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Detective quantum efficiency dependence on x-ray energy weighting in mammography

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An evaluation of the dependence of detective quantum efficiency (DQE) on the incident energy spectrum has been made for mammography. The DQE dependence on the energy spectrum has been evaluated for energy-integrating detectors, photon-counting detectors, and detectors that measure the energy of each photon. To isolate the effect of the x-ray energy spectrum the detector has been assumed to be ideal, i.e., all noise sources are assumed to be zero except for quantum fluctuations. The result shows that the improvement in DQE, if the energy-integrating detector is compared to a single-photon counting detector, is of the order of 10%. Comparing the energy-integrating detector and the detector measuring the energy for each photon the improvement is around 30% using a molybdenum anode spectrum typical in mammography. It is shown that the optimal weight factors to combine the data in the case the energy is measured are very well approximated if the weight factors are proportional to E^{-3}. Another conclusion is that in calculating the DQE, a detector should be compared to one that uses ideal energy weighting for each photon since this provides the best signal-to-noise ratio. This has generally been neglected in the literature. [S0094-2405(99)01312-7]

Key words: mammography, DQE, energy weighting, single-photon counting

I. INTRODUCTION

The detective quantum efficiency (DQE) is defined as

$$\mathrm{DQE}=\frac{(\mathrm{SNR_{out}})^2}{(\mathrm{SNR_{in}})^2}, \tag{1}$$

where $\mathrm{SNR_{in}}$ is the signal-to-noise ratio (SNR) for an ideal detector that preserves all information in the radiation stream without adding any noise.

The photon transmission rate through the breast drops at low energies and it is evident that the transmitted low energy photons carry more contrast information than transmitted photons of higher energy. Energy-integrating detectors will put a weight factor to each photon proportional to its energy and the weighting will thus be contrary to the information content. Energy integration and photon counting have been compared in detail using other criteria[1–3] while the impact of the x-ray energy spectrum on DQE is usually neglected. See, however, Tapiovaara and Wagner,[4] who discuss the use of ordinary filters to shape the energy spectrum. Those filters, consisting of, e.g., Al or Mo, are, however, very unselective and the spectrum is broad also after the filter. Moreover they cut the overall x-ray flux with about a factor of 2, something that is undesirable in photon-starved applications. For mammography careful evaluations of the DQE have been made[5,6] but we are aware of none that takes into account the energy spectrum; instead a monoenergetic beam is generally assumed. In this paper we will show that the effect of the x-ray energy spectrum is sizable.[7]

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II. OPTIMAL WEIGHTING OF THE X-RAY ENERGY SPECTRUM

Assume we have an x-ray source with spectrum $\Phi(E)$ photons per unit photon energy and an object embedded in a uniform material as in Fig 1. Imagine that we bin the x-ray energies into M bins. Now suppose we weight each bin by w_i and form the quantity

$$W=\sum_{i=1}^{M} n_i w_i, \tag{2}$$

where n_i is the number of photons in each energy bin. The expectation value for n_i when there is no target is

$$\langle n_i\rangle=\Phi(E_i)\Delta E e^{-\mu_2 L_2}, \tag{3}$$

where μ is the attenuation coefficient evaluated at E_i [see Eq. (11)]. Analogously, when the target is present the result is

$$\langle n_i\rangle=\Phi(E_i)\Delta E e^{-\mu_1 L_1-\mu_2(L_2-L_1)}. \tag{4}$$

Thus, writing

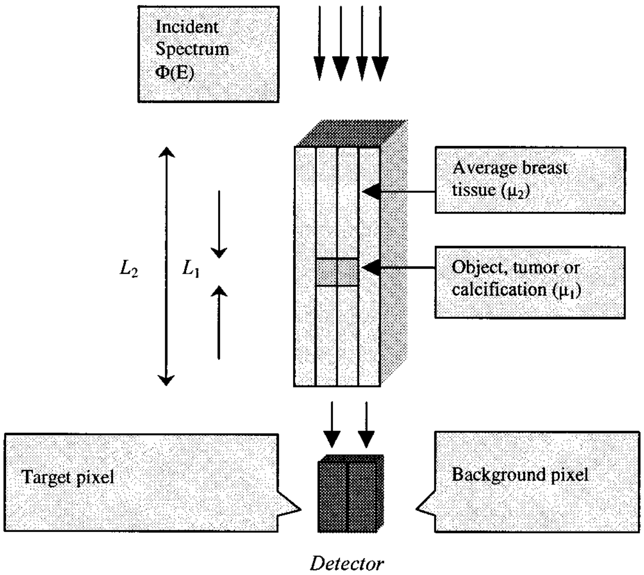

FIG. 1. Micro-calcification or tumor embedded in average breast tissue with notation used in the text.

$$X_i = X(E_i) = \mu_1(E_i)L_1 - \mu_2(E_i)L_1 \tag{5}$$

and assuming X_i is small, we can form the signal ΔW as the difference between the background pixel and the target pixel:

$$\langle \Delta W \rangle = \sum_{i=1}^{M} \langle n_i \rangle X_i w_i . \tag{6}$$

The square of the signal-to-noise ratio SNR is

$$\mathrm{SNR}^2 = \frac{2\langle \Delta W \rangle^2}{\sigma_w^2} = \frac{2(\Sigma_{i=1}^{M} \langle n_i \rangle X_i w_i)^2}{\Sigma_{n=1}^{M} \langle n_i \rangle w_i^2}. \tag{7}$$

To find the optimum SNR we can define the following expression:

$$\cos^2(\Theta) = \frac{\langle \Delta W \rangle^2}{\sigma_w^2 \Sigma_{j=1}^{M} \langle n_j \rangle X_j^2} = \frac{(\Sigma_{i=1}^{M} \langle n_i \rangle X_i w_i)^2}{\Sigma_{n=1}^{M} \langle n_i \rangle w_i^2 \Sigma_{j=1}^{M} \langle n_j \rangle X_j^2}. \tag{8}$$

We recognize that $\Sigma\, n_i a_i b_i$ constitutes a scalar product between the abstract vectors a_i and b_i and Θ is the angle between the directions of w_i and X_i. The maximum of $\cos^2(\Theta)$, and thus for SNR^2, occurs when w_i and X_i are parallel, i.e., when

$$w_i \propto X_i \propto \mu_1(E_i) - \mu_2(E_i). \tag{9}$$

Avoiding the above-mentioned approximations gives

$$w_i \propto (1 - e^{-(\mu_1(E_i) - \mu_2(E_i))L_1}). \tag{10}$$

In principle this result is what is found by Tapiovaara and Wagner,[4] who performed a similar investigation.

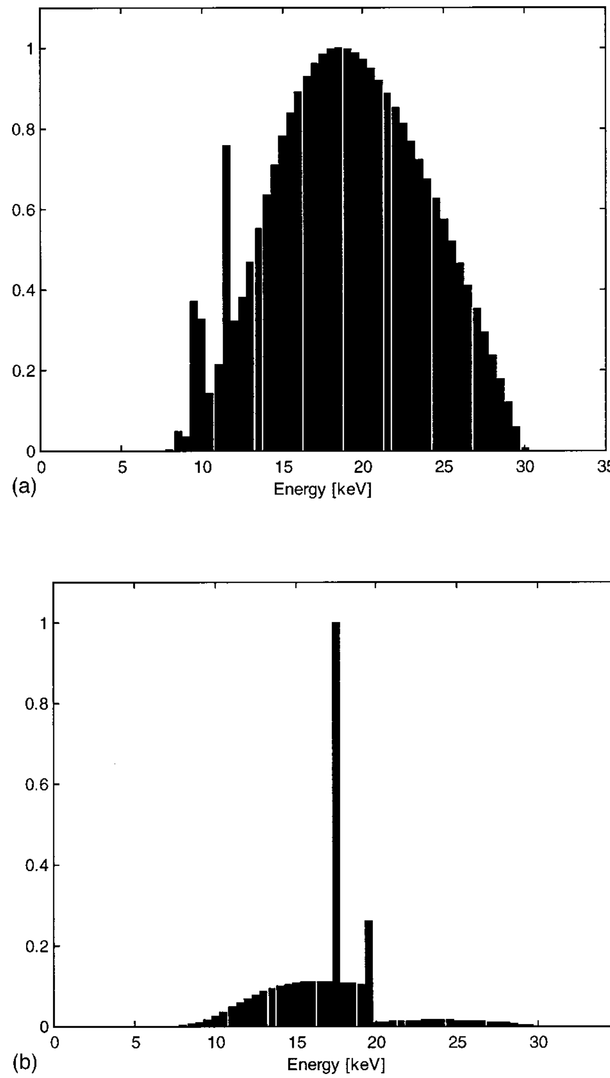

FIG. 2. (a) Wolfram anode spectrum at 30 kVp with 1 mm Be+0.5 mm Al filter in relative units from Ref. 8. (b) Molybdenum anode spectrum at 30 kVp with 1 mm Be+0.03 mm Mo filter in relative units from Ref. 8.

III. EVALUATION OF DIFFERENCES IN DQE DUE TO X-RAY ENERGY WEIGHTING FOR MAMMOGRAPHY SYSTEMS

Two different incident spectra have been used.[8] A 30 kVp W spectrum with 1 mm Be+0.5 mm Al filter that is similar to what has been used in the laboratory environment [Fig. 2(a)] and a 30 kVp Mo spectrum with 1 mm Be+0.03 mm Mo filter, which is a more common choice in mammography [Fig. 2(b)].

To calculate how the signal-to noise Ratios vary with weight functions, the entire spectrum after the breast needs to be estimated. The atomic number and the energy of the photon determine the cross section of all substances approximated[9] by

$$\tau(E) = 24.15 Z^{4.2} E^{-3} + 0.56Z, \tag{11}$$

where τ is the cross section in barn, Z is the atomic number, and E is the photon energy in kiloelectron volts. The attenuation coefficient is related to the cross section by

TABLE I. Parameters used for calculation of cross section for absorption. The tumor is assumed to have similar properties to muscular tissue. The data are from Refs. 11–13.

Tissue type	Effective Z	Effective A	Density (g/cm^3)
Microcalcification	14.76	26.88	2.2
Average breast tissue	6.56	11.48	0.98
Tumor	7.64	14.53	1.04

$$\mu(E)=\frac{\tau(E)\rho N_0}{A}, \tag{12}$$

where ρ is the density, A the atomic mass, and N_0 is Avogrado's number.

For integrating systems we take the weight factors to be proportional to the photon energy, one may note that relative to the optimum weighting there is a difference of the order E^4. In a single-photon-counting detector, all photons are given the same weight, i.e., $w_i=1$. For the case of optimum weighting the weight factors are taken according to Eq. (10). The difference in cross section in the medical energy range is close to proportional to E^{-3} and it is in this case a good approximation to take the factors w_i to be proportional to E^{-3} as well. The DQE is calculated using Eqs. (1) and (7) but with no approximations used in Eq. (7). We have used the values for Z, A, and ρ according to Table I. The results have been related to the optimum DQE.

The results in Tables II(a) and II(c) show that the energy response has a quite large impact on the DQE. Single-photon counting is an improvement compared to charge integration, but an even larger increase of around 30% can be achieved if the photon energy is used. The improvements accomplished with optimum energy weighting are larger for the Mo spectrum than for the W spectrum. This is explained by Fig. 3 where the two spectra are normalized and plotted overlay. The Mo spectrum shows stronger variations with energy, especially in the low energy region where the relative differences between the weight factors are larger. Therefore the gain in weighting the spectrum is larger for molybdenum compared to tungsten. The results in Tables II(a) and II(b) are fairly similar for tumours and microcalcifications. One may also note that the approximation $w_i \propto E^{-3}$ is very good compared to Eq. (10), the difference in Tables II(a) and II(b) is less than 2%.

With the fast single-photon-counting Application Specific Integrated Circuits currently being developed[10] it may soon be possible to implement single-photon counting combined with an energy measurement for each photon enabling the use of optimum weight factors and the following highest possible DQE. Another conclusion is that in calculating the DQE for a mammography system, a detector should be compared to one that uses ideal energy weighting for each photon since this provides the best signal-to-noise ratio.

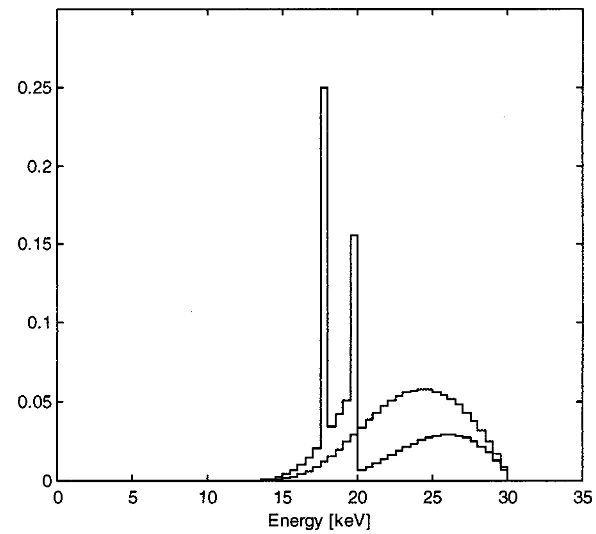

FIG. 3. Normalized Wolfram and Molybdenum anode spectra according to Fig. 2 after 45 mm of breast tissue. This is the spectra incident on the detector.

TABLE II. DQE for different objects in a mammography image for a W spectrum in (a) and for a Mo spectrum in (b). The DQE is indicated for charge integration, photon counting and also assuming weight factors proportional to E^{-3}, the latter is close to the optimum in mammography.

	Integrating	Photon counting	Weight$\propto E^{-3}$
(a)			
100 μm microcalcification	0.74	0.84	1.00
250 μm microcalcification	0.77	0.87	0.99
500 μm microcalcification	0.81	0.90	0.98
2.5 mm tumor	0.71	0.82	1.00
5.0 mm tumor	0.72	0.83	1.00
10.0 mm tumor	0.74	0.85	1.00
(b)			
100 μm microcalcification	0.70	0.84	1.00
250 μm microcalcification	0.72	0.86	1.00
500 μm microcalcification	0.76	0.89	0.99
2.5 mm tumor	0.68	0.83	1.00
5.0 mm tumor	0.68	0.83	1.00
10.0 mm tumor	0.70	0.85	1.00

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