Absorption and noise in x-ray phosphors

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When the signal in an x-ray image system is formed by integrating the scintillation pulses rather than by counting them, the signal-to-noise ratio is reduced by a factor which depends on the shape of the pulse-height distribution. The signal-to-noise ratio cannot be related directly to either quantum absorption or energy absorption, and a new quantity called noise-equivalent absorption is defined which bears a simple relationship to the signal-to-noise ratio. Quantum, energy, and noise-equivalent absorption are calculated as a function of thickness and x-ray energy for CsI, Gd_2O_2S, LaOBr, $Zn_{0.6}Cd_{0.4}S$, and $CaWO_4$.

INTRODUCTION

X-ray absorption is a parameter of primary importance for a radiographic or fluoroscopic phosphor: it directly determines both the relative brightness and the magnitude of the signal fluctuations referred to as "quantum mottle", or "scintillation noise".

In a scintillation counter the number of counts recorded in a given time follows Poisson's law and is characterized by the well-known relationship

$$\sigma = (NA_Q)^{1/2}. \tag{1}$$

Here σ, the standard deviation (or rms fluctuation) in the number of counts, may be taken as a measure of the scintillation noise; N is the average number of photons incident on the counter; and A_Q is the quantum absorption of the detector. Since the average signal is NA_Q, the signal-to-noise ratio is

$$\text{signal/noise} = (NA_Q)^{1/2}. \tag{2}$$

In an x-ray imaging system utilizing a scintillation phosphor, the detected x-ray pulses are usually integrated rather than counted, and the pulses in general are not of uniform size but are distributed according to some probability distribution. Three major factors contribute to the form of this distribution: (i) the incident x-ray energy distribution (XED); (ii) the absorbed energy distribution (AED) which results from variable absorption processes; and (iii) the optical pulse distribution (OPD) which results from unequal light propagation from different parts of the phosphor to the photodetector. As a result, the signal-to-noise relationship is changed to[1]

$$\text{signal/noise} = (NA_Q I)^{1/2}, \tag{3}$$

where

$$I = M_1^2/M_2M_0. \tag{4}$$

Here M_0, M_1, and M_2 are the respective moments of the scintillation pulse-height distribution. Although Eq. (3) is based on a pulse integration over a fixed time period, the use of a rate-measuring observational system introduces only a constant of proportionality.

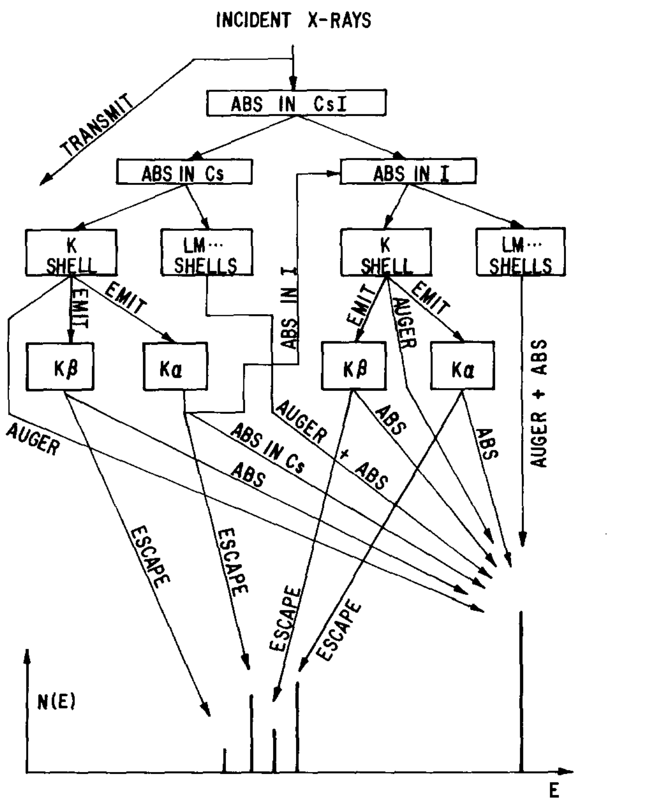

FIG. 1. Schematic diagram showing x-ray absorption events in CsI and the resulting absorbed energy spectrum for monochromatic x rays, within the approximations used in this paper.

TABLE I. CsI (density = 4.51 g/cm³).

Phosphor thickness [g/cm²(μ)]	Primary energy (keV)	Percent absorption A_Q quantum	A_N noise equiv.	A_E energy
0.02 (44.3)	20	39.2	39.2	39.2
	30	14.8	14.8	14.8
	I-K 33.17	11.4	11.4	11.4
		34.0	21.4	19.6
	Cs-K 35.98	28.3	19.6	17.3
		45.1	27.1	23.3
	40	36.2	25.0	20.5
	50	21.9	18.0	14.3
	60	13.9	12.3	9.87
	80	6.54	6.18	5.12
	100	3.58	3.47	2.96
0.05 (111)	20	71.2	71.2	71.2
	30	33.0	33.0	33.0
	I-K 33.17	26.2	26.2	26.2
		64.6	46.0	43.5
	Cs-K 35.98	56.5	42.9	39.5
		77.6	53.3	48.5
	40	67.6	50.9	44.8
	50	46.1	39.3	33.7
	60	31.2	28.2	24.2
	80	15.5	14.8	12.9
	100	8.72	8.46	7.55

TABLE II. CsI.

Phosphor thickness [g/cm²(μ)]	Primary energy (keV)	Percent absorption A_Q quantum	A_N noise equiv.	A_E energy
0.1 (222)	20	91.7	91.7	91.7
	30	55.2	55.2	55.2
	I-*K* 33.17	45.5	45.5	45.5
		87.4	68.9	66.4
	Cs-*K* 35.98	81.1	66.6	63.2
		95.0	72.6	68.3
	40	89.5	72.9	67.2
	50	70.9	63.0	57.0
	60	52.7	48.8	44.1
	80	28.7	27.6	25.2
	100	16.7	16.3	15.1
0.2 (443)	20	99.3	99.3	99.3
	30	79.9	79.9	79.9
	33.17	70.3	70.3	70.3
		98.4	83.7	81.8
	35.98	96.4	84.6	81.9
		99.8	82.0	78.7
	40	98.9	85.6	81.2
	50	91.5	84.5	79.6
	60	77.6	73.7	69.5
	80	49.2	47.8	45.4
	100	30.6	30.1	28.7

Use of Eqs. (3) and (4) implies knowledge of the overall pulse spectrum. It may be desirable to predict the performance of a complete system, based on knowledge of the individual three processes referred to above. For example, knowing the AED for various discrete x-ray energies, how does one calculate I for a given XED? The answer is that one must determine the moments of the AED as a function of incident x-ray energy, average these over the XED, and then compute I from Eq. (4) using the average moments.[1] It is incorrect to average I over the XED directly. One can extend this process to include all three subsystems. However, a special property of the OPD makes possible a great simplification. This is true because the OPD distorts a line distribution into a spectrum having the same shape for each line energy, merely stretching proportionally along the energy axis as the line energy changes. In this special case one has the following simple relation[1]:

$$I_{\mathrm{OPD+AED}} = I_{\mathrm{OPD}} I_{\mathrm{AED}}. \quad (5)$$

Because of the simplification afforded by Eq. (5), it makes sense to calculate and publish separate data on the AED and OPD of phosphors. The AED is determined by the atomic constituency and thickness of the phosphor. Its calculation for a group of useful phosphors is the subject of this paper. The OPD is determined by the optical properties of the phosphor. Its calculation is included in another report[2] which also includes optical properties not relevant to the present subject.

TABLE III. Gd_2O_2S (density = 7.34 g/cm³).

Phosphor thickness [g/cm²(μ)]	Primary energy (keV)	Percent absorption A_Q quantum	A_N noise equiv.	A_E energy
0.02 (26.9)	20	50.1	50.1	50.1
	30	20.7	20.7	20.7
	40	9.82	9.82	9.82
	50	5.38	5.38	5.38
	Gd-*K* 50.24	5.31	5.31	5.31
		26.1	13.2	11.0
	60	17.1	11.7	8.82
	80	8.26	7.07	5.26
	100	4.61	4.24	3.27
0.05 (67.3)	20	82.4	82.4	82.4
	30	44.0	44.0	44.0
	40	22.8	22.8	22.8
	50	12.9	12.9	12.9
	Gd-*K* 50.24	12.8	12.8	12.8
		53.1	30.2	26.6
	60	37.4	26.9	21.8
	80	19.4	16.7	13.3
	100	11.1	10.2	8.34

TABLE IV. Gd_2O_2S.

Phosphor thickness [g/cm²(μ)]	Primary energy (keV)	Percent absorption A_Q quantum	A_N noise equiv.	A_E energy
0.1 (135)	20	96.9	96.9	96.9
	30	68.6	68.6	68.6
	40	40.4	40.4	40.4
	50	24.2	24.2	24.2
	Gd-*K* 50.24	23.9	23.9	23.9
		78.0	50.1	45.9
	60	60.9	46.3	39.9
	80	35.0	30.8	26.0
	100	21.0	19.5	16.7
0.2 (269)	20	99.9	99.9	99.9
	30	90.1	90.1	90.1
	40	64.4	64.4	64.4
	50	42.5	42.5	42.5
	Gd-*K* 50.24	42.1	42.1	42.1
		95.2	69.1	65.5
	60	84.7	69.2	63.0
	80	57.8	52.3	46.8
	100	37.6	35.4	31.9

CALCULATION OF ABSORPTION DATA

A complete calculation of the AED would be very difficult and is not justified at this time by the accuracy of available x-ray coefficients, which is about 10% in the region of radiological interest. What are reported

TABLE V. LaOBr (density = 6.3 g/cm³).

Phosphor thickness [g/cm²(μ)]	Primary energy (keV)	Percent absorption A_Q quantum	A_N noise equiv.	A_E energy
0.02 (31.7)	20	50.3	49.0	48.0
	30	20.1	19.9	19.5
	La-*K* 38.92	10.3	10.2	10.0
		31.4	19.4	17.3
	40	29.6	18.9	16.6
	50	17.4	13.6	11.3
	60	10.9	9.42	7.71
	80	5.12	4.77	3.98
	100	2.81	2.69	2.31
0.05 (79.4)	20	82.6	81.5	80.8
	30	43.0	42.8	42.4
	La-*K* 38.92	23.8	23.7	23.5
		61.1	42.5	39.6
	40	58.4	41.6	38.3
	50	37.9	31.3	27.5
	60	25.1	22.2	19.3
	80	12.3	11.6	10.2
	100	6.88	6.63	5.92

TABLE VI. LaOBr.

Phosphor thickness [g/cm²(μ)]	Primary energy (keV)	Percent absorption		
		A_Q quantum	A_N noise equiv.	A_E energy
0.1 (159)	20	97.0	96.1	95.5
	30	67.5	67.3	67.0
	La-*K* 38.92	41.9	41.8	41.7
		84.9	65.5	62.6
	40	82.7	64.8	61.6
	50	61.5	53.3	49.0
	60	43.9	40.0	36.5
	80	23.1	22.0	20.2
	100	13.3	12.9	11.9
0.2 (317)	20	99.9	99.1	98.5
	30	89.4	89.3	89.0
	La-*K* 38.92	66.2	66.2	66.0
		97.7	82.0	79.7
	40	97.0	82.4	79.7
	50	85.2	77.5	73.8
	60	68.5	64.4	61.0
	80	40.9	39.5	37.5
	100	24.8	24.3	23.2

here are results of a calculation based on a simplified model which yields an accuracy of about 5% assuming exact coefficients.

In the present method, absorption is calculated assuming that photoelectric absorption is the only significant interaction. Neglect of the energy deposited by scattering is small for the phosphors and energies considered. The generation of *K* fluorescence and its escape from the phosphor is treated in detail. Higher radiations are assumed to be reabsorbed. Only atoms with $z > 28$ are considered to be fluorescent. When two high-z atoms are present, reabsorption of the higher-z fluorescence by the *K* shell of the lower-z atoms is considered, but a representative escape depth is used for the tertiary radiation. A diagram of the calculation for CsI, together with a schematic AED, is shown in Fig. 1.

ABSORPTION PARAMETERS

Instead of tabulating the moments of the AED, absorption parameters with more physical meaning have been used. If we normalize the AED to one incident photon, then the relationship between the absorption parameters and the moments of the AED are as follows:

$$A_Q = M_0,$$

$$A_E = M_1/E, \tag{6}$$

$$A_N = M_1^2/M_2. \tag{7}$$

TABLE VII. $Zn_{0.6}Cd_{0.4}S$ (density = 4.43 g/cm³).

Phosphor thickness [g/cm²(μ)]	Primary energy (keV)	Percent absorption		
		A_Q quantum	A_N noise equiv.	A_E energy
0.02 (45.1)	20	33.5	33.2	32.8
	Cd-*K* 26.71	16.5	16.4	16.2
		40.3	30.0	27.4
	30	31.3	25.1	22.4
	40	15.6	13.9	12.2
	50	8.72	8.16	7.18
	60	5.30	5.07	4.52
	80	2.38	2.33	2.11
	100	1.27	1.25	1.16
0.05 (113)	20	64.0	63.7	63.3
	Cd-*K* 26.71	36.4	36.3	36.1
		72.5	59.5	56.6
	30	60.9	52.4	49.0
	40	34.5	31.9	29.4
	50	20.4	19.4	17.9
	60	12.7	12.3	11.4
	80	5.84	5.74	5.40
	100	3.15	3.11	2.95

TABLE VIII. $Zn_{0.6}Cd_{0.4}S$.

Phosphor thickness [g/cm²(μ)]	Primary energy (keV)	Percent absorption		
		A_Q quantum	A_N noise equiv.	A_E energy
0.1 (226)	20	87.0	86.8	86.5
	Cd-*K* 26.71	59.5	59.4	59.3
		92.4	81.1	78.7
	30	84.7	76.7	73.8
	40	57.1	54.2	51.6
	50	36.6	35.4	33.8
	60	23.8	23.3	22.3
	80	11.3	11.2	10.8
	100	6.19	6.15	5.95
0.2 (451)	20	98.3	98.1	97.9
	Cd-*K* 26.71	83.6	83.5	83.4
		99.4	90.5	88.6
	30	97.7	91.4	89.2
	40	81.6	79.1	77.0
	50	59.8	58.7	57.2
	60	42.0	41.4	40.4
	80	21.4	21.2	20.8
	100	12.0	11.9	11.7

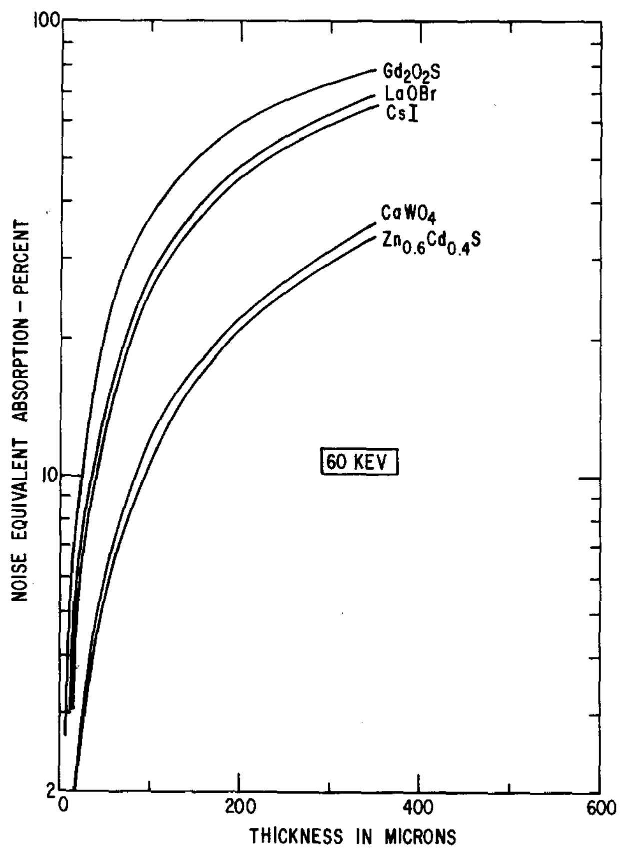

FIG. 2. Noise-equivalent absorption for five phosphors vs thickness at full density and an x-ray energy of 60 keV.

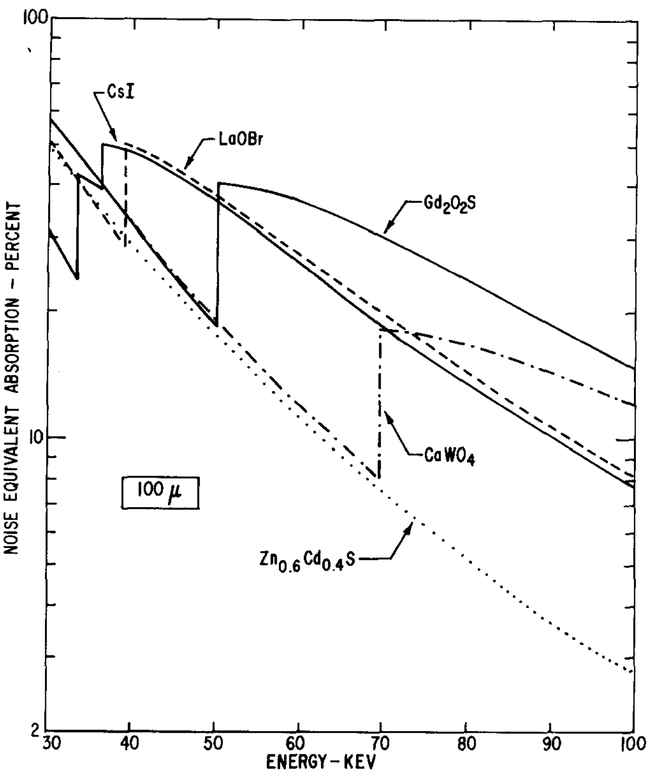

FIG. 3. Noise-equivalent absorption for five phosphors vs x-ray energy at a thickness of 100 μ (full density).

A_Q, the quantum absorption, is the number of detected photons per incident photon; A_E is the fractional energy absorption; and A_N is the quantity which should be used in Eq. (2) in the general case described here. Hence, it is referred to as the "noise-equivalent" absorption. E is the incident photon energy.

Absorption results for five phosphors at four mass thicknesses each are given in Tables I–X. Figure 2 shows the noise-equivalent absorption vs thickness at 60 keV, while Fig. 3 compares noise-equivalent absorption spectra at a thickness of 100 μ.

All the x-ray data used in these calculations were taken from Ref. 3.

APPENDIX

To derive Eq. (3) let the average number of pulses received in a given time be N, and consider for convenience that they can be grouped into a discrete number of size groups. The contribution of the kth subgroup to the signal is

$$S_k = N_k E_k \tag{8}$$

and the standard deviation of that signal is

$$\sigma_k = (N_k)^{1/2} E_k. \tag{9}$$

To obtain the total signal we sum the S_k and for the total noise we take the square root of the sums of squares of the σ_k. Hence,

$$\text{signal/noise} = \sum_k N_k E_k / (\sum_k N_k E_k^2)^{1/2} = M_1/(M_2)^{1/2}, \tag{10}$$

where M_j is the jth moment of the pulse size distribution Since $M_0 = N$ we can write

$$\text{signal/noise} = (NI)^{1/2}, \tag{11}$$

where

$$I = M_1^2 / M_2 M_0. \tag{12}$$

For the detector situation envisioned above, N is replaced by NA_Q to obtain Eq. (3).

To explain the method for obtaining the I factor for an x-ray spectrum, we must express the absorbed energy spectrum by the convolution integral

$$N(E) = \int_0^\infty F(E')G(E,E')\,dE', \tag{13}$$

where the kernel $G(E,E')$ gives the probability that a photon of energy E' will produce a pulse of magnitude E and $F(E')$ is the x-ray photon spectrum. If we take the jth moment of $N(E)$ and designate this as M_j we have

TABLE IX. $CaWO_4$ (density = 6.08 g/cm³).

Phosphor thickness [g/cm²(μ)]	Primary energy (keV)	Percent absorption A_Q quantum	A_N noise equiv.	A_E energy
0.02 (32.9)	20	57.1	57.1	57.1
	30	24.5	24.5	24.5
	40	12.1	12.1	12.1
	50	6.71	6.71	6.71
	60	4.07	4.07	4.07
	W-K 69.52	2.71	2.71	2.71
		12.9	6.18	4.96
	80	9.09	5.81	4.22
	100	5.11	4.13	2.92
0.05 (82.2)	20	87.9	87.9	87.9
	30	50.4	50.4	50.4
	40	27.5	27.5	27.5
	50	15.9	15.9	15.9
	60	9.87	9.87	9.87
	W-K 69.52	6.64	6.64	6.64
		29.3	15.1	12.7
	80	21.2	14.0	10.8
	100	12.3	9.98	7.45

TABLE X. $CaWO_4$.

Phosphor thickness [g/cm²(μ)]	Primary energy (keV)	Percent absorption A_Q quantum	A_N noise equiv.	A_E energy
0.1 (164)	20	98.5	98.5	98.5
	30	75.4	75.4	75.4
	40	47.4	47.4	47.4
	50	29.3	29.3	29.3
	60	18.8	18.8	18.8
	W-K 69.52	12.8	12.8	12.8
		50.0	28.4	24.9
	80	37.9	26.1	21.4
	100	23.1	19.0	15.0
0.2 (329)	20	99.98	99.98	99.98
	30	94.0	94.0	94.0
	40	72.3	72.3	72.3
	50	50.1	50.1	50.1
	60	34.0	34.0	34.0
	W-K 69.52	24.0	24.0	24.0
		75.0	48.1	44.0
	80	61.4	45.4	39.4
	100	40.8	34.5	29.1

$$M_j=\int_0^\infty E^j \int_0^\infty F(E')G(E,E')\,dE'\,dE, \quad (14)$$

$$M_j=\int_0^\infty F(E')G_j(E')\,dE', \quad (15)$$

where $G_j(E')$ is the jth moment of $G(E,E')$ in the variable E. Equation (15) states that the jth moment of the pulse spectrum with a distribution of x-ray energies is obtained by averaging the corresponding "monochromatic moment", $G_j(E')$ over the x-ray spectrum, $F(E')$.

To derive Eq. (5), let $G(E,E')$ represent the probability that a pulse of *absorbed energy* E' will produce an optical pulse at the detector of magnitude E, and let $F(E')$ be the *absorbed energy* distribution. A convolution integral of the form (14) will then give the resultant pulse distribution.

Let

$$G(E,E')=\sum_{i=1}^{\infty} A_i\delta(E-\alpha_i E'), \quad (16)$$

where A_i and α_i are constants and $\delta(E-\alpha_i E')$ is a normalized Dirac δ function. This series expansion (or its extension into an integral) can be used to construct any shape $G(E,E')$ for a given E', but when E' is changed the function will expand or contract linearly along the E axis and maintain constant relative shape and constant area. This is exactly the property normally possessed by the OPD. It is easily shown that the jth moment in E is given by

$$G_j(E')=K_j(E')^j, \quad (17)$$

where

$$K_j=\sum_{i=1}^{\infty} A_i(\alpha_i)^j. \quad (18)$$

As a consequence of Eq. (17), the I factor of the OPD

$$I_{\text{OPD}}=\frac{G_1^2}{G_0G_2}=\frac{K_1^2}{K_0K_2} \quad (19)$$

is independent of E'. This result shows that the I factor of a distribution is dependent only on the relative shape of the distribution, and not on any scale factors. Substituting (16) into (14) one obtains

$$M_j=K_jF_j, \quad (20)$$

where F_j is the jth moment of $F(E')$. Then since $I_{\text{OPD+AED}}=M_1^2/M_0M_2$, one obtains on substitution of (19) and (20), the result given in Eq. (5).

[1]See Appendix for a derivation of these results.
[2]R. K. Swank, Appl. Opt. (to be published).
[3]E. Storm and H. I. Israel, Los Alamos Scientific Report No. LA-3753, 1967 (unpublished).