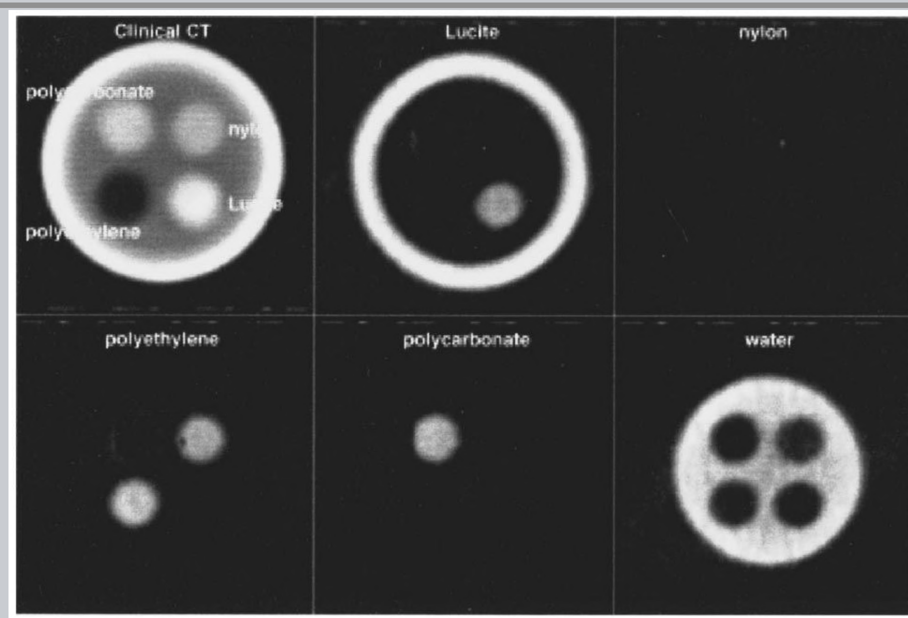


When plotted vs angle, θ, coherent scattering is seen to be very forward peaked.

- Coherent scattering from molecules and compounds is more complex that for atoms.
- King2011 – King BW et. al. , Phys. Med. Biol, 56, 2011.



- This has been investigated as a way to obtain material specific images.
- Westmore1997 – Westmore MS et.al., Med.Phys,24,1997

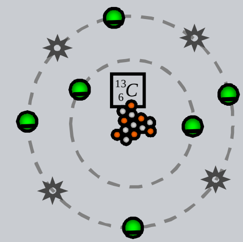


D) X-ray generation

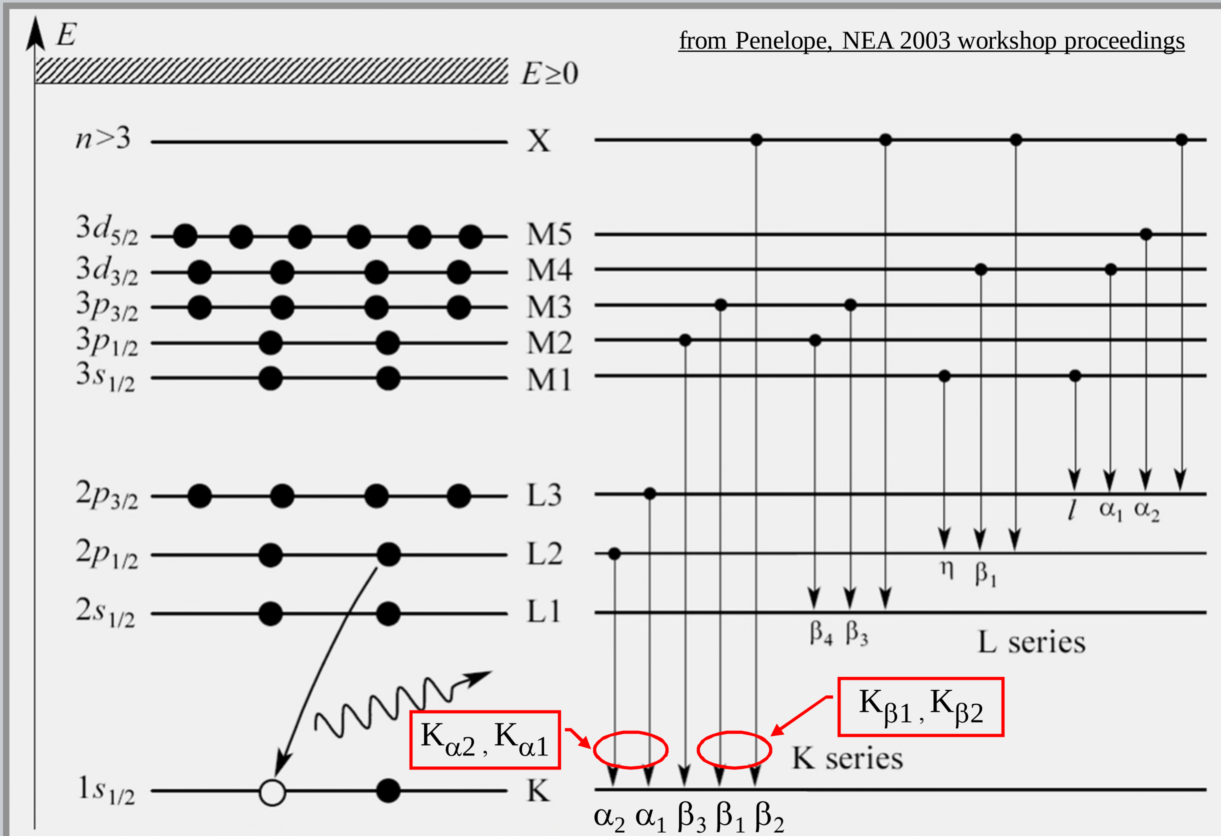
1) Atoms and state transitions
2) Bremmsstralung production

II.D.1 – Atomic levels

- Each atomic electron occupies a single-particle orbital, with well defined ionization energy. https://en.wikipedia.org/wiki/X-ray_notation
- The orbitals with the same principal and total angular momentum quantum numbers and the same parity make a shell.

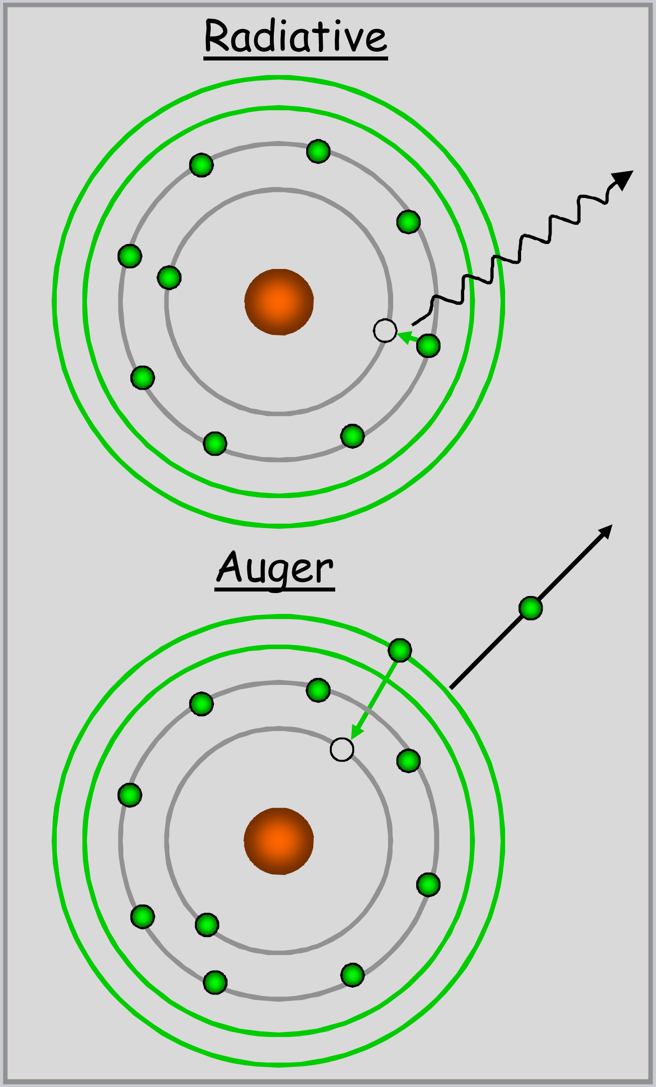


Each shell has a finite number of electrons, with ionization energy Ui.

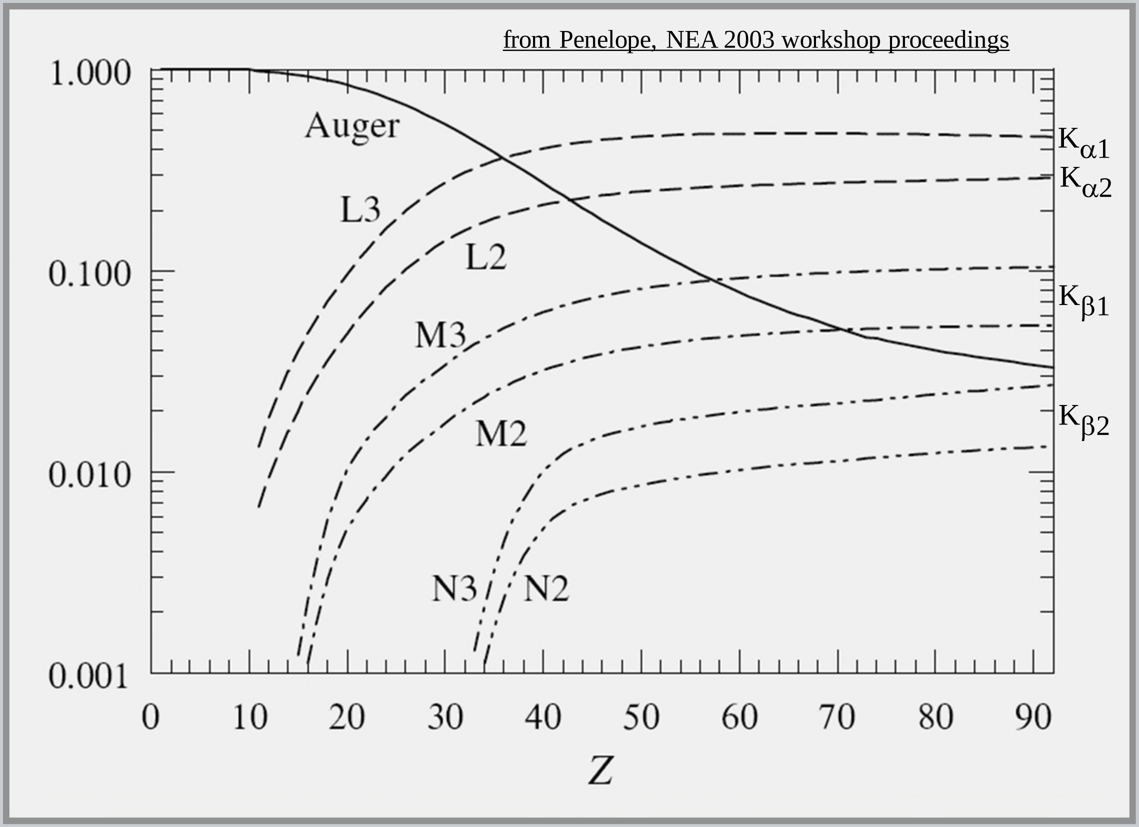


II.D.1 – Atomic relaxation

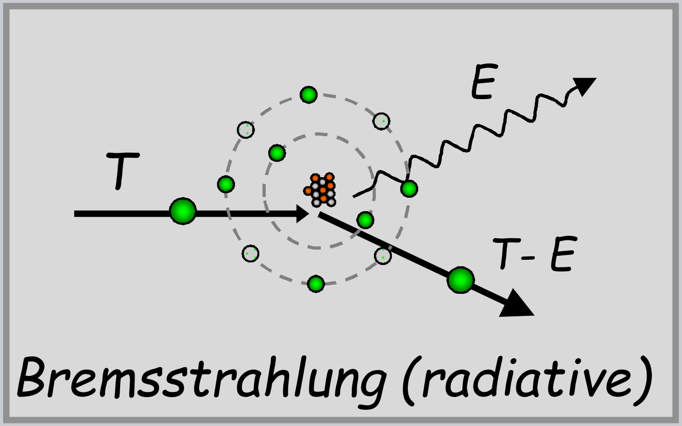
- Excited ions with a vacancy in an inner shell relax to their ground state through a sequence of radiative and non-radiative transitions.
- In a radiative transition, the vacancy is filled by an electron from an outer shell and an x ray with characteristic energy is emitted.
- In a non-radiative transition, the vacancy is filled by an outer electron and the excess energy is released through emission of an electron from a shell that is farther out (Auger effect).
- Each non-radiative transition generates an additional vacancy that in turn, migrates "outwards".



Relative probabilities for radiative and Auger transitions that fill a vacancy in the K-shell of atoms.



- In a bremsstrahlung event, a charged particle with kinetic energy T generates a photon of energy E, with a value in the interval from 0 to T.



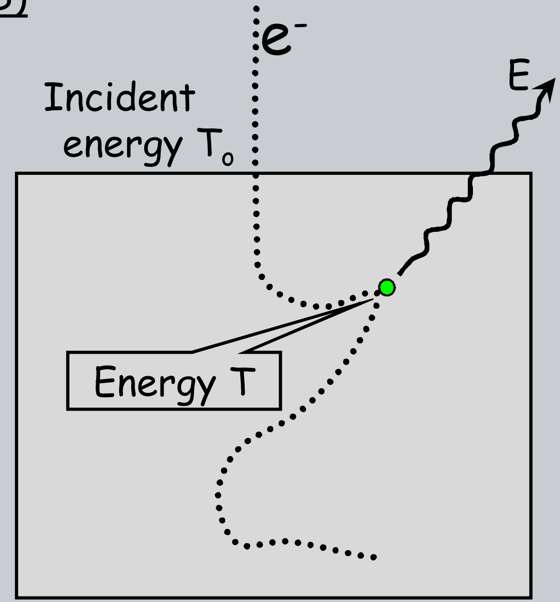
- Bremsstrahlung (braking radiation) production results from the stong electrostatic force between the nucleus and the incident charged particle.
- The acceleration produced by a nucleus of charge Ze on a particle of charge ze and mass M is proportional to Zze^2/M. Thus the intensity (i.e. the square of the amplitude) will vary as

$$Z^2z^2/M^2$$

- For the same energy, protons and alpha particles produce about 10^{-6} as much bremsstrahlung radiation as an electron.

II.D.2 – Brems. Differential Cross Section (DCS)

- The probability per atom that an electron traveling with energy T will produce an x-ray within the energy range from E to E+dE is known as the radiative differential cross section (DCS) , $d\sigma_r/dE$.
- Theoretic expressions indicate that the bremsstrahlung DCS can be expressed as;



$$\frac{d\sigma_r}{dE} = f_r(T,E,Z)\frac{Z^2}{\beta^2}\frac{1}{E}$$

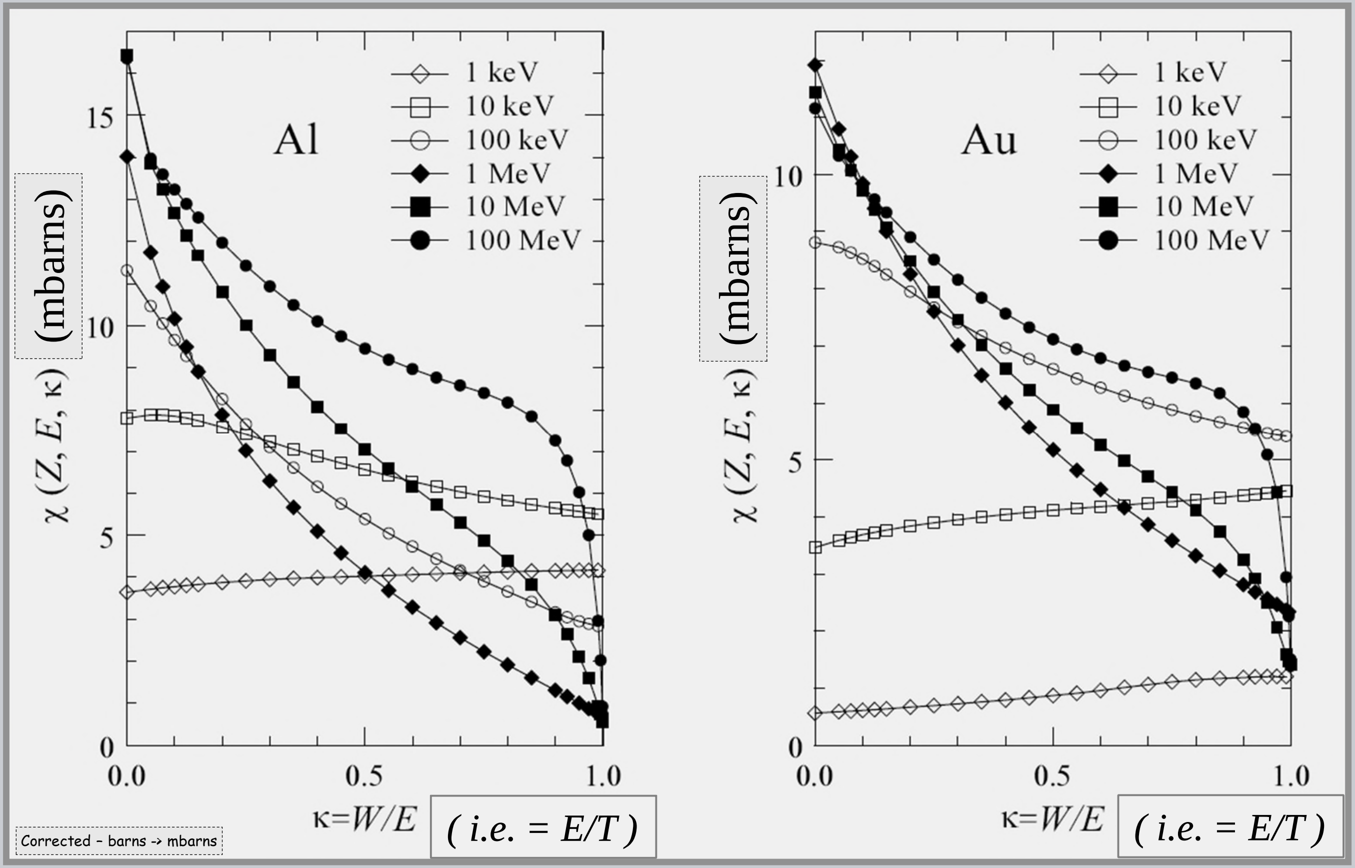
$$\beta^2 = 1 - \frac{1}{(1+\tau)^2}$$

$$\tau = T/m_e c^2$$

- Where β is the velocity of the electron in relation to the speed of light.
- The slowing varying function, $f_r(T,E,Z)$, is often tabulated as the scaled bremsstrahlung DCS.

Seltzer SM & Berger MJ, Atomic Data & Nucl. Data Tables, 35, 345-418(1986).

Numerical scaled bremsstrahlung energy-loss DCS of Al and Au as a function of x-ray energy relative to electron energy, W/E (E/T).



Seltzer and Berger, 1986, from Penelope, NEA 2003 workshop proceedings

II.D.2 – angular dependence of DCS

For convenience, the radiative DCS is written as,

$$\sigma_r \equiv {d\sigma_r}/{dE}$$

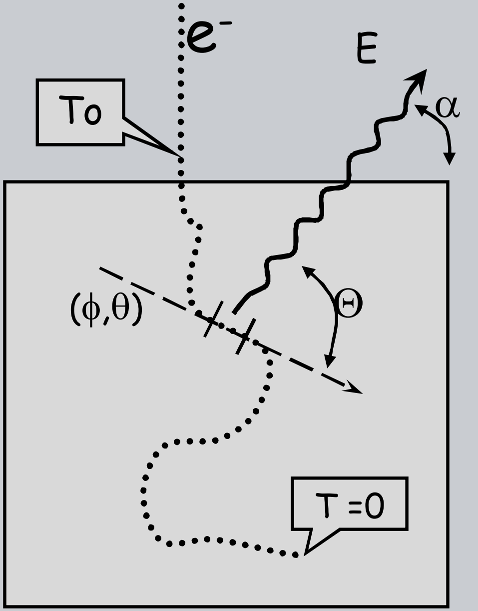
A doubly differential cross section is used for the interaction probability differential in x-ray energy, E, and solid angle, Ω , in the direction θ ,

$$\sigma_{r\Theta} \equiv {d^2\sigma_r}/{dEd\Omega}$$

The shape function for atomic-field bremsstrahlung is defined as the ratio of the cross section differential in photon energy and angle to the cross section differential only in energy.

$$S(T,E,\Theta) = \frac{\sigma_{r\Theta}}{\sigma_r}$$

And thus, $\sigma_{r\Theta} = S(T,E,\Theta)\sigma_r$



$\sigma_{r\Theta}$ - Barns/nuclei/keV/sr

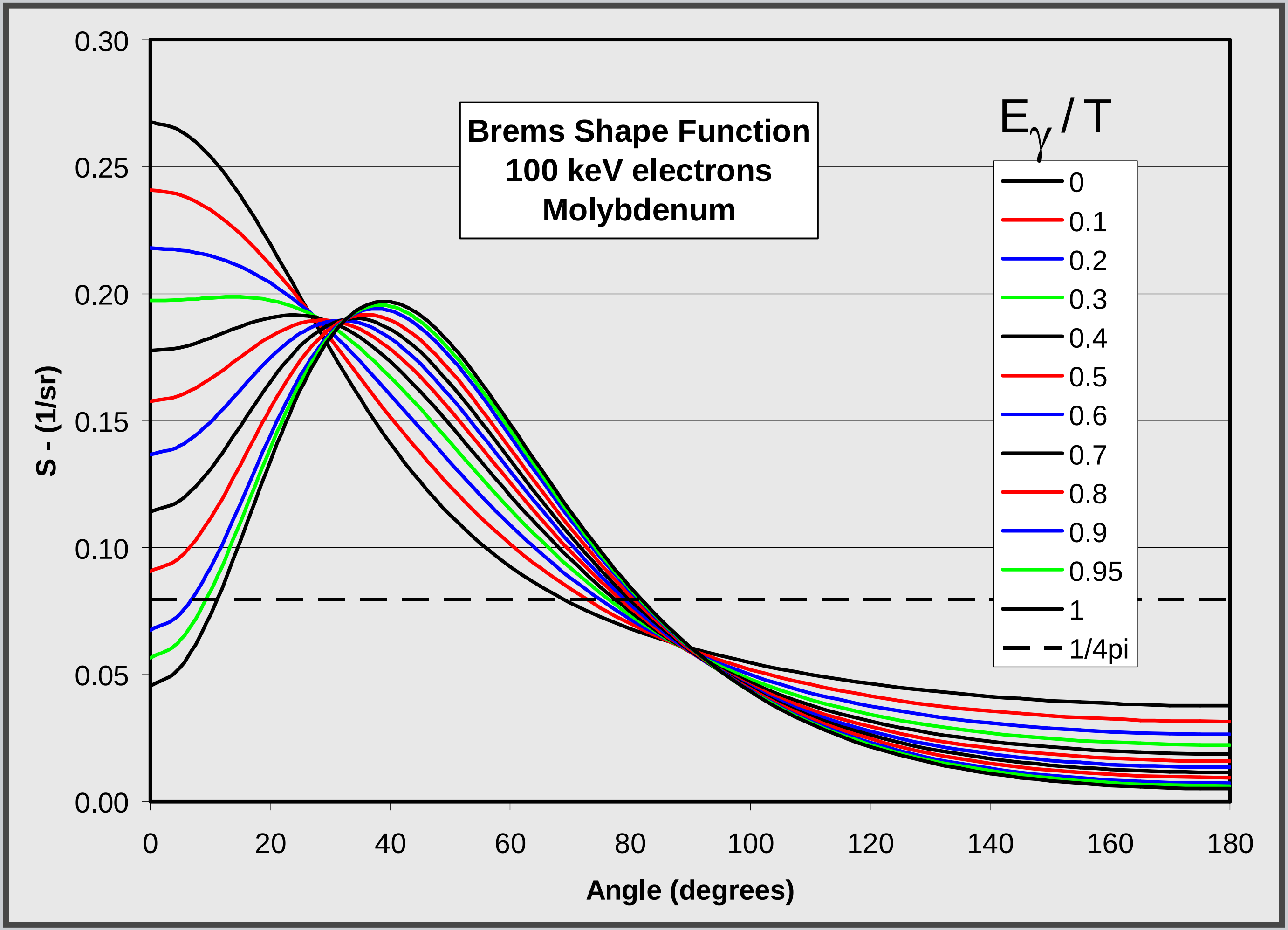
σ_r - Barns/nuclei/keV

Θ - electron (φ,θ) – xray (α)

α - xray takeoff angle

(φ,θ) – electron vector direction

II.D.2 – Kissel shape function, S(T,E)



In lecture 04 will consider the nuclear processes associated with the generation of gamma radiation.

a. n/p stability

b. beta emission

c. electron capture

d. positron emission