A HYBRID MAXIMUM LIKELIHOOD POSITION COMPUTER FOR SCINTILLATION CAMERAS.

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Abstract

Maximum likelihood (ML) estimators offer advantages of improved spatial resolution and linearity over traditional position estimates in position sensitive detectors. We have constructed a two board, multibus based hybrid position computer capable of performing the ML estimate at SPECT countrates. In addition, the board can implement any estimate linear in the photomultiplier tube outputs.

Introduction

Techniques from communication theory have been applied extensively in recent years to the problem of scintillation event localization in position sensitive detectors [1–5]. Most efforts have focused on the development of maximum likelihood estimators which assume relatively simple forms under a set of reasonable restrictions. These estimators possess the asymptotic properties of being unbiased and efficient. We expect that applications of these estimators to practical systems would result in superior spatial linearity and resolution — at least in the limit of an arbitrarily large number of events.

Although the estimators are mathematically simple they require the maximization of the likelihood functional with respect to event position. In all but the simplest cases this leads to a set of nonlinear equations to solve for the estimate — a difficult task to perform in a system with the required countrate capability. Successful realtime implementations have been limited to discrimination among relatively few positions from a few photomultiplier tubes (PMT). Burnham, *et al.* use four crystals/PMT and a series of summing amplifiers with weighting coefficients realized as precision resistors [2]. Casey and Nutt use a similar system although their weights are based on a likelihood ratio scheme [3]. Milster, *et al.* have reported on the use of digital look up tables for ML estimator implementation with PMT outputs quantized to six bits [4]. Our detector system is a two dimensional array with 20 phototubes and about 2000 locations to distinguish. To implement the first approach would require 40000 precision resistors and to implement the look up table approach would require 2^{120} words of memory!

Since our detector system is designed for SPECT, we do not need the extreme countrate capability that PET demands and have settled on an approach that maximizes the log-likelihood functional by performing successive hybrid inner products of analog sums of PMT outputs and weighting coefficient vectors stored digitally in RAM. The position computer boards solve for the x and y components of event location separately and are implemented with inexpensive electronic components. The two board set forms an extremely flexible system capable of performing any position estimate linear in the PMT outputs.

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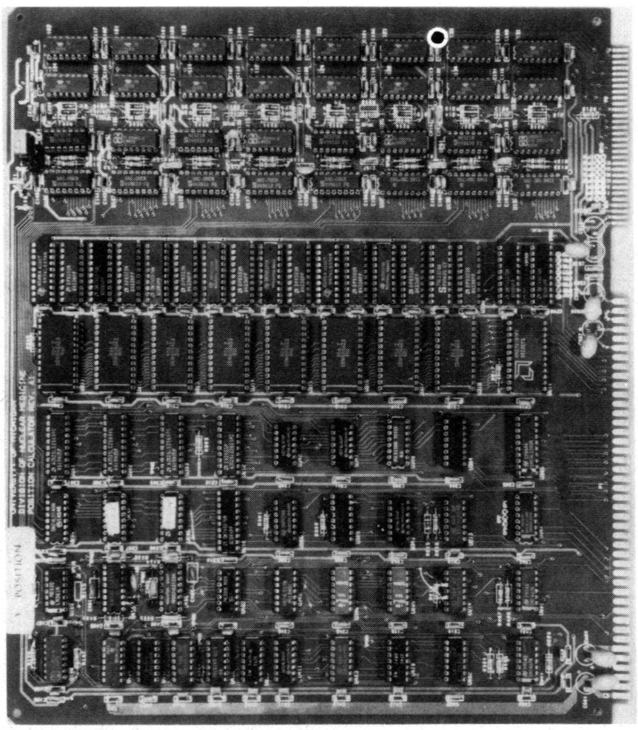

Figure 1. Multibus based hybrid position computer.

Theory

For a two-dimensional detector the position likelihood functional should be maximized with respect to the position vector, (x, y). Since an inexpensive hardware realization of this presently seems intractible, we have separated the vector estimation problem into two one-dimensional problems. We first form independent position sums over the rows or columns of the PMT array. If our observation vector, $\underline{M}$, consists of N independent position sums, the elements of $\underline{M}$ are conditionally independent given event position. Assuming the noise process in the sums follows a Poisson probability law we write the likelihood as

$$Pr\,[\underline{M}|x] = \prod_{i=1}^{N} \frac{S_i(x)^{M_i} e^{-S_i(x)}}{M_i!}, \tag{1}$$

where $S_i(x)$ represents the mean number of photons detected by the ith row of PMTs when the scintillation occurs at any point along the line, x. We see that these mean position sum responses are determined by the intrinsic spread of scintilla-

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tion light in the detector and by the size and shape of the PMT, determining how the light is integrated into signal.

Performing the maximization with respect to the log-likelihood, we obtain

$$\frac{\partial}{\partial x} ln\, Pr\,[\underline{M}|x] = \sum_{i=1}^{N} M_i \frac{\partial S_i(x)/\partial x}{S_i(x)} - \frac{\partial}{\partial x}\sum_{i=1}^{N} S_i(x) = 0\,. \quad (2)$$

Solving (2) for x yields the most likely position. The first term in the middle expression is seen to be an inner product of the observed position sums and weighting coefficients derived from the properties of the mean postion sum response functions, $S_i(x)$. The second term reflects changes in light collection or energy signal with respect to position. In position sensitive detectors having good light collection properties we would expect this term to be small.

By using the fact that $E[M_i|x] = S_i(x)$, we distribute the second term among the observed position sums, M_i, normalizing (2) with respect to γ-ray energy and yielding an equation linear in the position sums. The resulting approximation is

$$\frac{\partial}{\partial x} ln\, Pr\,[\underline{M}|x] \simeq \sum_{i=1}^{N} M_i w_i(x) = 0\,. \quad (3)$$

The resulting position sum weights, $w_i(x)$, are given by

$$w_i(x) = \left[\frac{\partial S_i(x)/\partial x}{S_i(x)} - \frac{\sum_{j=1}^{N} \partial S_j(x)/\partial x}{\sum_{k=1}^{N} S_k(x)}\right]. \quad (4)$$

We note that only the relative values of the mean responses are needed to calculate the weights and not the absolute number of photons collected.

Estimator performance is usually assessed by examining the mean and variance of the estimates which relate to spatial linearity and resolution in the present context. A lower bound on the variance of any estimator can be derived knowing only the joint probability distribution in equation (1). This is the Cramer-Rao bound and takes a particularly simple form for our problem, [3, 5].

$$\sigma_{lb}^2(x) = \left[\sum_{i=1}^{N} \frac{(\partial S_i(x)/\partial x)^2}{S_i(x)}\right]^{-1}. \quad (5)$$

An estimator achieving this lower bound is called *efficient*. Although our estimate is asymptotically efficient it does not attain the Cramer-Rao bound for finite number of events. The variance of our estimator is given approximately as

$$\sigma^2(x) \simeq \sum_{i=1}^{N} S_i(x) w_i^2(x) \left[\sum_{j=1}^{N} S_j(x) w_j(x)\right]^{-2}. \quad (6)$$

We note that in both the Cramer-Rao bound and our variance expression that other than the implicit assumption of Poisson statistics the shape of the PMT response functions with respect to source position is the most important factor in determining the estimator variance and ultimately the intrinsic spatial resolution of the detector system.

The Modular Detector

The position computer was designed for a modular detector system that will be closely packed into a ring geometry single photon emission tomograph. Figure 2 is a photograph of the modular detector, lightguide, and associated electronics. For high detection efficiency, the module must have usable resolution and linearity to the edge of the detector in the circumferential (x) direction. To accomplish this each detector module is comprised of 43 long bars of NaI(Tl) scintillator 3mm wide (x) x 150mm long (y) x 12.5mm thick. The bars are separated by reflective paper, optically coupled to a 5mm glass exit window, and packed into a two dimensional detector 140mm x 150mm in the x and y directions respectively. This configuration produces an intrinsically narrow spread of scintillation light in the x direction which can be broadened with a light guide to match PMT diameter for maximum circumferential resolution. The bars also tend to reduce uncontrolled light propagation near the module edge resulting in increased resolution and linearity at the edges. Detector requirements are relaxed somewhat in the axial (y) direction since edge effects can be reduced by making the detector longer than the necessary active length.

A closely packed hexagonal array of 20 Hamamatsu R980 38mm round photomultipliers is afixed to the detector/lightguide package. The gain of each PMT is computer controlled by adjusting the potential of the sixth dynode. Output pulses from the PMTs are integrated and shaped by 20 preamps then summed to form independent row and column position sums. This summation results in five position sums for the x direction and eight for y.

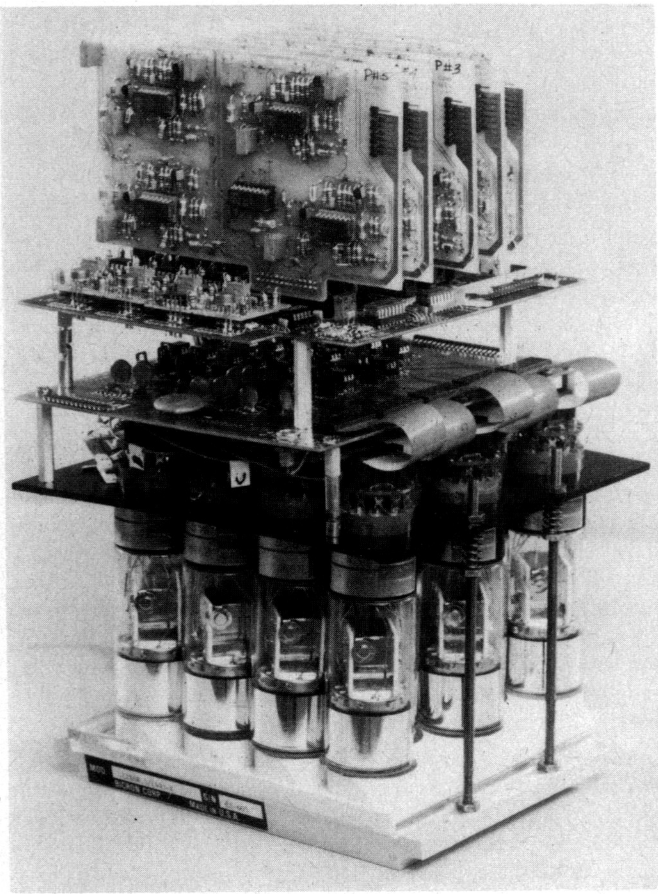

Figure 2. Modular detector and electronics. The 20 preamps are located at the top. Board just below forms row and column sums of PMT array. The bottom board is for PMT gain control and voltage distribution. Detectors will be closely packed into a ring for SPECT imaging.

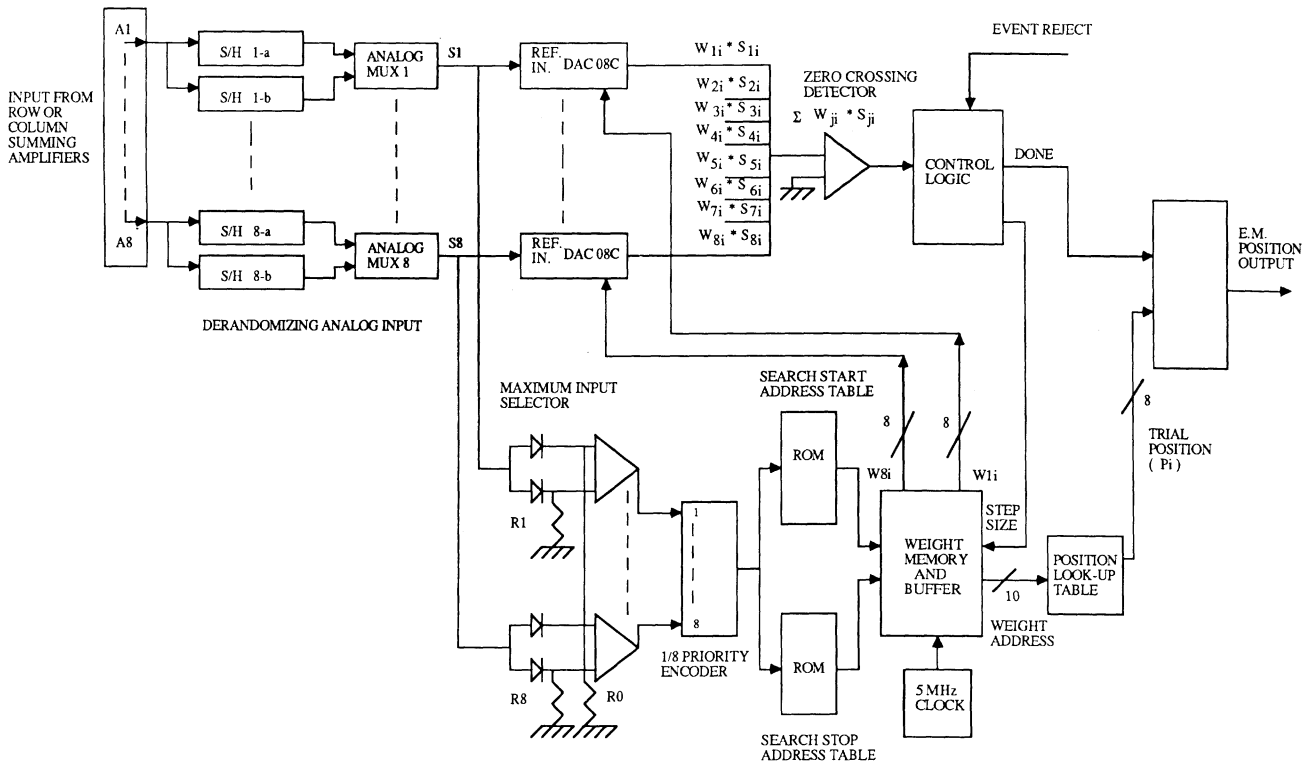

Figure 3. Block diagram of the position board.

To calculate the weighting coefficients for the ML estimate we must determine the expected response of each position sum as a function of source position. We translate a collimated ^{99m}Tc fan-beam source across the direction of interest in 1mm increments, acquire several thousand γ-ray events at each position with a 12 bit ADC and test computer system, and estimate the mean of the resulting position sums. After acquisition, the response functions are fit piecewise with quadratic polynomials with forced first derivative continuity at the endpoints. The weighting coefficients for each position sum are calculated from the fitted data.

Position Board Architecture

The modular detector requires two position boards for the calculation of the x and y position components. The boards are multibus based with the addition of a high-speed auxiliary bus for control signals and for the transfer of position data without bus contention overhead.

Figure 3 is a block diagram of the position board. The row or column position sums are presented to a pulse-stretcher/analog buffer circuit consisting of sample/hold amplifiers and high-speed analog multiplexors. Pulses from each scintillation are held at their peak value for the duration of the position search. The two stage analog buffer reduces system deadtime by retaining position sums from a scintillation occurring while the position board is active.

The range of trial positions searched is confined to a neighborhood of the maximum position sum. It is necessary to limit the search range not only to increase the computation speed but also to avoid selecting local extrema of the likelihood function. Referring to figure 3, the stretched position sums are directed to pairs of diodes. The maximum position sum (minus the diode junction voltage drop) appears across R0 and also at one input of a bank of comparators. The comparator corresponding to the maximum sum will change states since both diodes conduct and are biased in such a way that the voltage drop across R0 is smaller than R1-R8. The comparator outputs are encoded into a three bit address which is mapped through high-speed PROMs selecting the first and last positions to check.

The position search algorithm begins once the search range has been determined. Stretched position pulses are applied to the reference inputs of DAC-08E high-speed four quadrant multiplying D/A convertors. The weighting coefficients corresponding to each trial position are quantized to seven bits plus sign and stored in the 2Kx8 bit coefficient RAMs. Each DAC/RAM combination computes a product term of (4). The four quadrant multiplication feature of the DACs allows both positive and negative weighting coefficients to be used. The true and complementary current outputs are summed over the position sums and the two resulting sums are routed to a differential current-to-voltage convertor to perform the summation indicated in (4).

An oscillograph of the search process, representing the result of (4) vs. trial position, is shown in figure 4. The graph was made using a collimated ^{99m}Tc point source and the detector module shown previously. The board steps through the trial positions within the search range until a zero-crossing is detected indicating that the most likely position has been passed. The scan direction is then reversed in fine steps until a second zero-crossing is attained for a better estimate of position. The coarse to fine step ratio is 4:1. The address corresponding to the most likely position is translated to a

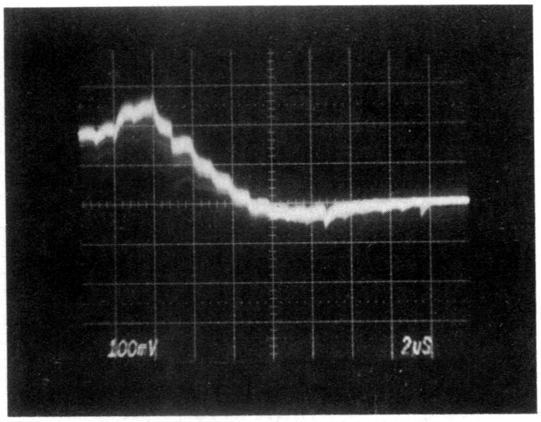

Figure 4. Oscillograph of the search algorithm showing calculated output of equation (4) vs. position estimate.

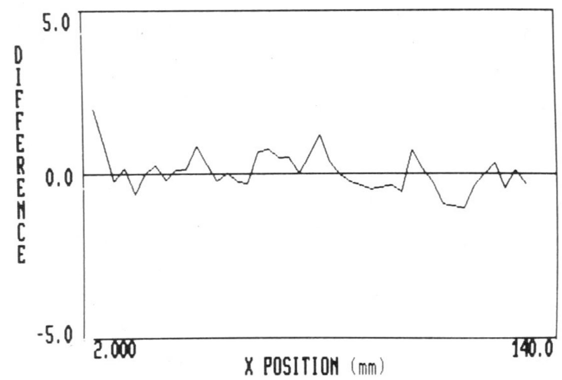

Figure 6. Linearity differences (mm) between position board and software calculation of ML estimate.

detector location through an output PROM and stored in a pipeline register. If the output registers are not full another calculation may begin immediately. In the event that the position sums are inconsistant due to, perhaps, pulse pileup the last address of the search range may be exceeded. If this occurs a flag is set indicating an erroneous position and the search is terminated.

Results

The performance of our implementation of the ML estimate was evaluated by calculating the Cramer-Rao bound for our mean position sum responses, $S_i(x)$, and comparing to the variance computed using equation (6). Figure 5 shows that our estimate very nearly achieves the maximum attainable resolution. The variance for the traditional estimate of determining the centroid of the scintillation light distribution was also calculated by first unbiasing the estimator to achieve spatial linearity and then applying (6). The ML estimate produces about a 30% increase in resolution relative to the plotted centroid estimate (CT).

Position computer performance was assessed by comparing results to those obtained by digitizing position sum data to 12 bits with a test computer system and implementing the ML estimate in software. A typical plot of the linearity differences between the board and the software estimate is shown in figure 6, demonstrating that the board performs very well with respect to the expected linearity. We notice larger deviations at the detector edge but feel this can be corrected with the adjustment of the position search ranges.

We might expect that separating the vector estimation problem would result in loss of spatial linearity since only

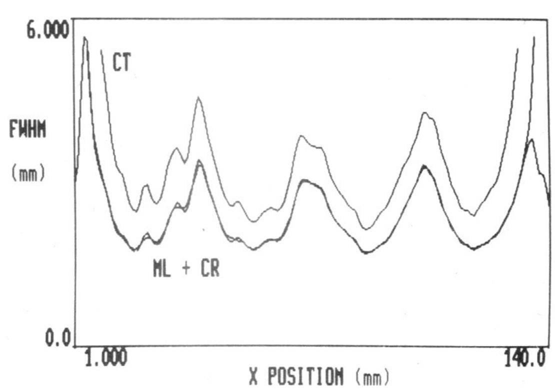

Figure 5. Predicted FWHM resolution given responses, $S_i(x)$, and energy resolution of 12% FWHM at center of detector. Maximum likelihood (ML) nearly achieves resolution limit (CR). Traditional centroid estimate (CT) is about 30% worse.

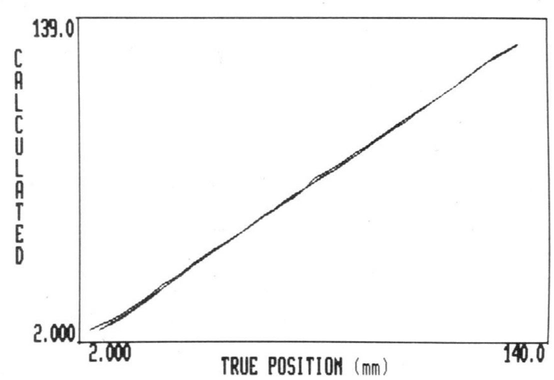

Figure 7. Calculated x position (mm) vs. true source position at y levels of 35mm, 65mm, and 95mm.

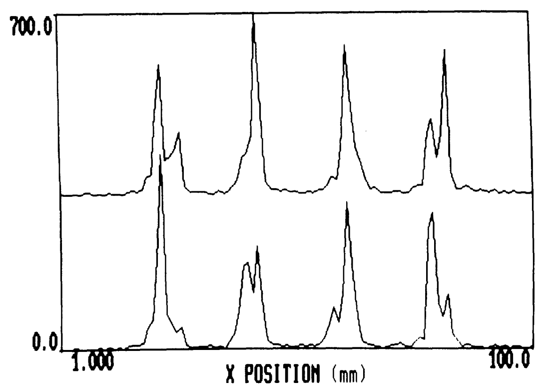

Figure 8. Lower curve. Response to point sources located at x positions 24mm, 44mm, 64mm, and 84mm. Upper curve. Response generated by moving point sources 1mm to 25mm, 45mm, 65mm, and 85mm.

one set of weighting coefficients, $w_i(x)$, can be used regardless of the orthogonal component of event location. To illustrate that the approximation is reasonable, we scanned a point source in the x direction at several y positions and calculated ML estimates with the postion board. Figure 7 shows the results of these scans, demonstrating that good linearity is achieved using just one set of weights.

Figure 8 shows several point sources acquired from the detector and position board. The lower curve is the response to four point sources separated by 20mm. In the upper curve each point source has been moved 1mm to the right. We note that the spatial resolution of the detector/position board combination is very good. The discrete nature of the detector is evident indicating which of the 3mm bars are being excited. Since the detector is unshielded, a significant amount of background activity is present.

Currently, the position boards are operating at one-third of the designed clockrate. Preliminary tests indicate slew-rate limiting in the current-voltage convertor circuit. As designed the worst case event search time is 5 μs but we are presently achieving search times on the order of 15 μs. Since these are the prototype boards we expect that with slight redesign we will be able to achieve the design speed.

Conclusion

A hybrid computer to calculate ML scintillation position estimate has been described and is in the preliminary test phase. The board has the advantage of being an inexpensive solution to the problem of computing the ML estimate when it is necessary to distinguish many positions. The computer is capable of performing any position estimate linear in the PMT outputs, and since the weighting functions can be altered at any time, it may be possible to implement recursive position estimators based on prior knowledge of the source distribution. The computer is not yet running at its design speed of 5 μs/event but we expect that this problem can be handled with slight redesign. The modularity of the computer suggests miniaturization.

Acknowledement

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