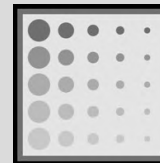


# NERS/BIOE 481

## Lecture 11 B Computed Tomography (CT)

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*Henry Ford*  
Health System

RADIOLOGY RESEARCH



## VII - Computed Tomography

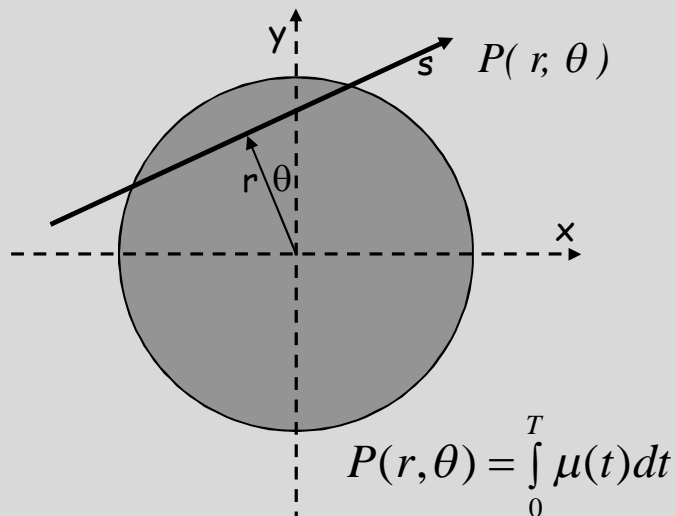
- A) X-ray Computed Tomography ...(L11)
- B) CT Reconstruction Methods ...(L11/L12)



### B) CT Reconstruction (L12)

- 1) Projection geometry (5 slides)
- 2) Fourier Domain Solution
- 3) Convolution / Backprojection
- 4) Cone beam reconstruction
- 5) Iterative Reconstruction

## VII.B.1 - X-ray projection measurements



For an object with a variable attenuation coefficient  $\mu(x,y)$ , the transmitted x-ray intensity is given by the projection;

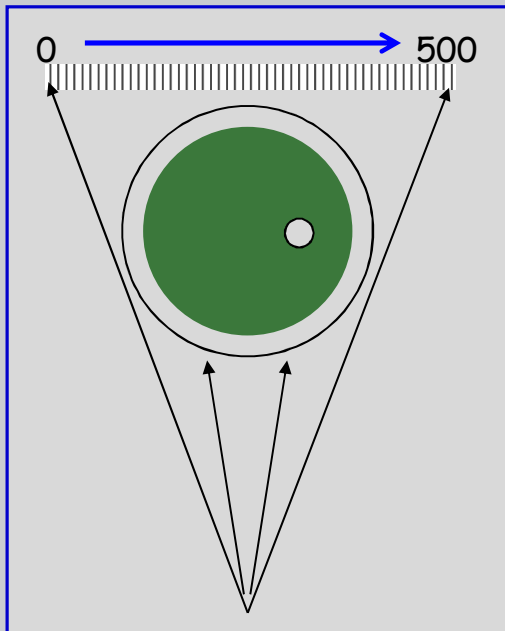
$$I(r, \theta) = I_o \exp[ - P(r, \theta) ]$$

Thus the projection can be deduced by measuring the transmission;

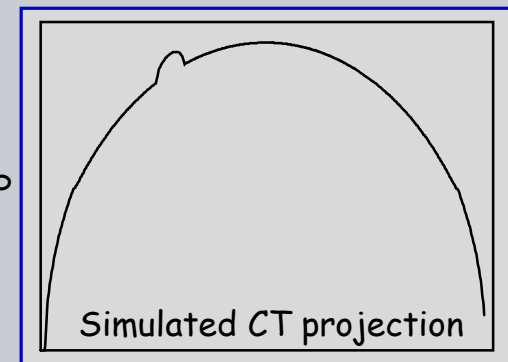
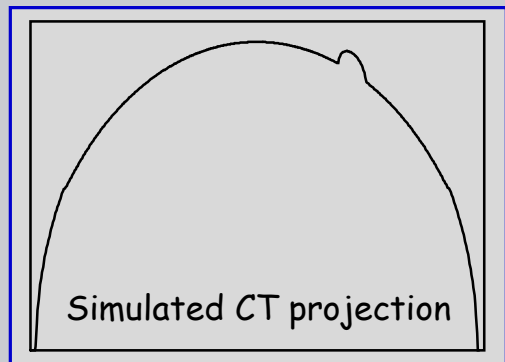
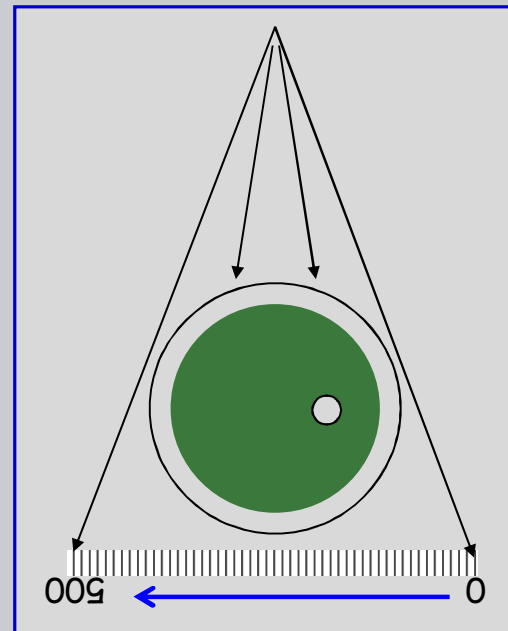
$$P(r, \theta) = -\text{Log}_{\text{nat}}[ I(r, \theta) / I_o ]$$

- CT scanner devices are periodically calibrated using a phantom to determine the reference signal  $I_o$ .
- The projection,  $P( r, \theta )$ , is determined using correction factors for x-ray spectral hardening and scattered radiation.

# VII.B.1 - Fan beam projection views - 0 & 180 degrees



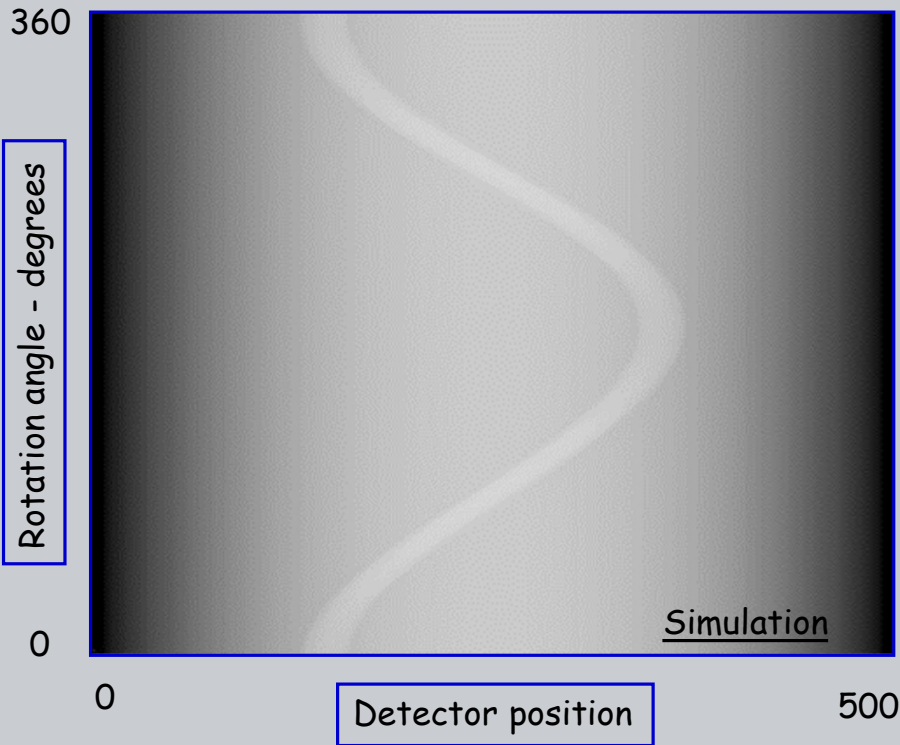
As a CT gantry rotates, the projection of a small target is recorded on the detector at positions that shift from one side to the other.



## VII.B.1 - Projection views: $0^\circ$ to $360^\circ$

### Sinogram:

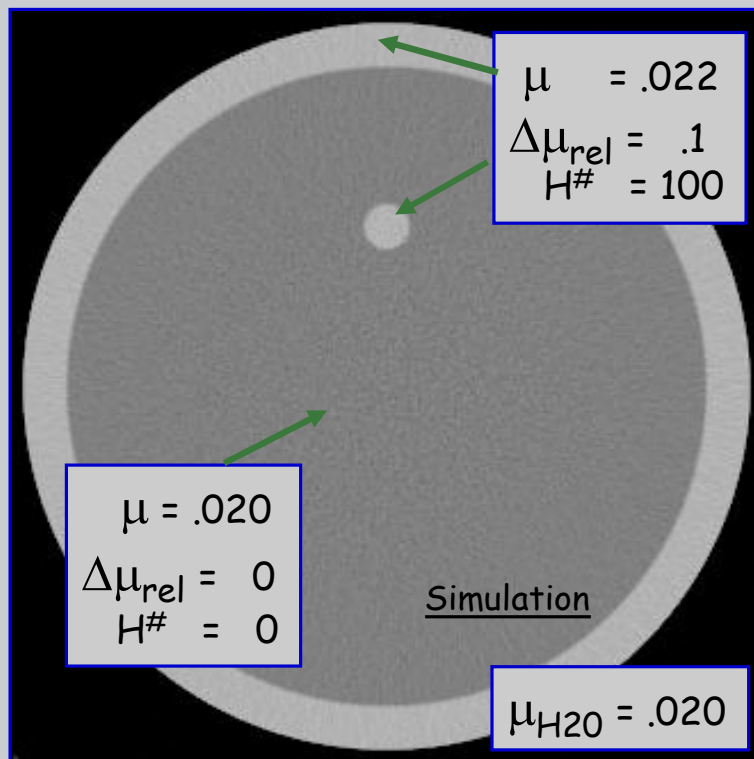
- An image with the projection values organized as rotation angle versus detector position is referred to as the sinogram.
- The sinogram depicts all of the transmission data used to perform a reconstruction of the object attenuation values.



sinogram of a more complex object



## VII.B.1 - Inverse solution (computed tomogram)



- Attenuation values:  
Image reconstruction results in the value for the material attenuation coefficient.
- Hounsfield Units (HU):  
Medical standards define the Hounsfield number as the reconstructed attenuation coefficient relative to water,

$$\Delta\mu_{rel}(x,y) = (\mu(x,y) - \mu_{H2O}) / \mu_{H2O}$$

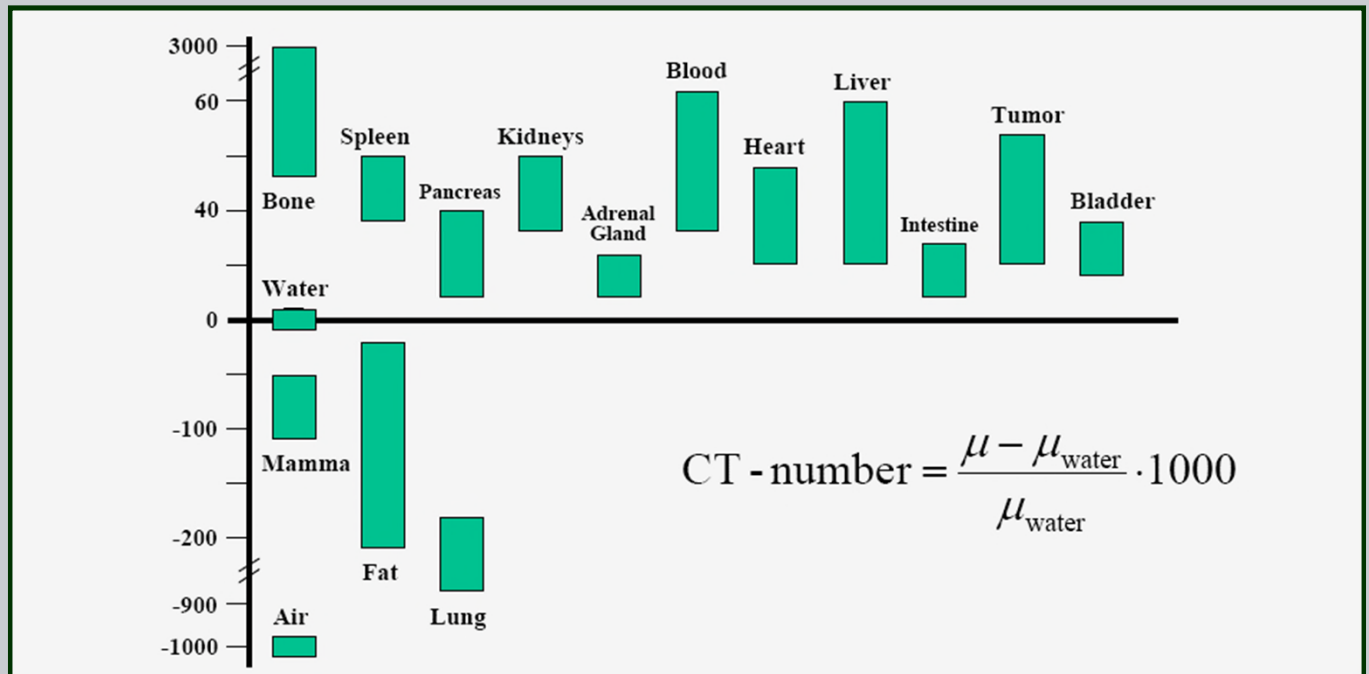
$$H^\# = 1000 \Delta\mu_{rel}(x,y)$$

$$H^\# \text{ water} = 0$$

$$H^\# \text{ air} = -1000$$

## CT numbers for Medical CT images

- For soft tissues, the Hounsfield numbers are between 0 and 100.
- This corresponds to a 1% range of attenuation coefficient values.
- Air (~-1000) and bone (> 1000) provide high contrast.



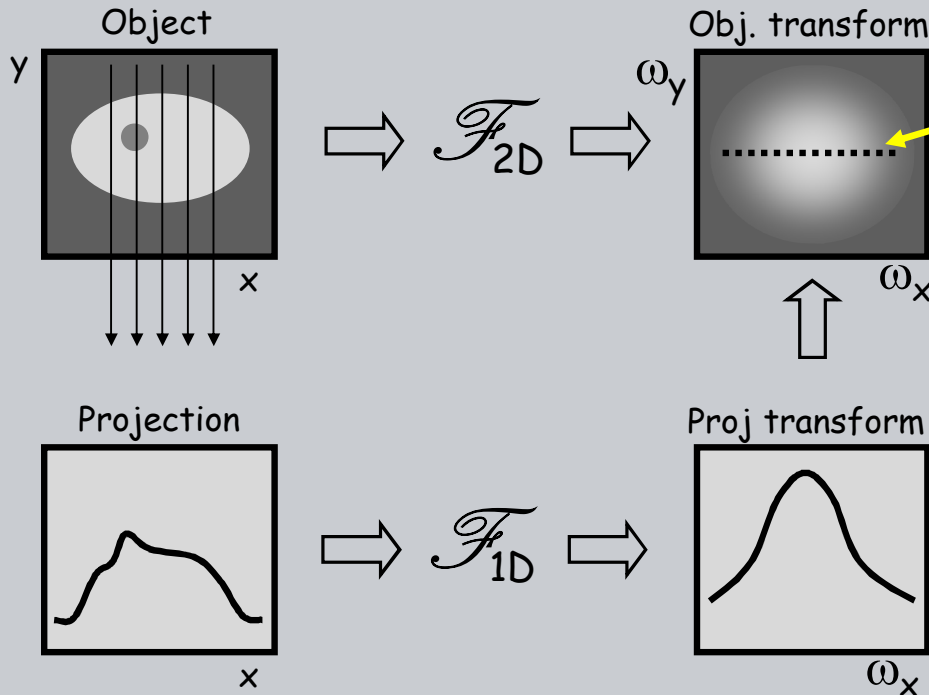




### B) CT Reconstruction

- 1) Projection geometry
- 2) Fourier Domain Solution (9 slides)
- 3) Convolution / Backprojection
- 4) Cone beam reconstruction
- 5) Iterative Reconstruction

The central slice theorem from Fourier analysis provides a method to easily demonstrate that an object can be reconstructed from projections.



The values of the 1D transform of an object projection are equal to the values of the 2D transform of the object along a line through the  $(0,0)$  coordinate that is perpendicular to the projection direction.



The central slice theorem is easily proven by considering the values of the Fourier transform of an object,  $O(x,y)$ , along the  $\omega_y = 0$  axis,

$$\mathfrak{F}(\omega_x, \omega_y) = \iint O(x, y) \cdot e^{-2\pi i(\omega_x x + \omega_y y)} dx dy$$

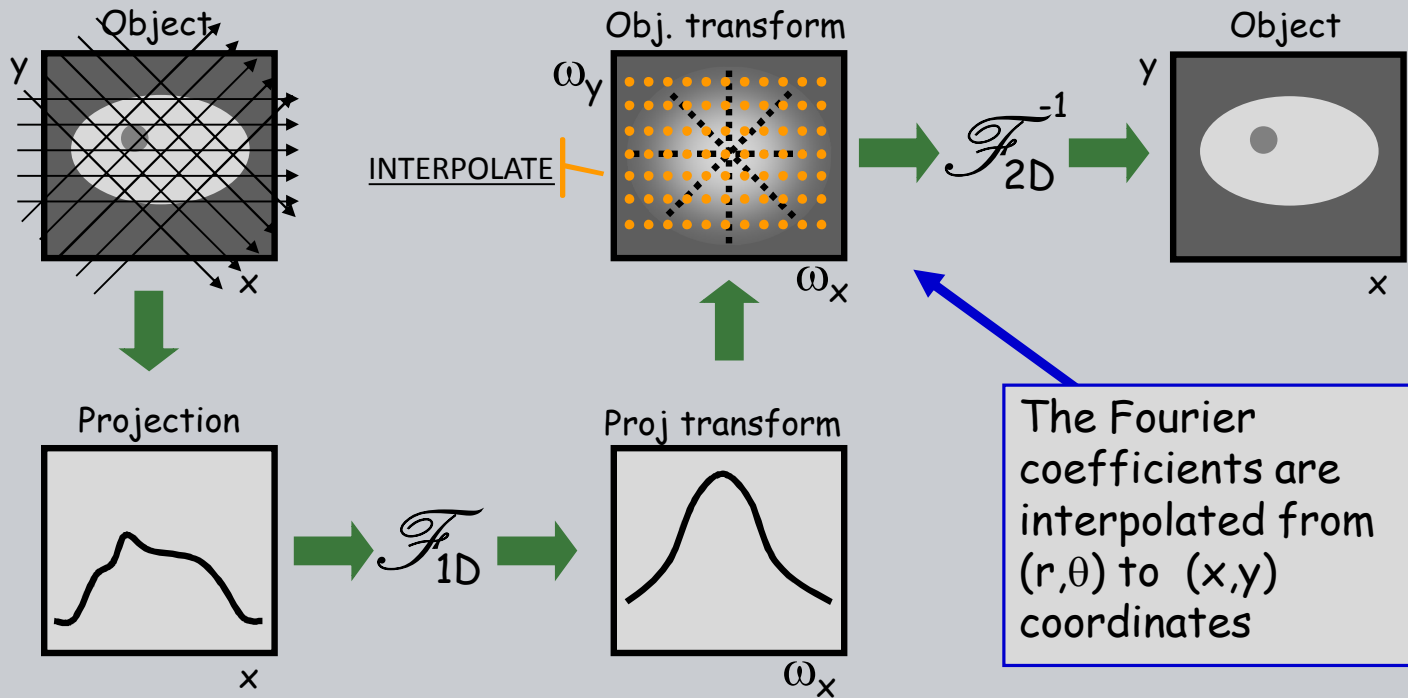
$$\mathfrak{F}(\omega_x, 0) = \int \left( \int O(x, y) dy \right) \cdot e^{-2\pi i(\omega_x x)} dx$$

$$\mathfrak{F}(\omega_x, 0) = \int (P(r, 0)) \cdot e^{-2\pi i(\omega_x x)} dx$$

The inner integration reduces to the projection in a direction parallel to the y axis ( $\theta=0$ ). Other directions can be considered by a simple rotation of the object.

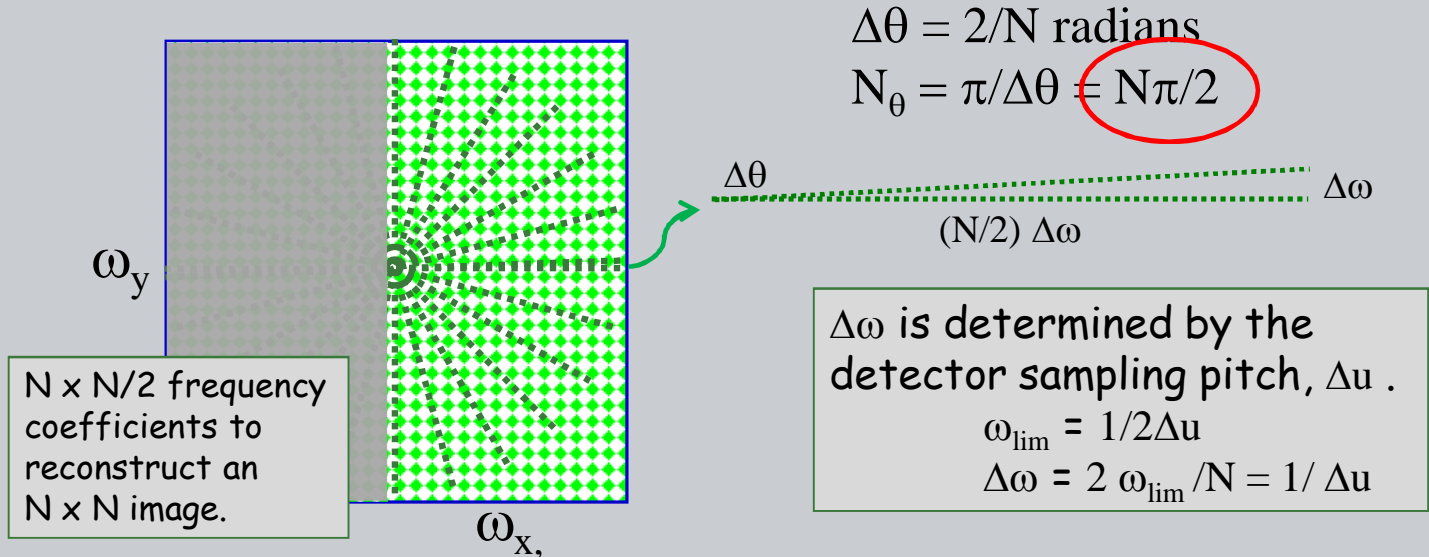
## VII.B.2 - Fourier reconstruction method.

Projections measured from many directions are transformed to describe the 2D Fourier transform of the object. The Object material properties are estimated using the 2D inverse Fourier transform



## VII.B.2 - Angular sampling requirement

Full sampling of the Fourier domain requires that the radial frequency coefficients be closely spaced in the high frequency portion of the domain.



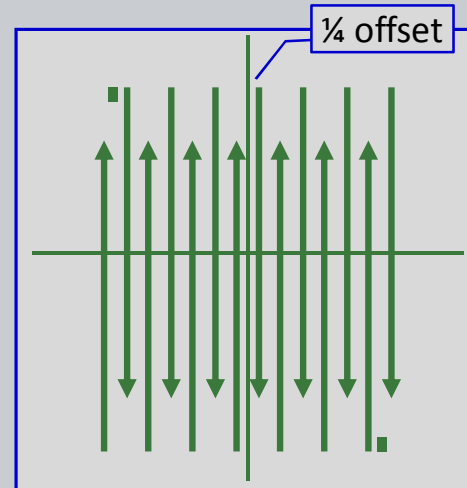
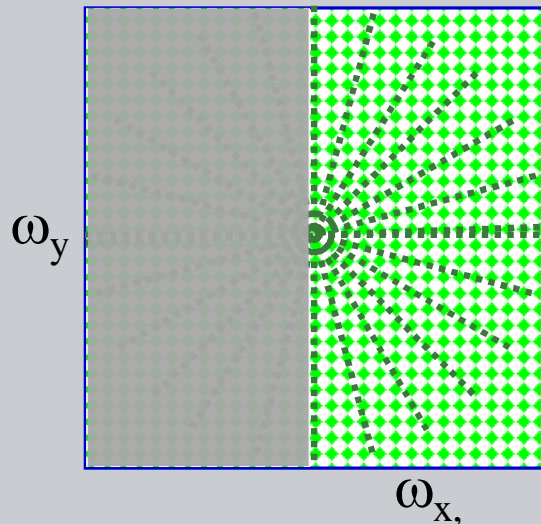
Angular sampling may be doubled to overlap the detectors element for each projection sample

### Views required to reconstruct a 512x512 image

800 views	180 °	
1600 views	360°	quarter offset geometry
3200 views	360°	$\frac{1}{4}$ offset + double sampling

## VII.B.2 - quarter-quarter offset

- Angular sampling over 180 degrees is sufficient to describe an object in the Fourier domain.
- However, 360 degree sampling is commonly done with the rotation center offset by  $(\frac{1}{4}, \frac{1}{4})$  of the sample increment,  $\Delta\mu$ .

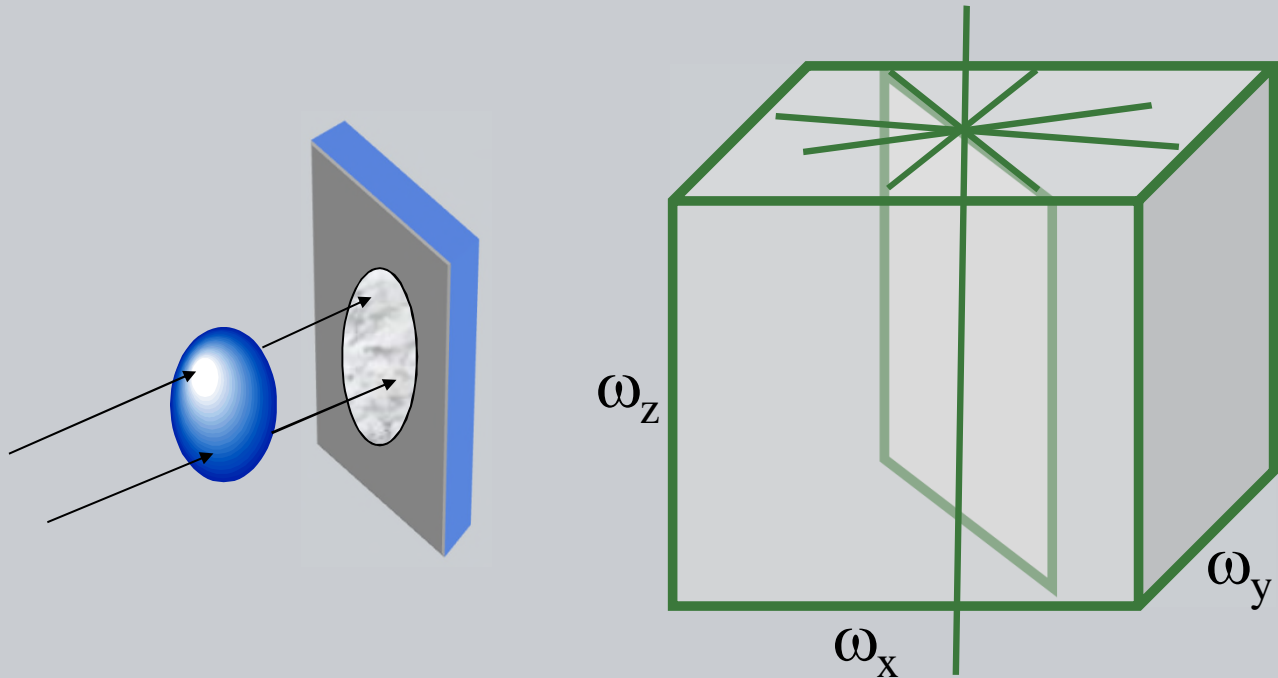


$\frac{1}{4}, \frac{1}{4}$  offset sampling improves resolution by decreasing the effective sampling increment,  $\Delta\mu$ , by a factor of two.

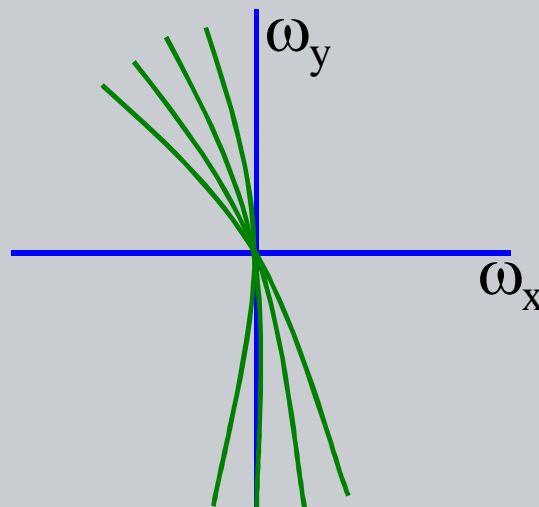
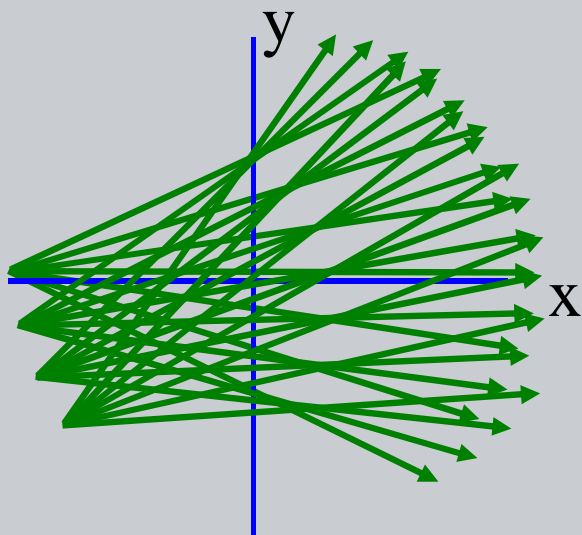


## VII.B.2 - Parallel beams, circular orbits

A parallel beam of radiation used to acquire  $P(u,v)$  using circular rotational sampling completely samples the 3D Fourier domain.

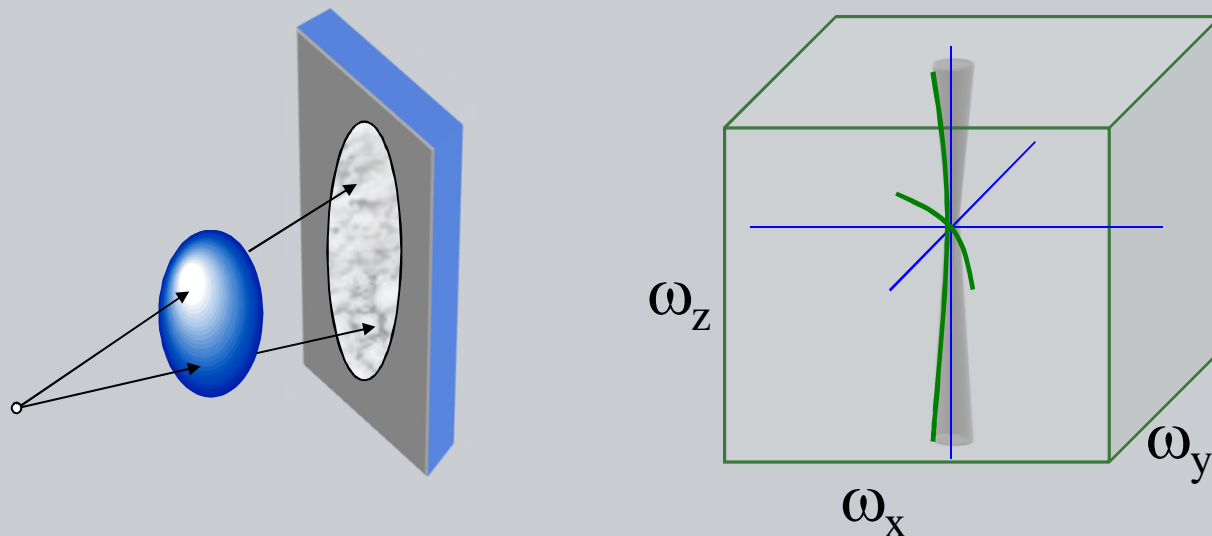


A fan beam of radiation used to acquire  $P(u)$  with angular sampling produces frequency samples in the 2D Fourier domain in arcs through the 0,0 axis.





A cone beam of radiation used to acquire  $P(u,v)$  with angular sampling DOES NOT FULLY SAMPLE the 3D Fourier domain in the region of the axis.



Each projection is associated with a dish shaped surface of fourier coefficients going through the 3D frequency domain. When rotated, there is a void of coefficients along the axis of rotation.

- The Radon values of all planes intersecting the object have to be known in order to perform an exact reconstruction. The Tuy sufficiency condition (Tuy 1983) states that exact reconstruction is possible if all planes intersecting the object also intersect the source trajectory at least once.
- The circular trajectory does not satisfy the Tuy-Smith condition as illustrated. It is therefore necessary to extend the trajectory with an extra circle or line if exact reconstruction is required.

Tuy, H. (1983). An inversion formula for cone-beam reconstruction. *SIAM Journal of Applied Mathematics* 43, 546-552.

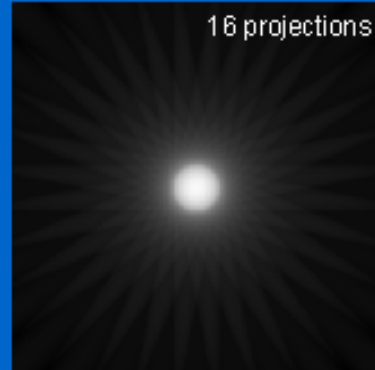
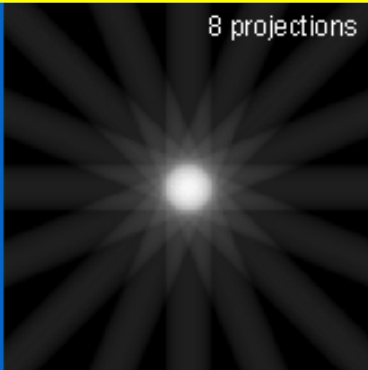
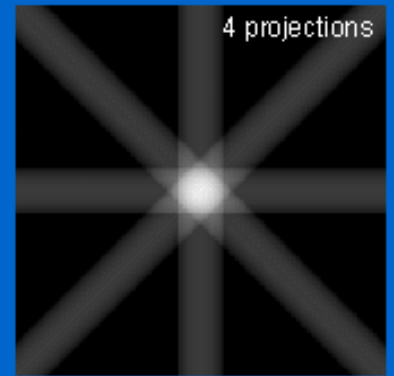
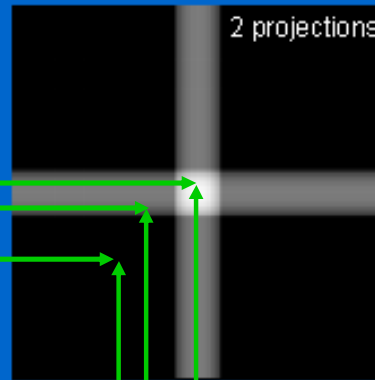
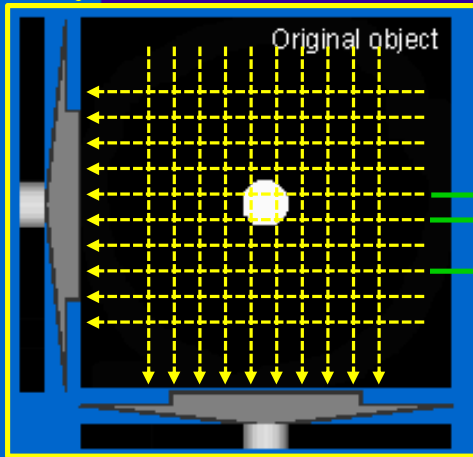


### B) CT Reconstruction

- 1) Projection geometry
- 2) Fourier Domain Solution
- 3) Convolution / Backprojection (11 slides)
- 4) Cone beam reconstruction
- 5) Iterative Reconstruction

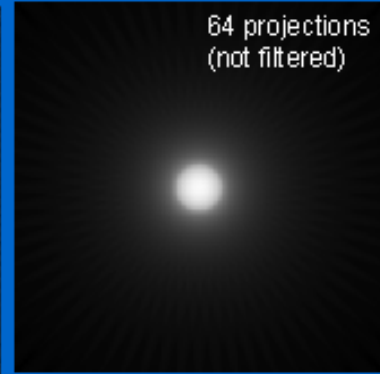
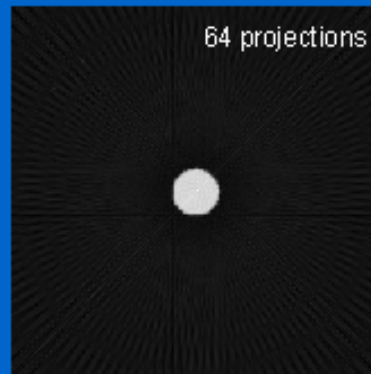
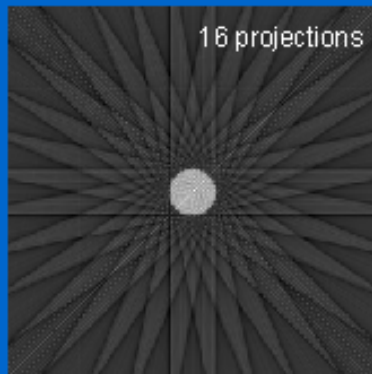
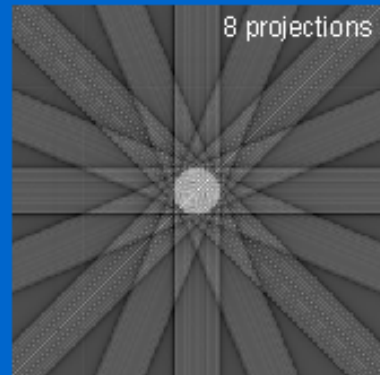
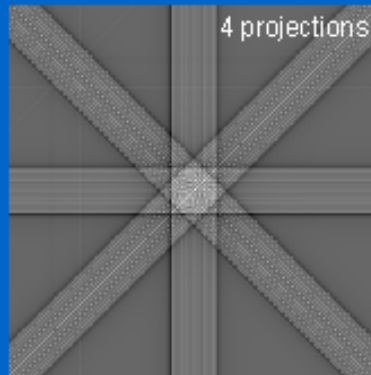
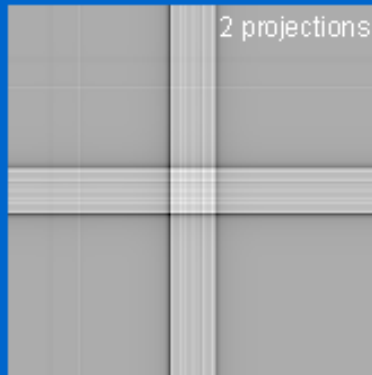
# Back Projection

Projection



# Filtered back projection

FPB



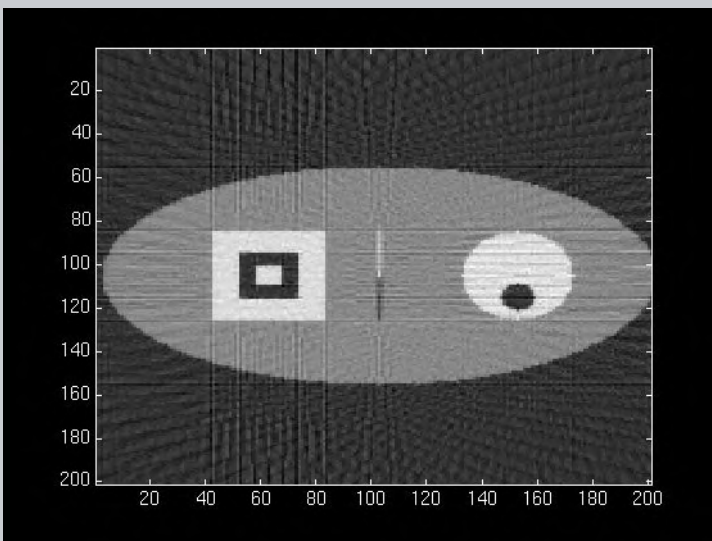
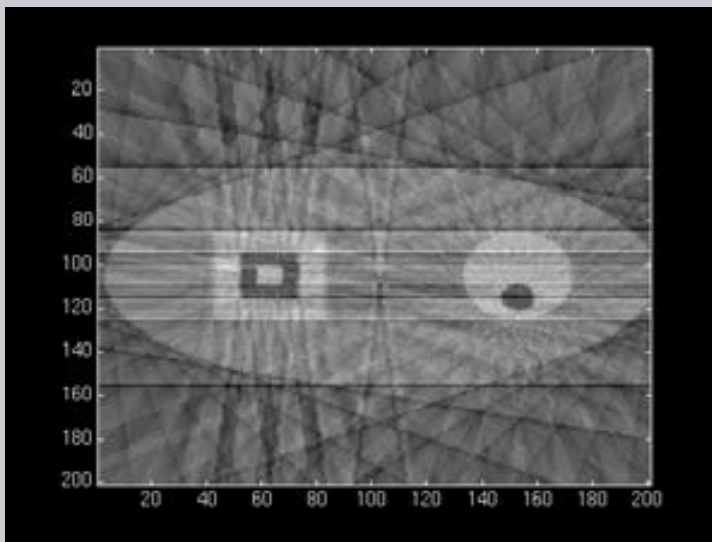


## VII.B.3 - Girod example

### Filtered - Backprojection

1. Measure projections.
2. Filter projections.
3. Backproject.

For every point in the reconstruction image, the value for each filtered projection is interpolated and added to the the image.

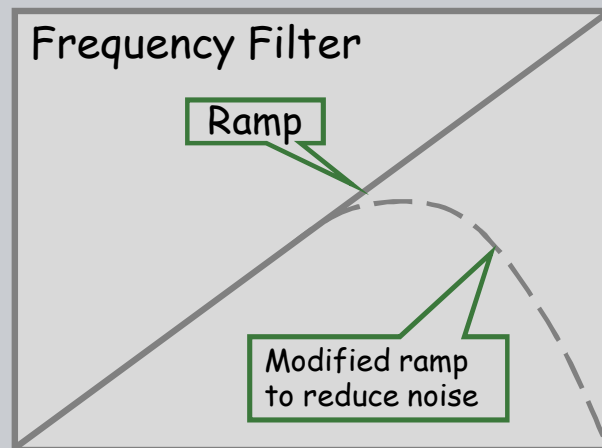
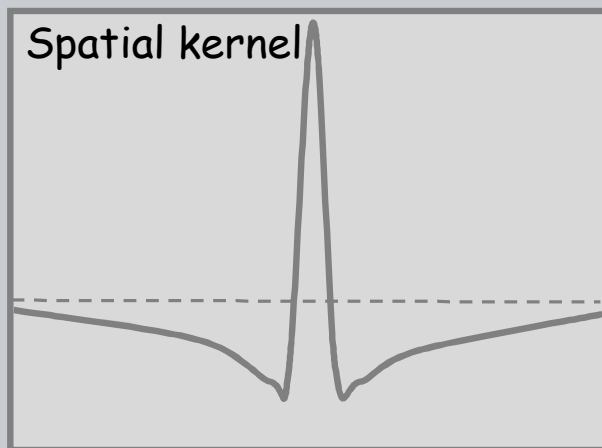




## VII.B.3 - Filter shape

Projections are filtered either by

- convolution with a spatial kernel or
- Fourier transformations with a filter function

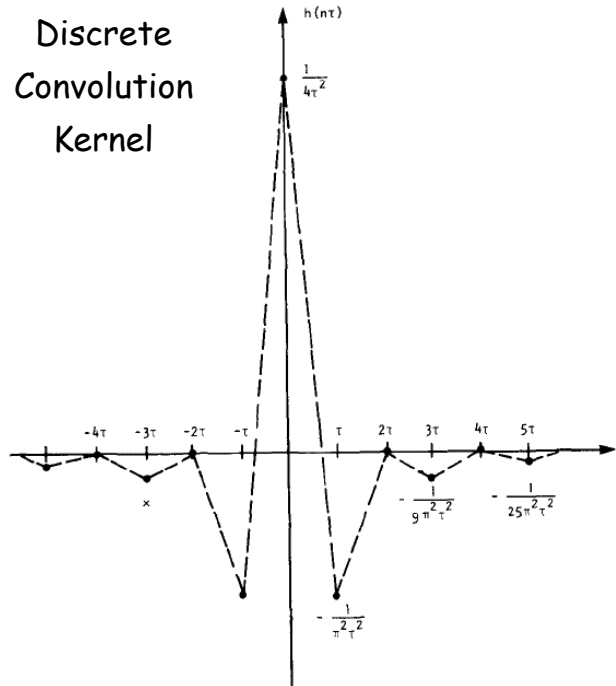


Equivalent:

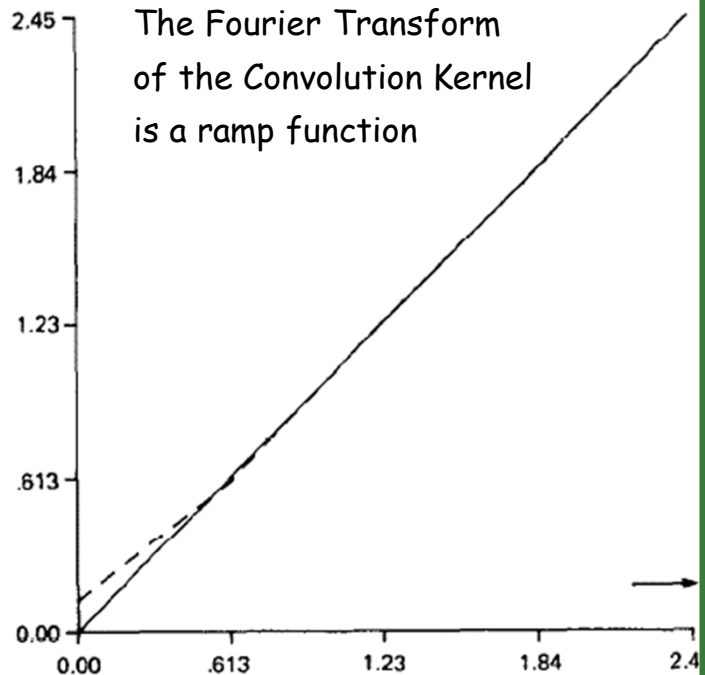
- Convolution Backprojection
- Filtered Backprojection

## VII.B.3 - Discrete kernals/filters

Discrete  
Convolution  
Kernel



The Fourier Transform  
of the Convolution Kernel  
is a ramp function



- A. C. Kak and Malcolm Slaney,  
Principles of Computerized Tomographic Imaging, IEEE Press, 1988.
- <http://www.slaney.org/pct/pct-toc.html>



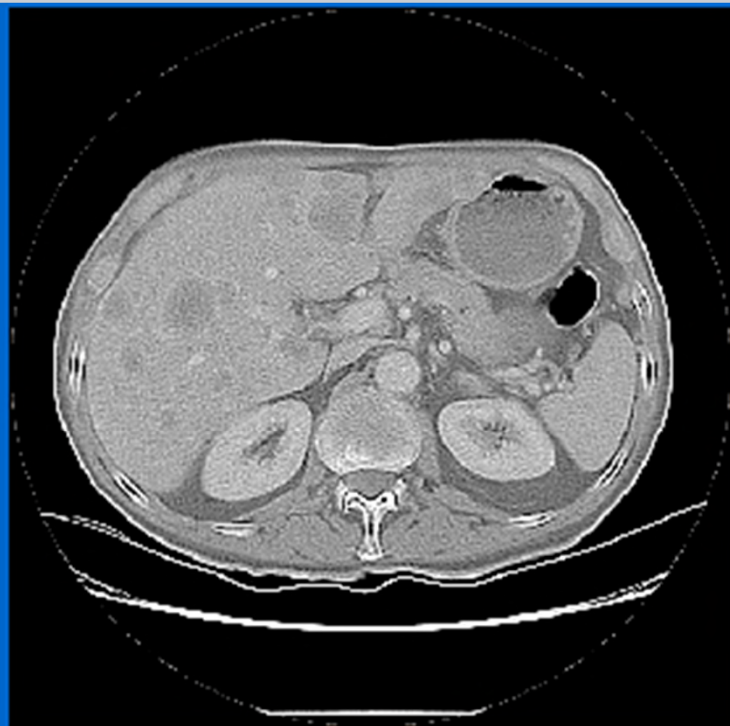
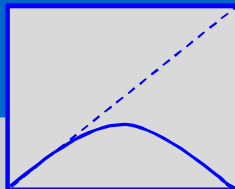


## VII.B.3 - Modified filters

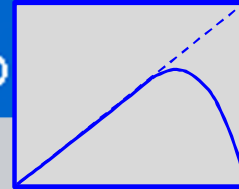
The ideal filter (ramp) is usually modified to smooth noise or sharpen edges.



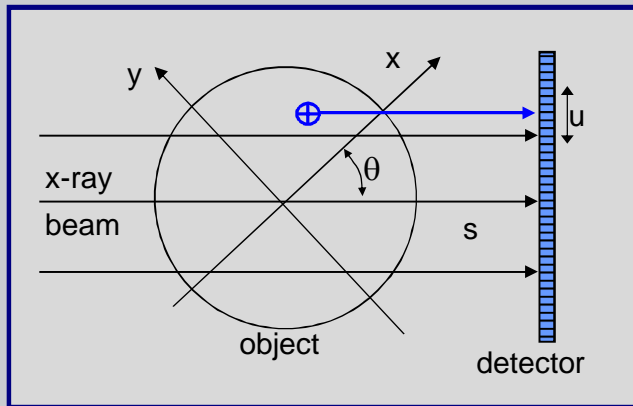
Smooth



Sharp



## VII.B.3 - FBP reconstruction, 2D parallel, integral notation



- Consider  $(x, y)$  positions in a plane of an object rotated about the  $z$  axis thru  $\theta$  degrees.
- The projection thru this point to the position  $u$  on the detector is,

$$P(u, \theta) = \int \mu(s) ds$$

$$u(x, y, \theta) = x \sin(\theta) + y \cos(\theta)$$

For each point  $(x, y)$ , the value of  $\mu$  is equal to the integral of the convolved projection over all angles

where:

- $u = u(x, y, \theta)$
- The convolution Kernel is the inverse Fourier transform of the ramp function,  $|\omega|$ , and any addition smoothing filter,  $F(\omega)$ .

$$\mu(x, y) = \int P^*(u, \theta) d\theta$$

$$P^*(u, \theta) = P(u, \theta) * K(u) \\ = \int K(u' - u) P(u', \theta) du'$$

$$K(u) = \int F(\omega) |\omega| e^{i\omega u} d\omega$$



## VII.B.3 - FBP reconstruction, 2D parallel, discrete notation

- For a detector with discrete elements spaced at a distance of  $\Delta u$ , the convolution can be written as a sum over the discrete kernel.
- While this sum is written with infinite limits, it is bounded by the object beyond which  $P$  is zero.
- The reconstruction can be similarly written as a discrete sum with a constant corresponding to the angular range.

$$P^*(u, \theta_k) = \sum_{l=-\infty}^{+\infty} K(l\Delta u - u)P(l\Delta u, \theta_k)\Delta u$$

$$\mu(x, y) = \frac{\pi}{N_\theta} \sum_{k=1}^{N_\theta} P^*(u, \theta_k)$$

If we write the solution as a double sum for the convolution and the backprojection, we can see that the noise of the results will be determined by the noise in the projection values at each position and angle.

$$\mu(x, y) = \frac{\pi}{N_\theta} \Delta u \sum_{k=1}^{N_\theta} \sum_{l=-\infty}^{+\infty} K(l\Delta u - u)P(l\Delta u, \theta_k)$$

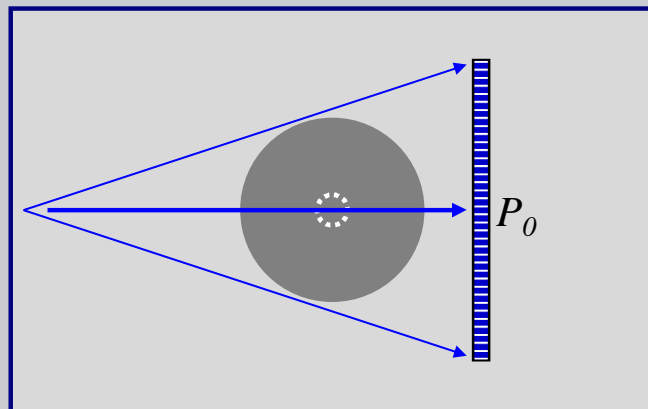
$$\sigma_{\mu(x, y)}^2 = \left\{ \frac{\pi}{N_\theta} \Delta u \right\}^2 \sum_{k=1}^{N_\theta} \sum_{l=-\infty}^{+\infty} \{K(l\Delta u - u)\}^2 \sigma_{P(l\Delta u, \theta_k)}^2$$



## VII.B.3 - FBP reconstruction, 2D parallel, discrete notation

- A useful solution for the noise in a reconstruction can be found for the special case of a cylindrical homogenous object.
- We then consider only the noise of the reconstruction in the center which is influenced by the noise in the central ray projection,  $P_0$ .
- The central ray projections are rotationally similar with a noise of  $\sigma_P$ .

Note: The fan beam solution (central cone) is the same as the parallel beam for the central ray (see VII.B.4).



- If the projection noise does not vary with angle,  $\sigma_P$ , then projection variance,  $\sigma_P^2$ , can be taken out of the summations.
- The angular summation is now trivial and results in an  $N_\theta$  term that cancels one in the denominator.

$$\sigma_{\mu(x,y)}^2 = \frac{\{\pi\Delta u\}^2}{N_\theta} \alpha^2 \sigma_P^2$$

$$\alpha^2 = \sum_{l=-\infty}^{+\infty} \{K(l\Delta u - u)\}^2$$

## VII.B.3 - FBP reconstruction, 2D parallel, discrete notation

- A common smoothing function used to modify the ramp filter is the sinc function,  $\sin(\omega)/\omega$ .
- For this the  $\alpha^2$  term is a function of the limiting spatial frequency,

$$\omega_{lim} = 1/(2\Delta u)$$

Note: See the lecture notes on CT noise propagation for the derivation of  $\alpha^2$ .

$$\begin{aligned} \alpha^2 &= \frac{\omega_{lim}^2}{2\pi^2} && \text{sinc filter} \\ \sigma_\mu^2 &= \frac{\omega_{lim}^2}{2N_\theta} \sigma_P^2 \\ &= \frac{1}{2^3 N_\theta (\Delta u)^2} \sigma_P^2 \end{aligned}$$

We recall now that the Projection is proportional to the natural log of the detected signal and thus the projection noise is equal to the relative noise of the detector signal.

$$P = -\ln(S/S_o) = \ln(S_o) - \ln(S), \quad \sigma_P = \sigma_S/S$$

In terms of the noise equivalent quanta,  $Q_{eq}$ , the projection noise is thus,

$$\sigma_P^2 = 1/SNR^2 = 1/(Q_{eq} A_d) = 1/(Q_{eq} \Delta u S_w)$$

where  $A_d$  = detector area,  $S_w$  = slice width.

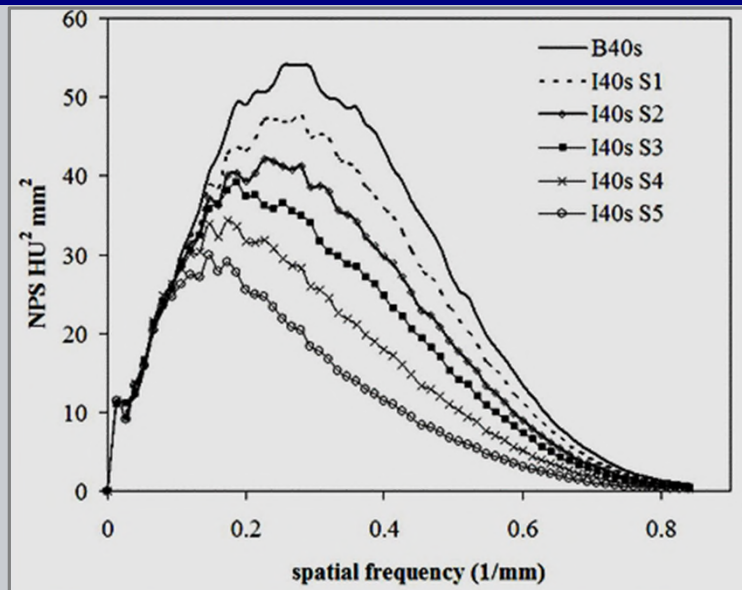
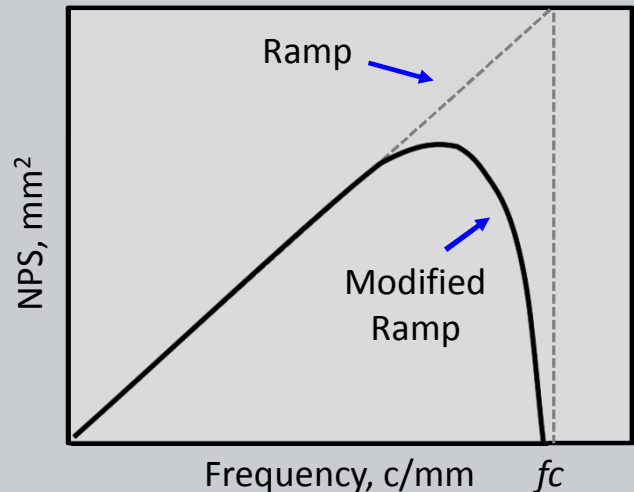
sinc filter

$$\sigma_\mu^2 = \frac{1}{2^3 N_\theta (\Delta u)^3 S_w} \left( \frac{1}{Q_{eq}} \right)$$

Note:

- For cone beam CT,  $S_w = \Delta u$ , and the CT noise is inversely proportional to the pixel area.
- Noise is inversely proportional to  $(\text{mas})^{1/2}$  due to  $Q_{eq}$ .
- CT SNR =  $\mu / \sigma_\mu$

- The Noise Power Spectrum (NPS) for CT reflects the modified ramp function used to filter the projections.
- As expected, the variance of reconstructed image values will be proportional to the area under the NPS.



Ghetti2013, JACMP

NPS measured for a SOMATOM Definition Flash CT scanner (Siemens).

- B40s is a standard FBP filter.
- I40s S[1-5] are filters used with the SAFIRE reconstruction algorithm.



### B) CT Reconstruction

- 1) Projection geometry
- 2) Fourier Domain Solution
- 3) Convolution / Backprojection
- 4) Cone beam reconstruction (11 slides)
- 5) Iterative Reconstruction



# Practical cone-beam algorithm

L. A. Feldkamp, L. C. Davis, and J. W. Kress

*Research Staff, Ford Motor Company, Dearborn, Michigan 48121*

Received November 11, 1983; accepted February 28, 1984

The cone-beam reconstruction algorithm developed at FMC is still widely used for both laboratory and clinical systems. The original paper has been cited over 5000 times (google scholar, 2017)

The original mCT system  
from Ford Motor Co.

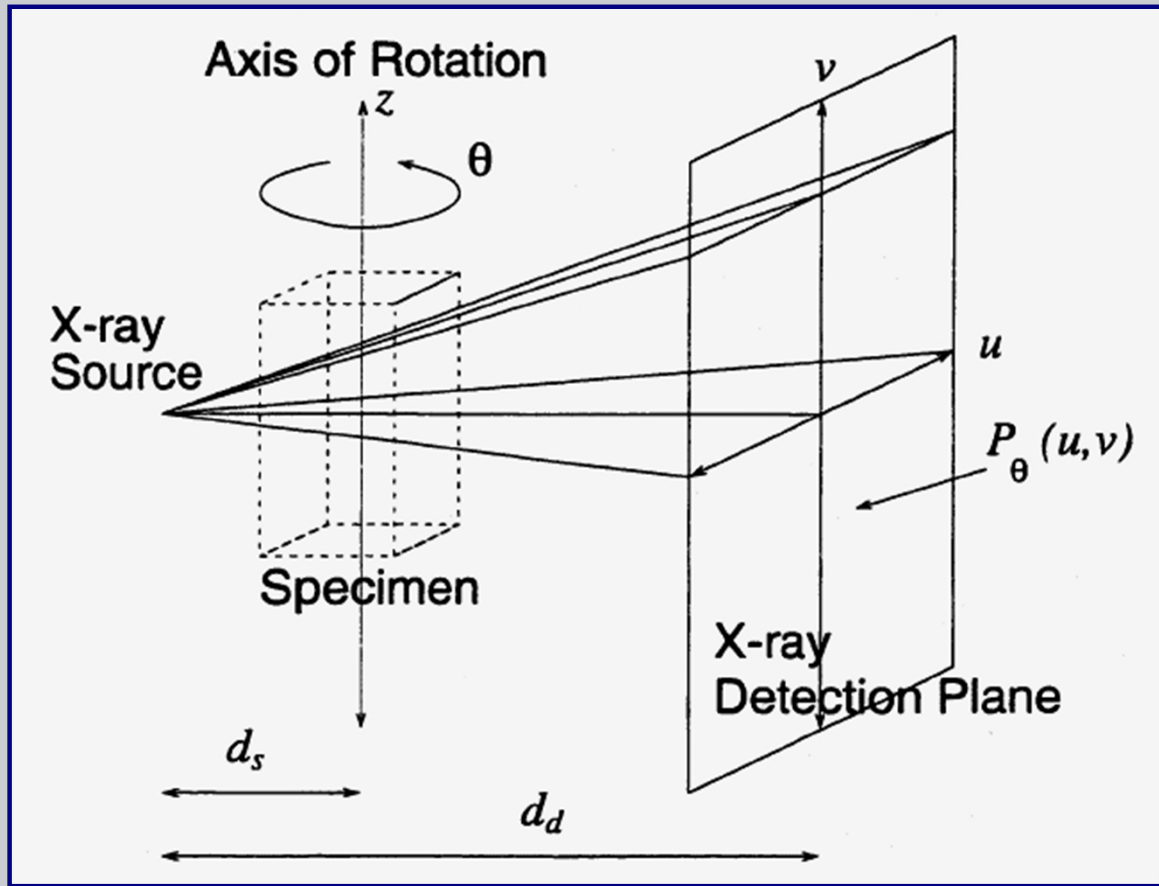
Fein Focus source

Image intensifier

(relocated to HFHS)







Reimann, WSU thesis, 1998

- A 3D solution for a parallel beam is a simple extension of the 2D solution.
- Each plane is an independent 2D solution.

- Resample detector data for alignment with rotation axis

$$P_{\theta}(u, v)$$

- Convolve the Projection Data in the direction of rotation

$$P_{\theta}^{*}(u, v) = P_{\theta}(u, v) * h(u)$$

- Backproject the convolved Projection Data

$$\mu(x, y, z) = \int_0^{2\pi} P_{\theta}^{*}(x', z') d\theta$$

The Feldkamp solution weights the projection data and scales the backprojection

- Weight Projection Data

$$P'_\theta(u, v) = \frac{d_s}{\sqrt{d_s^2 + v^2 + u^2}} P_\theta(u, v)$$

- Convolve Weighted Projection Data

$$P_\theta^*(u, v) = P'_\theta(u, v) * h(u)$$

- Backproject the convolved weighted Projection Data

$$\mu(x, y, z) = \int_0^{2\pi} \frac{d_s^2}{(d_s - y)^2} P_\theta^* \left( \frac{d_s x'}{d_s - y'}, \frac{d_s z'}{d_s - y'} \right) d\theta$$

## VII.B.4 - FKD pseudocode A - process the projection views

```
1  for each vertical detector position  $v$  ( $N_v$  positions):
2    for each horizontal detector position  $u$  ( $N_u$  positions):
3      precompute weights  $w(u, v) = d_s / \sqrt{d_s^2 + u^2 + v^2}$ 
4    end  $u$  loop
5  end  $v$  loop
6
7  for each  $\theta$  ( $N_\theta$  views):
8    Weight Projection
9    for each vertical detector position  $v$  ( $N_v$  positions):
10     for each vertical detector position  $u$  ( $N_u$  positions):
11        $p'_\theta(u, v) = p_\theta(u, v)w(u, v)$ 
12     end  $u$  loop
13   end  $v$  loop
14
15   Fourier Filter Zero Padded Projections 2 rows at a time
16   for every other vertical detector position  $v$  ( $N_v/2$  positions):
17      $P(u) = FFT\{p'_\theta(u, v) \text{ and } p'_\theta(u, v + 1)\}$ 
18     for each zero padded horizontal detector position  $u$  ( $2N_u$  positions):
19        $P^*(u) = P(u)h(u)$ 
20     end  $u$  loop
21      $p^*_\theta(u, v) \text{ and } p^*_\theta(u, v + 1) = FFT^{-1}\{P^*(u)\}$ 
22   end  $v$  loop
23
```

## VII.B.4 - FKD pseudocode B - Backproject

```
24   for each  $y$  position ( $N_y$  positions):
25       for each  $x$  position ( $N_x$  positions):
26           compute  $x' = x \cos \theta + y \sin \theta$ 
27           compute  $y' = y \cos \theta - x \sin \theta$ 
28           compute  $M = d_s / (y' - d_s)$ 
29           compute  $u = x' M + u_0$ 
30           compute  $M^2$ 
31           compute  $M^2 c_2 = M^2 (\lfloor u \rfloor)$ 
32           compute  $M^2 c_1 = M^2 (1 - \lfloor u \rfloor)$ 
```

Backproject each view  
column by column.

```
33       For each  $z$  position ( $N_z$  positions):
34           compute  $v = z M + v_0$ 
35           compute  $c_4 = \lfloor v \rfloor$ 
36           compute  $c_3 = 1 - \lfloor v \rfloor$ 
37           interpolate  $M^2 p_\theta^*(u, v) = c_4 (c_2 P(\lfloor u + 1 \rfloor, \lfloor v + 1 \rfloor)$   

            $+ c_1 P(\lfloor u \rfloor, \lfloor v + 1 \rfloor)) + c_3 (c_2 P(\lfloor u + 1 \rfloor, \lfloor v \rfloor) + c_1 P(\lfloor u \rfloor, \lfloor v \rfloor))$ 
38           increment  $I(x, y, z)$  by  $M^2 p_\theta^*(u, v)$ 
39           end  $z$  loop
40       end  $x$  loop
41   end  $y$  loop
42 end  $\theta$  loop
```

# VII.B.4 - FKD pseudocode, C computation overhead

Line Number	Floating Point Operations Required per Step	Total Times Step is Executed	Total Floating Point Operations Required
3	7	$N_v N_u$	$7N_v N_u$
11	1	$N_\theta N_v N_u$	$N_\theta N_v N_u$
17	$9(2N_u \log_2(2N_u))$	$N_\theta(N_v/2)$	$9N_\theta N_v N_u \log_2(2N_u)$
19	2	$N_\theta(N_v/2)(2N_u)$	$2N_\theta N_v N_u$
21	$9(2N_u \log_2(2N_u))$	$N_\theta(N_v/2)$	$9N_\theta N_v N_u \log_2(2N_u)$
26	3	$N_\theta N_y N_x$	$3N_\theta N_y N_x$
27	3	$N_\theta N_y N_x$	$3N_\theta N_y N_x$
28	2	$N_\theta N_y N_x$	$2N_\theta N_y N_x$
29	2	$N_\theta N_y N_x$	$2N_\theta N_y N_x$
30	1	$N_\theta N_y N_x$	$N_\theta N_y N_x$
31	1	$N_\theta N_y N_x$	$N_\theta N_y N_x$
32	2	$N_\theta N_y N_x$	$2N_\theta N_y N_x$
34	2	$N_\theta N_z N_y N_x$	$2N_\theta N_z N_y N_x$
36	1	$N_\theta N_z N_y N_x$	$N_\theta N_z N_y N_x$
37	8	$N_\theta N_z N_y N_x$	$8N_\theta N_z N_y N_x$
38	1	$N_\theta N_z N_y N_x$	$N_\theta N_z N_y N_x$

The heavy lifting is in the column backprojection

$N_\theta N_z N_y N_x$   
 $2N_\theta N_z N_y N_x$   
 $N_\theta N_z N_y N_x$   
 $8N_\theta N_z N_y N_x$   
 $N_\theta N_z N_y N_x$



To reconstruct an  $N \times N \times N$  volume,

$$N_{\theta} = N\pi/2$$

$$FLOPS = N^4 6\pi + \delta$$

Thus,

$$\underline{512^3 \rightarrow 1300 \times 10^9 \text{ FLOPS}}$$

FLOPS - Floating Point Operations Per Second

3 GHz Xeon processors are rated by Intel at 50 GFLOPS.

However, for reconstruction problems speed is often limited by memory i/o rates.

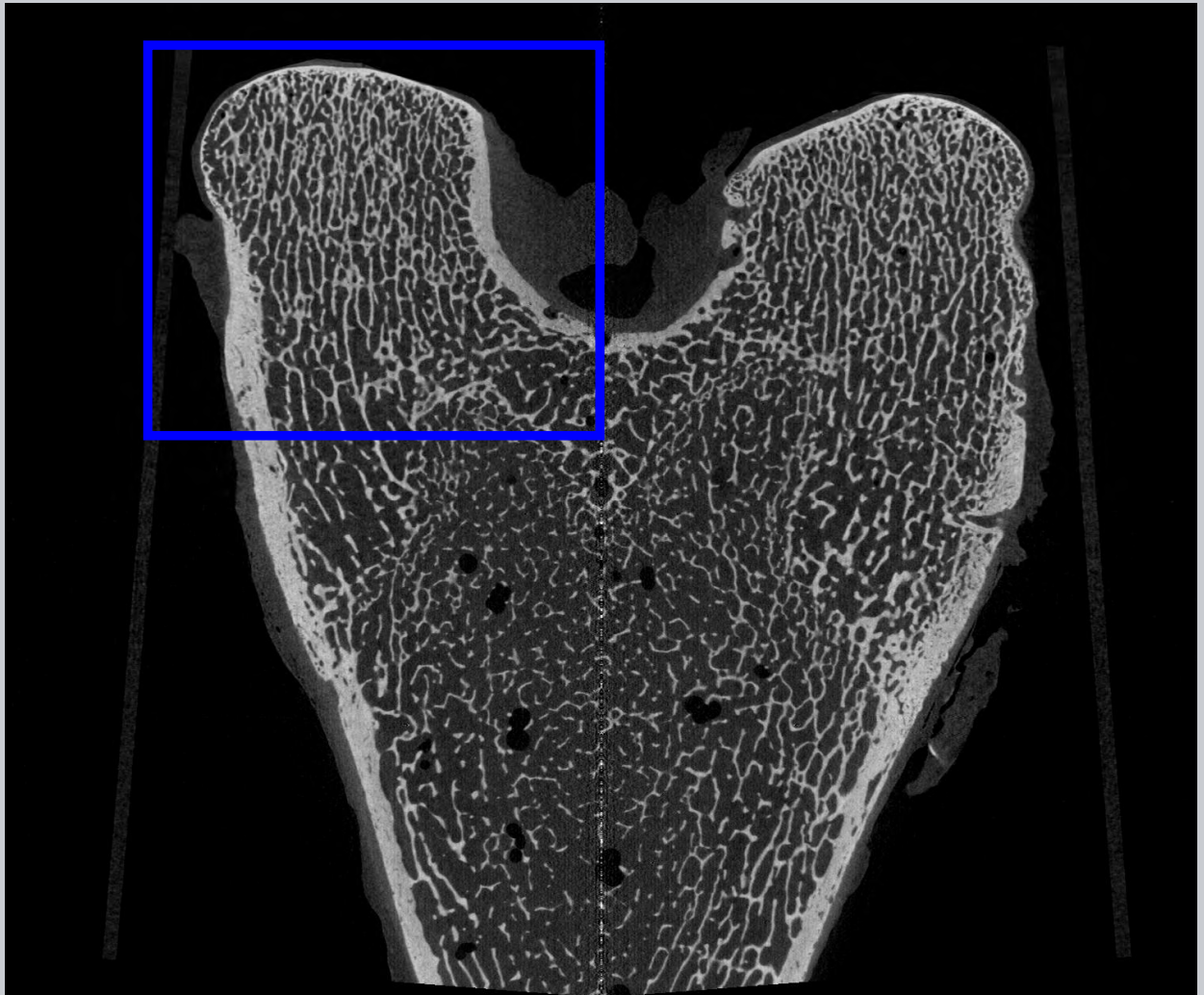


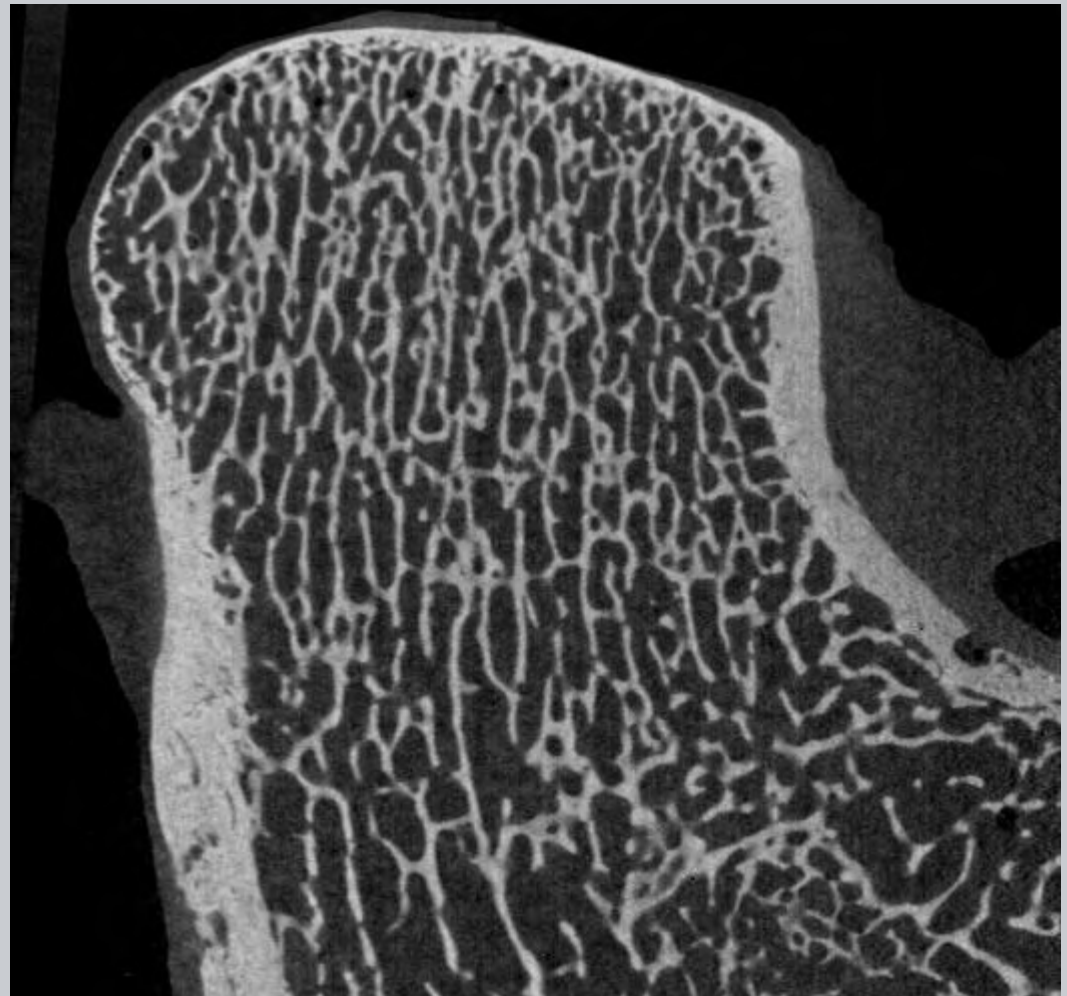
$N$	64	128	256	512	1024
Trad.	$1 \cdot 10^8$	$2 \cdot 10^9$	$3 \cdot 10^{10}$	$5 \cdot 10^{11}$	$9 \cdot 10^{12}$
Fast	$2 \cdot 10^8$	$2 \cdot 10^9$	$2 \cdot 10^{10}$	$2 \cdot 10^{11}$	$1 \cdot 10^{12}$

**Table 3.4** *Number of FLOPs on projection data in traditional and fast backprojection as a function of  $N = N_x = N_y = 2N_z = N_t = N_q = N_\theta/2$ . The estimate for fast backprojection when  $N = 1024$  is extrapolated.*

- Henrik Turbell, *Cone-Beam Reconstruction Using Filtered Backprojection*, PhD Dissertation no. 672, Linköping University, Sweden, February, 2001
- <http://www.cvl.isy.liu.se/ScOut/Theses/>







- 1536 x 1920 Acq.  
PaxScan 2520
- Cubic spline resample  
Thevenaz, 'interpol'
- Compress proj data  
JPEG2000 8-1  
Kakadu
- Filtered Backproj.  
MPI cluster (6/12)
- DICOM VCT data  
JPEG2000 8-1  
DCMTK (OFFIS)



### B) CT Reconstruction

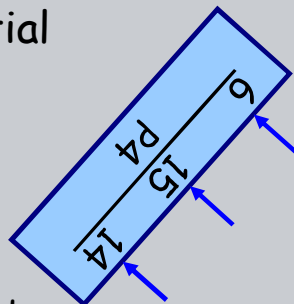
- 1) Projection geometry
- 2) Fourier Domain Solution
- 3) Convolution / Backprojection
- 4) Cone beam reconstruction
- 5) Iterative Reconstruction (9 slides)

In general, iterative reconstructions make an initial guess as to the tomographic solution, then

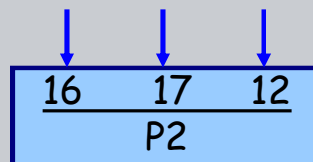
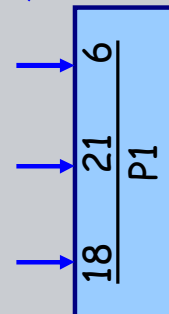
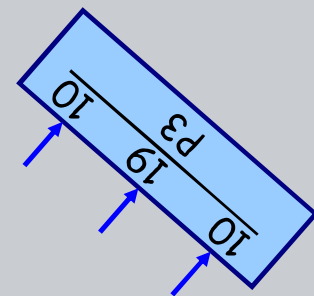
- reproject in a particular direction
- examine the difference between the reprojected estimate and an actual measurement.
- distribute the difference back to the solution estimate

Consider the simple example from the Webb reading assignment where a 3 x 3 tomograph is considered.

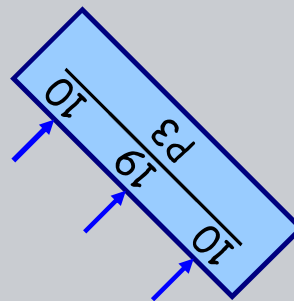
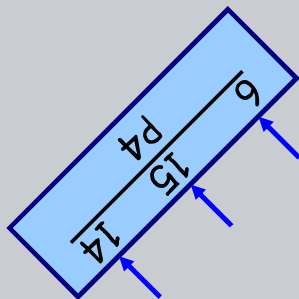
- The actual object has values from 1 to 9.
- 4 projection measurements are made.



1	2	3
8	9	4
7	6	5

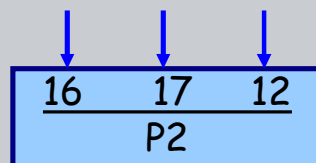
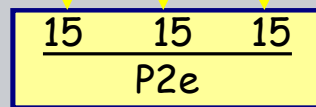


# VII.B.5 - Iterative Reconstruction

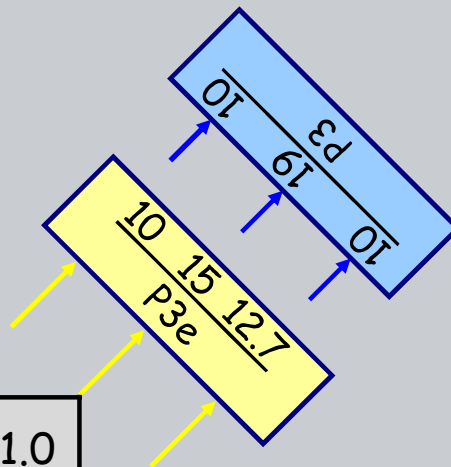
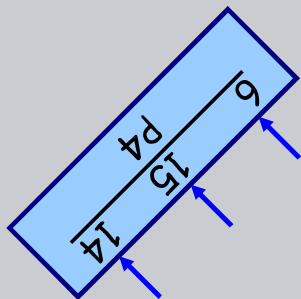


- We now start by distributing the P1 values horizontally across a 3 x 3 solution matrix.
- These are then reprojected in the P2 direction

2	2	2
7	7	7
6	6	6



# VII.B.5 - Iterative Reconstruction



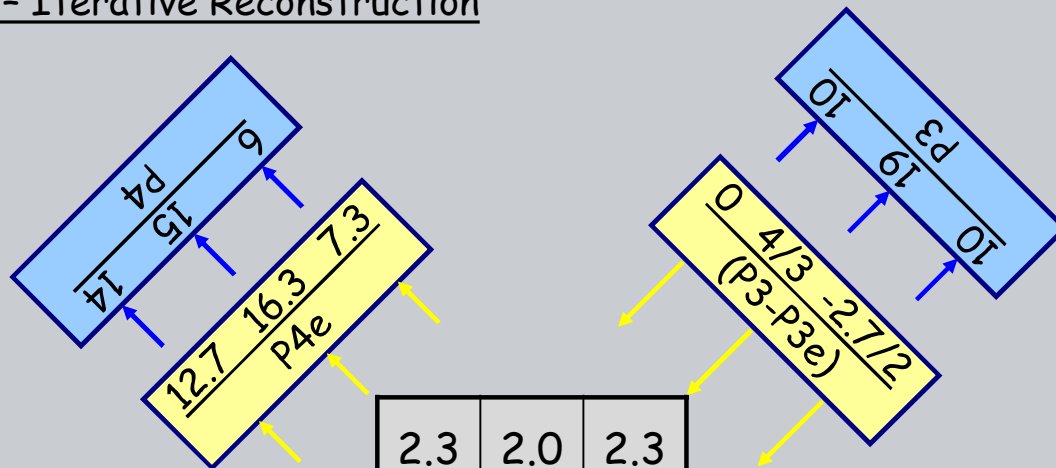
- We then take the difference between the original projection,  $P_2$ , and the estimate,  $P_{2e}$ , and backproject it to get a second estimate.
- The is then reprojected in the  $P_3$  direction.

2.3	2.7	1.0
7.3	7.7	6.0
6.3	6.7	5.0

$$\frac{1/3 \quad 2/3 \quad -3/3}{(P_2 - P_{2e})}$$

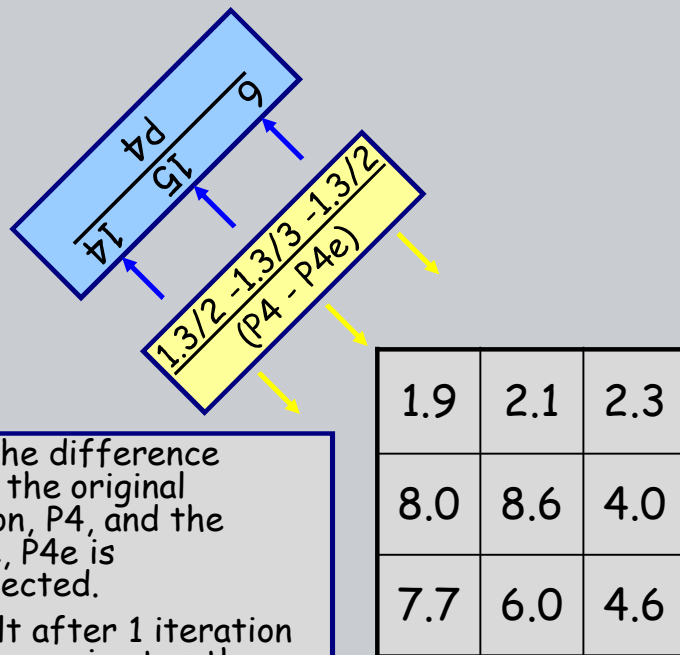
16	17	12
P <sub>2</sub>		

# VII.B.5 - Iterative Reconstruction



- Similarly, the difference between the original projection,  $P_3$ , and the estimate,  $P_{3e}$  is backprojected, and
- the result is reprojected in the  $P_4$  direction.

2.3	2.0	2.3
7.3	9.0	4.7
7.7	5.3	5.0



- Finally, the difference between the original projection,  $P_4$ , and the estimate,  $P_{4e}$  is backprojected.
- the result after 1 iteration closely approximates the original values.

Original

1	2	3
8	9	4
7	6	5



## VII.B.5 - Maximum Likelihood CT reconstruction

- Maximum Likelihood (ML) reconstruction methods offers the possibility to include the Poisson statistics of the photons in the reconstruction. Since the projections,  $i$ , are independent, the log-likelihood,  $L$ , can be written as,

$$L = \sum_i \left( -d_i e^{-\sum_j A_{ij} \mu_j} - Y_i \sum_j A_{ij} \mu_j \right) + c_1$$

Equation 5  
Ziegler 2007

- where

- $d_i$  is the expected number of photons leaving the source along the  $i$ th projection,
  - $Y_i$  are the observed photon counts along projection  $i$ ,
  - $\mu_j$  is the absorption coefficient of the  $j$ th supporting grid point,
  - $A_{ij}$  are the elements of the system matrix, and  $c_1$  is an irrelevant constant
- An approximate solution of maximizing  $L$  leads to an iterative step  $n$  to  $n+1$  of,

$$\mu_j^{n+1} = \mu_j^n + \mu_j^n \frac{\sum_i A_{ij} [d_i e^{-\langle A_i, \mu^n \rangle} - Y_i]}{\sum_i A_{ij} \langle A_i, \mu^n \rangle d_i e^{-\langle A_i, \mu^n \rangle}}$$

Equation 6  
Ziegler 2007

- Using an ordered subset method, Ziegler demonstrated that ML reconstruction can result in a signal to noise improvement of about 3 for equal resolution relative to filtered backprojection methods (FBP).



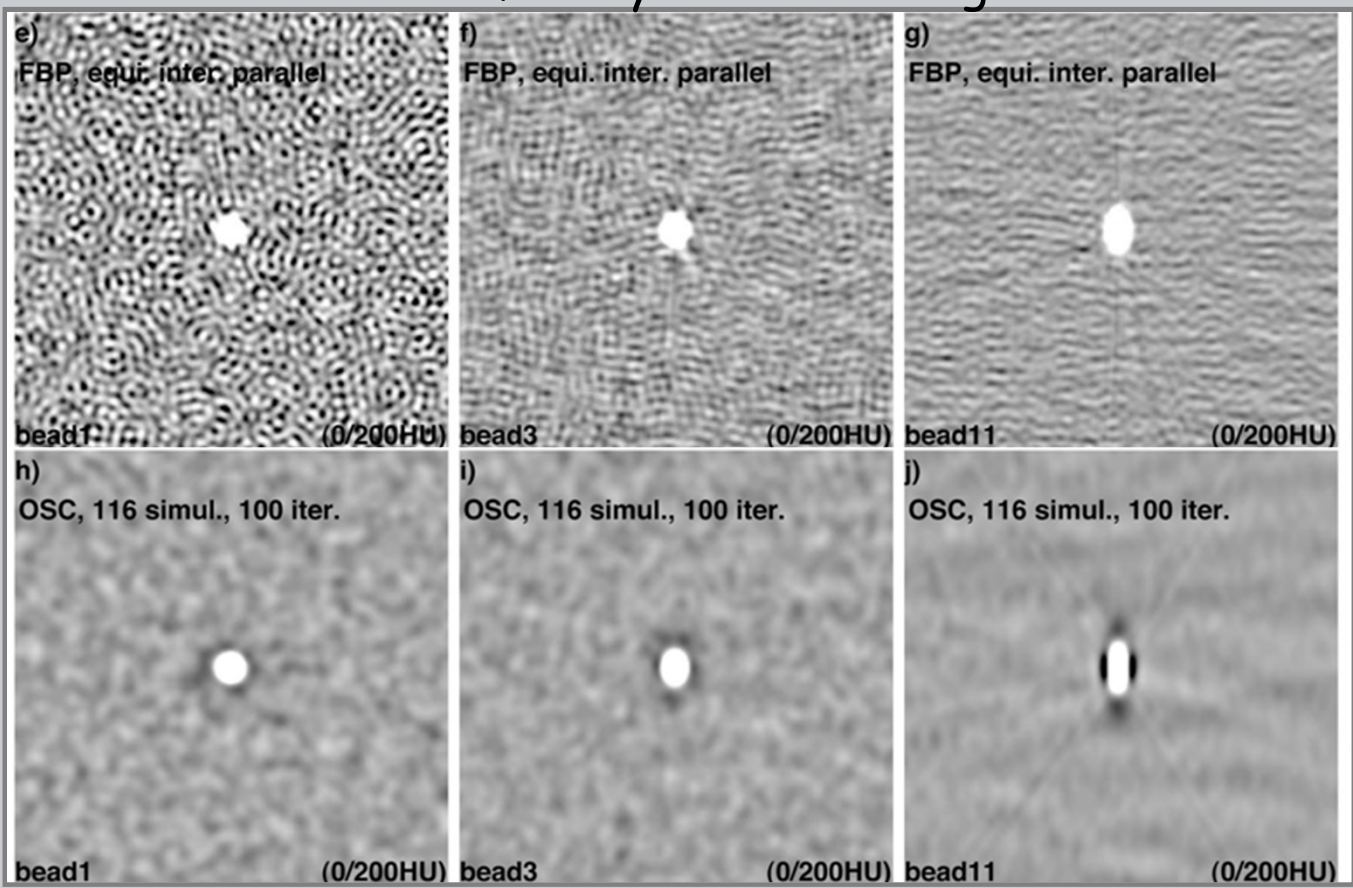
Center

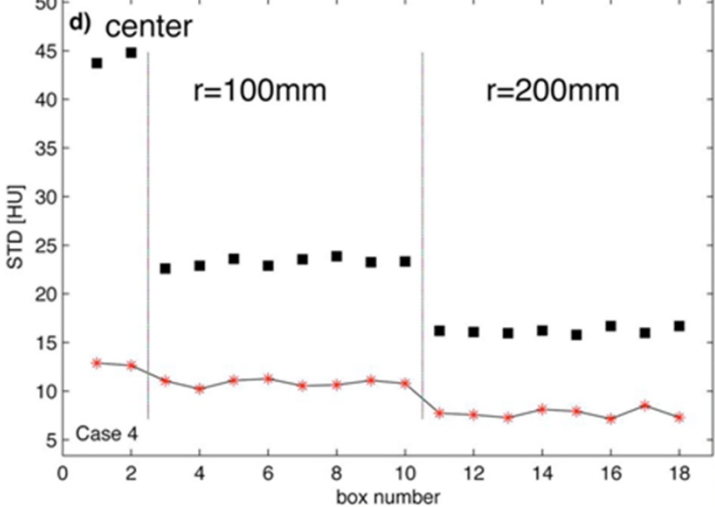
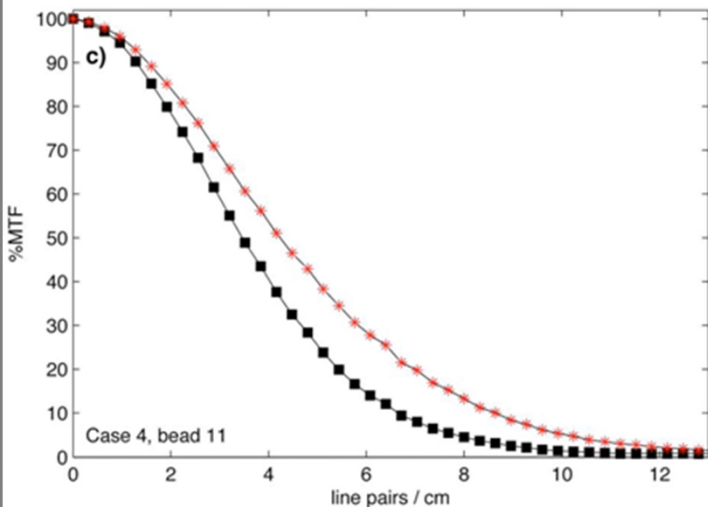
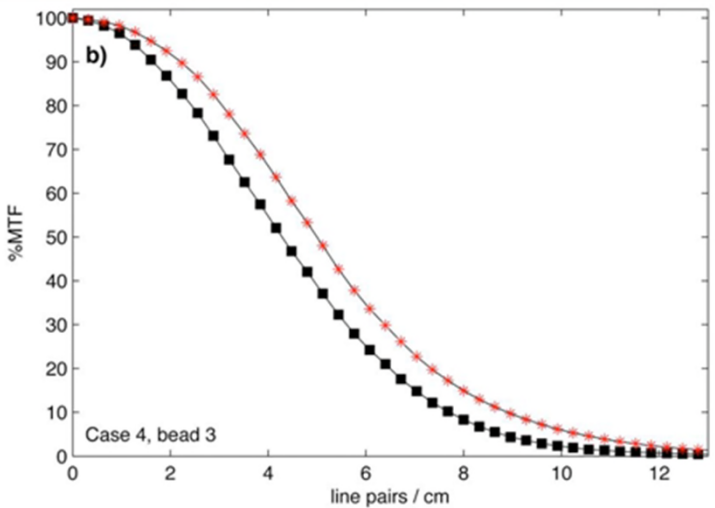
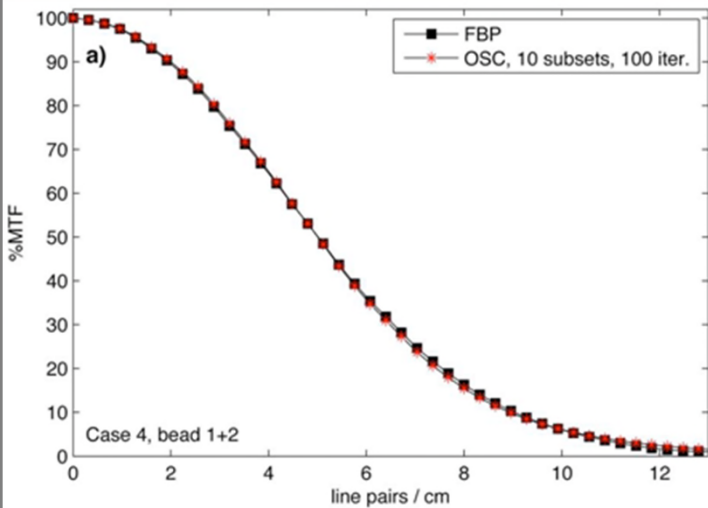
midway

edge

FPB =>

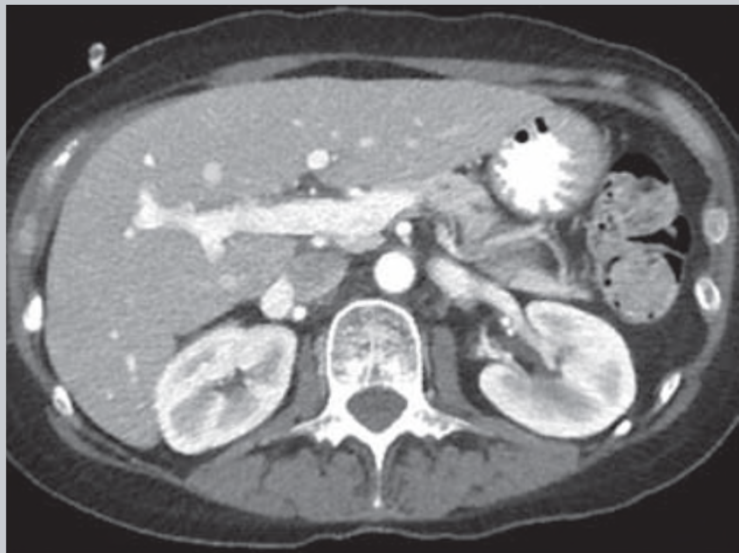
ITR =>  
(OSC)







Adaptive Statistical Iterative Reconstruction,  
GE Medical Systems.



Dose Index = 8

ASIR



Dose Index = 22

Traditional Reconstruction



### GE Medical Systems.

2008 - ASIR : Adaptive Statistical Iterative Reconstruction

2010 - VEO : Model based (computationally intensive)

2013 - ASIR-V : Hybrid ASIR-VEO

### Philips Medical Systems.

2010 - iDose4 : Adaptive Statistical Iterative Reconstruction.

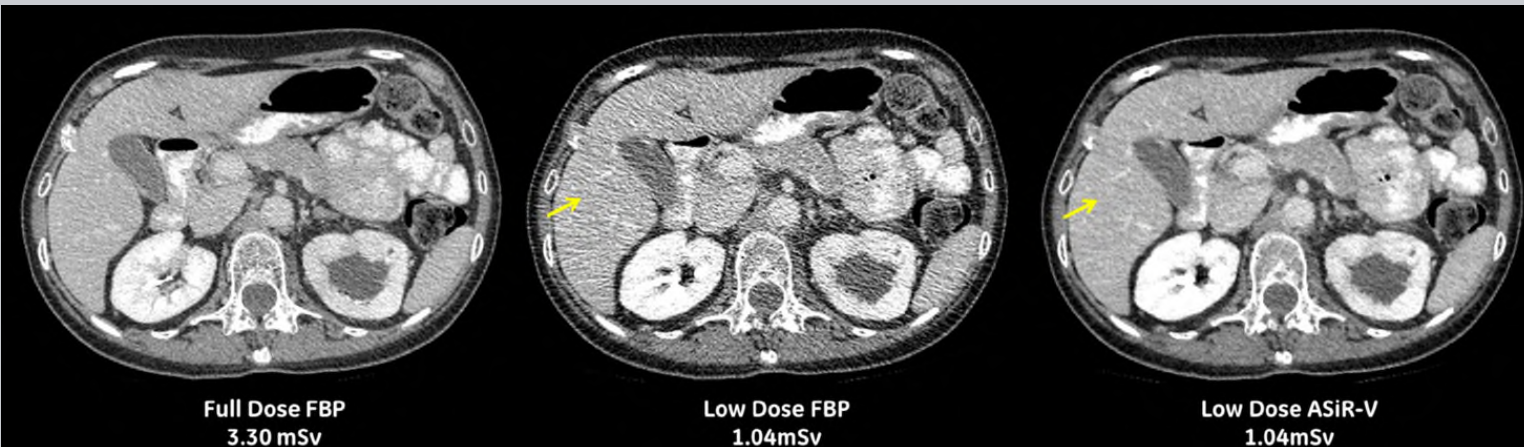
2013 - IMR : Model based reconstruction.

### Siemens Medical Systems.

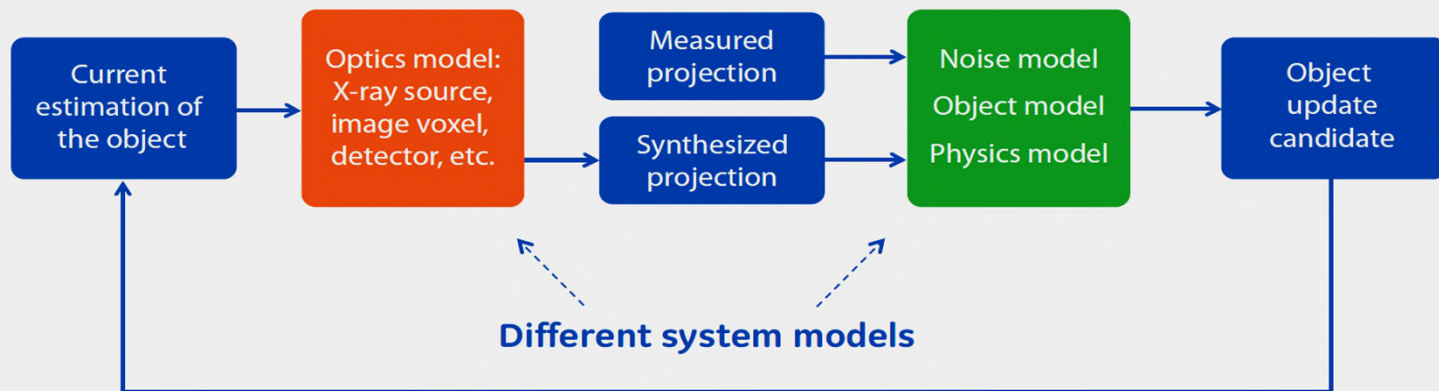
2008 - IRIS : Iterative reconstruction in image space.

2010 - SAFIRE : Sinogram affirmed iterative reconstruction

2015 - ADMIRE : Advanced modeled iterative reconstruction.

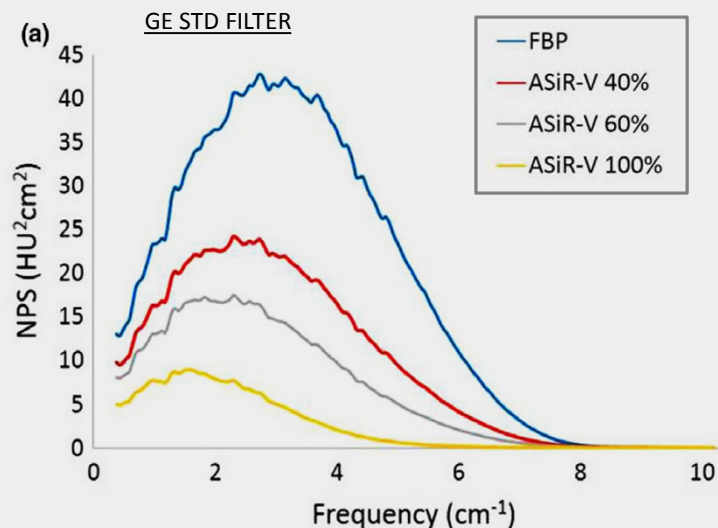


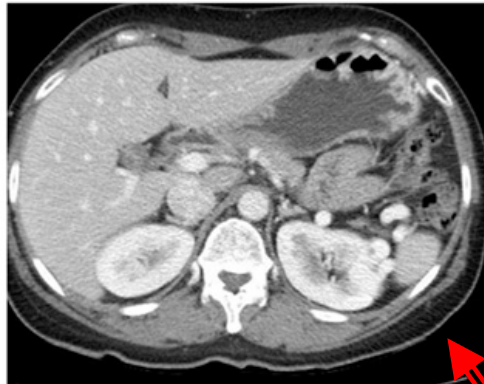
## VII.B.5 - Model based IR



### Model based IR:

- Rather than treating all measurements with equal weighting, a statistical model allows differing degrees of credibility among data.
- Three dimensional models describe the data acquisition process (source, gantry geometry, active detector) including the radiation interactions in a 3D model of the subject.
- Most IR methods specify a parameter that influences the amount of noise reduction.





8.4 mGy ASIR 40%



4.7 mGy FBP



4.7 mGy ASIR 40%



4.7 mGy ASIR-V 30%

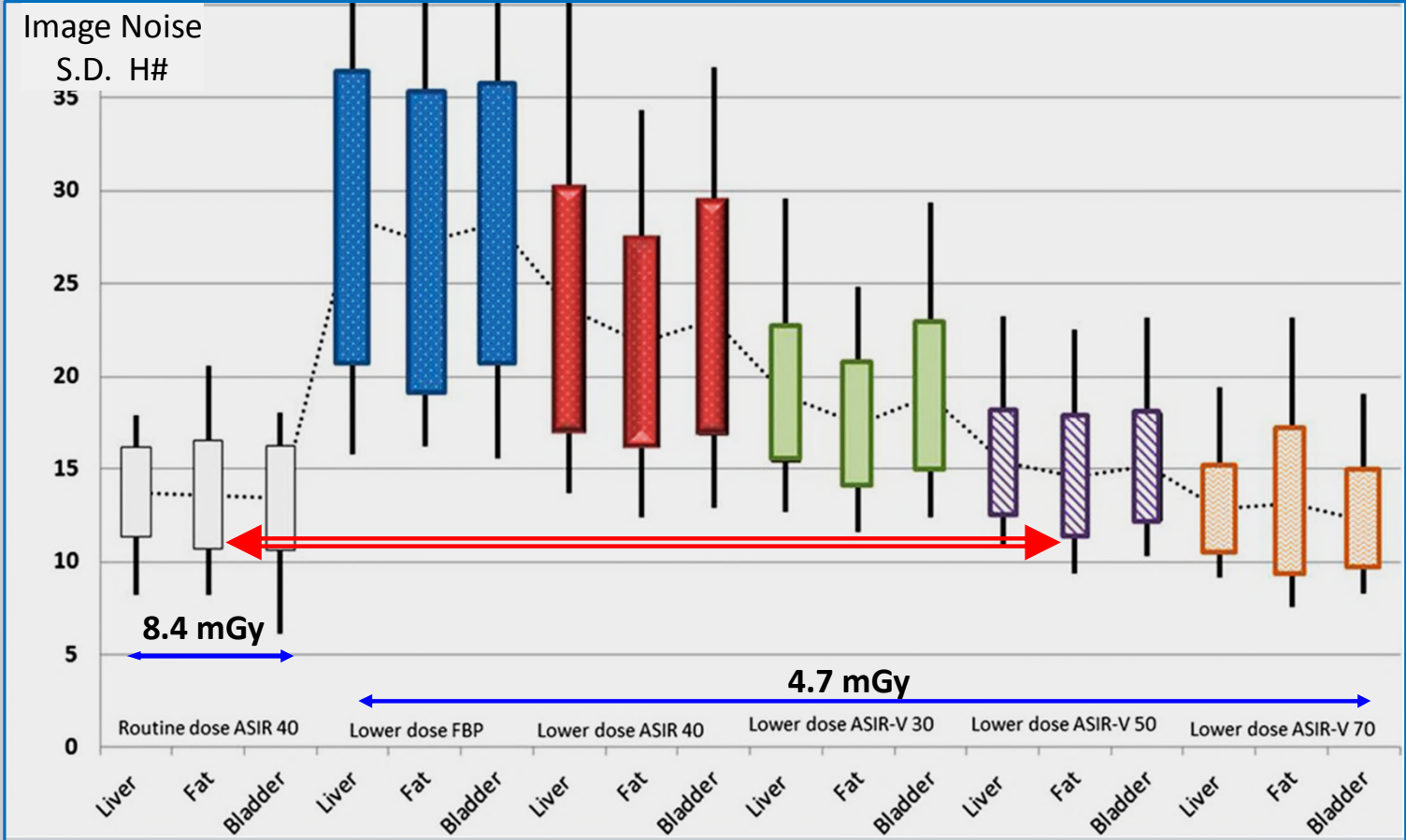


4.7 mGy ASIR-V 50%



4.7 mGy ASIR-V 70%

The authors conclude that the 4.7 mGy ASIR-V 50% images are nearly identical in image noise, sharpness and diagnostic acceptability to the 8.4 mGy ASIR 40%



The authors conclude that the 4.7 mGy ASIR-V 50% images are nearly identical in image noise, sharpness and diagnostic acceptability to the 8.4 mGy ASIR 40%



