Christensen's
Introduction to the Physics of Diagnostic Radiology

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Geometry of the Radiographic Image

We must briefly consider some geometric and trigonometric factors that influence the quality of the image of a radiograph. We will only consider examples of x rays originating at the x-ray tube and directed at the patient (or test object) and an x-ray film. We will assume that any object placed in the x-ray beam will absorb all the x-ray photons that hit the object. This is a very unlikely situation, but it makes examples easier to draw and understand. Because x rays travel in straight lines, the x-ray beam can be drawn as a straight line that hits the object or the film. The fact that x rays are emitted in all directions from the target of the x-ray tube has been previously discussed. In our examples we will assume the ideal situation, in which proper collimation has produced a beam that is perfectly cone-shaped, and the object is placed in the x-ray beam at a varying distance from the film. We will observe the beam from one side so its shape can be drawn as a triangle, with the focal spot of the x-ray tube as the apex (origin of x rays) and the object or the film as the base of the triangle (Fig. 12–1). In Figure 12–1, h represents the focal spot-object distance, and H is the focal spot-film distance. This type of diagram will allow us to consider two triangles, one with the object as its base and the other with the film as its base (Fig. 12–2). The two triangles have the same shape but are of different sizes, and are known as **similar triangles. The sides and altitudes of similar triangles are proportional.** This means that, in Figure 12–2,

$$\frac{a}{A} = \frac{b}{B} = \frac{c}{C} = \frac{h}{H}$$

The altitude of a triangle is a perpendicular dropped from a vertex to the opposite side or to an extension of the opposite side.

By using simple line drawings to represent the conditions of our idealized x-ray object-film conditions, and by applying the rule of similar triangles, it is possible to develop an understanding of the basic

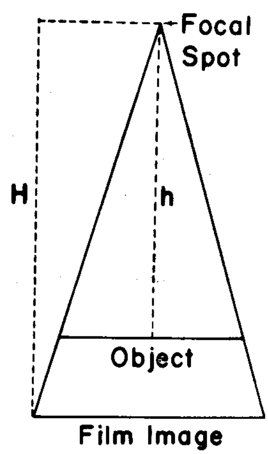

Figure 12–1 Schematic drawing of the relationship between an x-ray beam, the object being examined, and the image of the object on the film

principles of magnification, distortion, and penumbra.

MAGNIFICATION

When an object is placed in the x-ray beam, it will cast a "shadow" on the film that will show some degree of enlargement. Because it is assumed that the object absorbs all the x rays that hit it, the developed film will show a clear area corresponding to the shape of the object, surrounded by blackened (exposed) film. If the object is round and flat, shaped like a coin, its magnified image will be round but larger than the coin. The image has been magnified, and the amount of magnification (M) can be defined as

$$M = \frac{\text{size of the image}}{\text{size of the object}}$$

In the clinical situation the object may be a structure or foreign body within the patient that is not available for measurement. It is usually possible to determine the distance of the source of x rays (focal spot of the x-ray tube) from the film, however, as well as the distance of the object from the film, and the image size can be measured directly. By determining the degree of magnification, the true size of the object can be calculated. Refer to Figure 12–1, which shows a simple line drawing of an object placed some distance from the x-ray film, causing its image to be magnified by the diverging x-ray beam. The altitude of the large triangle (H) represents the distance from the focal spot of the x-ray tube to the film, often termed **focus-film distance.** The altitude of the smaller triangle (h) represents the distance from the focal spot to the object, or the **focus-object distance.** Because the sides and altitudes of similar triangles are proportional,

$$\frac{h}{H} = \frac{\text{object size}}{\text{image size}}$$

Because focus-film distance, image size, and focus-object distance can be measured directly, object size is easy to calculate. Notice that calculating simple magnification problems does not require that any formulas be learned. Consider a simple example. Using a 40-inch focus-film distance, the image of an object that is known to be 8 inches from the film measures 10 cm in length. What is the true length of the object, and how much magnification is present? Using Figure 12–1, we see that H = 40, h = 32 (40 − 8 = 32), and image size = 10 cm. Setting up the proportion:

$$\frac{40}{32} = \frac{10\text{ cm}}{\text{object size}}$$

Solving for object size:

$$\text{Object size} = \frac{(32)(10)}{40} = 8\text{ cm}$$

$$M = \frac{\text{size of the image}}{\text{size of the object}} = \frac{10}{8} = 1.25$$

What if the object is not placed directly

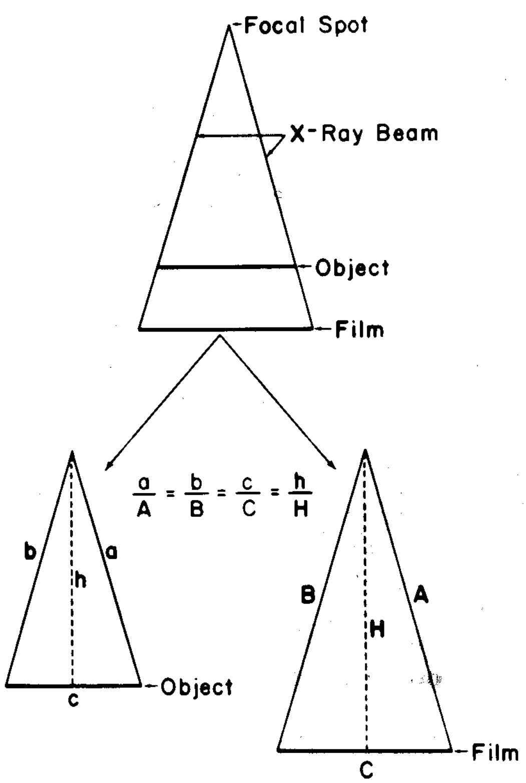

Figure 12–2 The proportional relationships of similar triangles

beneath the focal spot, but is displaced to one side so that the more oblique x rays are used to form an image? If the object is flat, and remains parallel to the film, magnification will be exactly the same for the object whether central or oblique x rays are used. Consider Figure 12–3, in which the coin is not directly beneath the x-ray tube but is still parallel to the film. The altitude of the two triangles is exactly the same as it was when the object was directly under the focal spot (see Fig. 12–1), so the ratio of object to image size will be the same in each circumstance. In Figure 12–4 the three coins are parallel to, and the same distance above, the film. The image of each coin will be a circle, and all the circles will be the same size.

Under usual radiographic situations, magnification should be kept to a minimum. Two rules apply: (1) **Keep the object as close to the film as possible,** and (2) **Keep the focus-film distance as large as possible.**

Consider Figure 12–1 again. Because magnification is determined by the ratio of

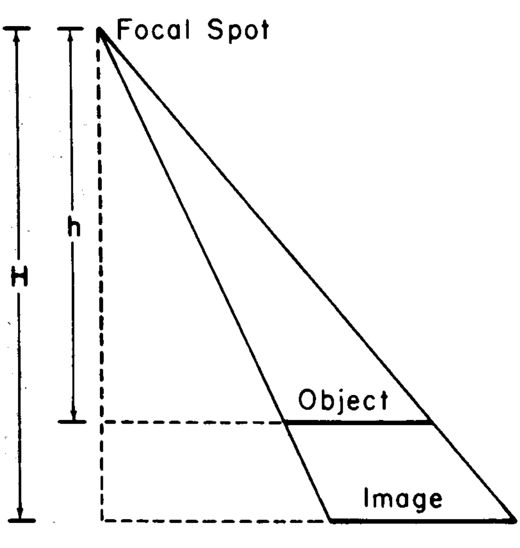

Figure 12–3 Magnification of an object by oblique rays

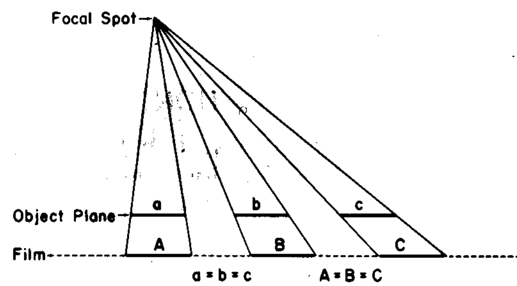

Figure 12–4 The magnification of three coins (a, b, c) all parallel to the film and the same distance above the film produces three round images (A, B, C) of equal size

the altitude of the two triangles, we may write

$$M = \frac{H}{h}$$

If H:h = 1.0, there is no magnification. The closer the object is to the film, the closer the magnitude of altitude h will approach the value of altitude H. An increase in focus-film distance (increasing H while leaving object-film distance unchanged) will also bring the ratio H:h closer to 1. To illustrate, assume an object is 6 inches from the film. What will be the magnification if the focus-film distance is (1) 40 inches, and (2) 72 inches? At 40 inches focus-film distance:

$$M = \frac{H}{h} = \frac{40}{34} = 1.18$$

At 72 inches focus-film distance:

$$M = \frac{H}{h} = \frac{72}{66} = 1.09$$

To review, magnification depends on two factors: **object-film distance** and **focus-film distance.**

DISTORTION

Distortion results from unequal magnification of different parts of the same object. Consider Figure 12–5, in which one coin (*dashed line*) is parallel to the film and the other coin (*solid line,* AB) is tilted with respect to the plane of the film. The tilted

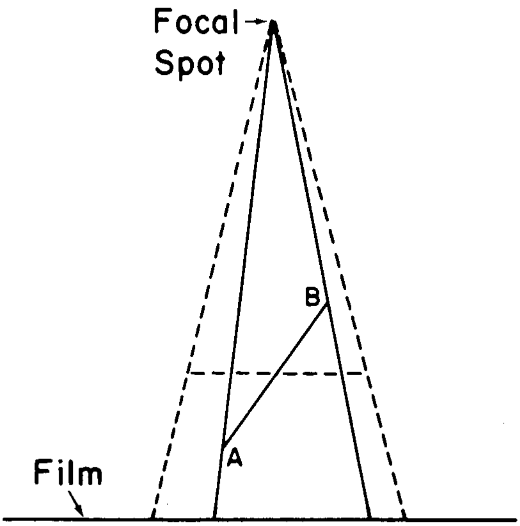

Figure 12–5 Distortion of an object (line AB) that is not parallel to the film

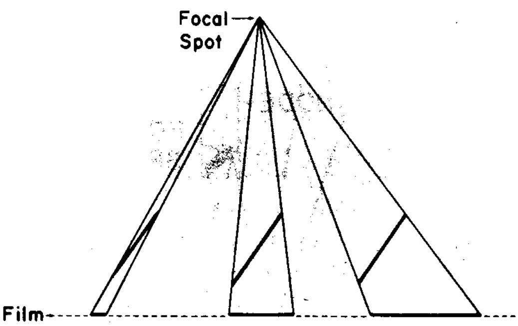

Figure 12–6 Distortion of the shape and size of the image of a tilted object depends on the position of the object in the x-ray beam

coin undergoes distortion because of unequal magnification (side A is closer to the film than side B). The shape of the image of the tilted coin will be an ellipse, and its exact size and shape will vary with the amount of tilting.

Distortion of the image of an object will be different in different parts of the x-ray beam. Figure 12–6 shows how distortion of the shape and size of three equally tilted coins will vary in different parts of the x-ray beam.

Distortion of thick objects occurs if they are not directly in the central part of the x-ray beam. Because different parts of thick objects are different distances from the x-ray film, each part will be magnified by a different amount. This will cause the shape of the image of most thick objects to be distorted. Only the part of a thick object that is parallel to the film will be undistorted. Figure 12–7 illustrates the relative size and shape of the image of three spheres (such as steel ball bearings) that are in different parts of the x-ray beam. The sphere that lies in the center of the beam will exhibit a round (undistorted) magnified image. The image of each of the two laterally placed spheres will be an ellipse because the x-ray beam "sees" a diameter of each of these spheres that is not parallel to the film. The x-ray beam "sees" a laterally placed spherical object in the same way it "sees" a round flat object (a coin) that is tilted with respect to the plane of the film. Notice the similarity between Figures 12–6 and 12–7.

Distortion of the relative position of the image of two objects may occur if the objects are at different distances from the film. For example, in Figure 12–8 two opaque objects, A and B, are present inside a circle. Object A is more medial than B, but A is farther from the film. The film image of object A will be lateral to the image of object B. This is because the distance between A and the midline (line a) has been magnified much more than has the distance between B and the central beam (line b). Distortion of position is minimal when the object is near the central part

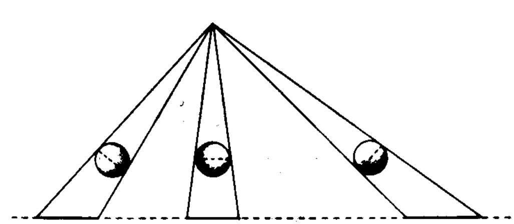

Figure 12–7 The size and shape of the image of a spherical object depends on the position of the object in relation to the central part of the x-ray beam

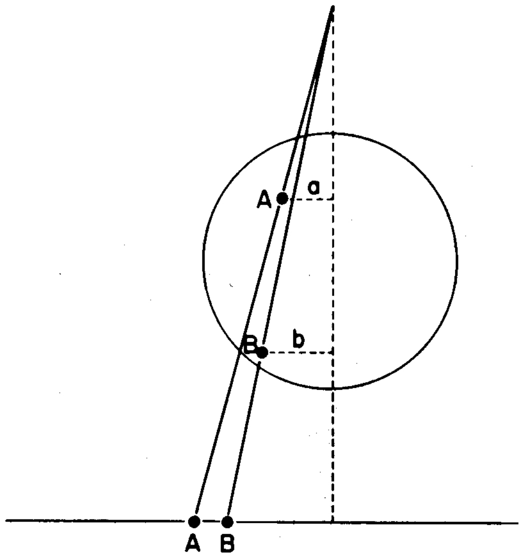

Figure 12–8 Distortion of position

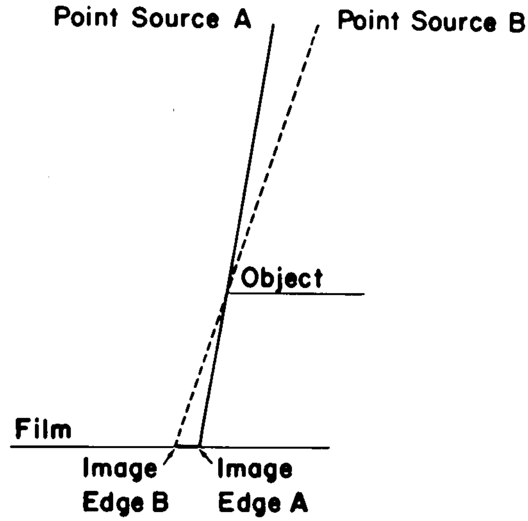

Figure 12–9 The focal spot acts as if it were composed of many point sources of x rays

of the x-ray beam, and the object is placed as close to the film as possible.

PENUMBRA

Penumbra (from the Latin *pene,* meaning almost, and *umbra,* meaning shadow), often termed **edge gradient,** is defined as the region of partial illumination that surrounds the umbra, or complete shadow. In the discussion of magnification and distortion, it was assumed that the source of x rays (focal spot) was a point source. Actually, the focal spot is not a point. It has finite dimensions, usually ranging from 0.3 to 2.0 mm square. The focal spot acts as if it were composed of many point sources of x rays, with each point source forming its own image of an object. The edges of each of these images will not be in exactly the same spot on the film. In Figure 12–9 the edge of an object, as formed by two x-ray sources (A and B), which represent the opposite ends of a focal spot, is shown. The x rays must travel in a straight line from each point source to the film. Image edges A and B are not in the same place, and an image formed in this way has a fuzzy, or unsharp, margin. This zone of unsharpness is called **geometric unsharpness, penumbra,** or **edge gradient,** and represents the area at which the margins caused by the many "point sources" of x rays in the focal spot overlap. Figure 12–10 shows how the zone of penumbra is formed from

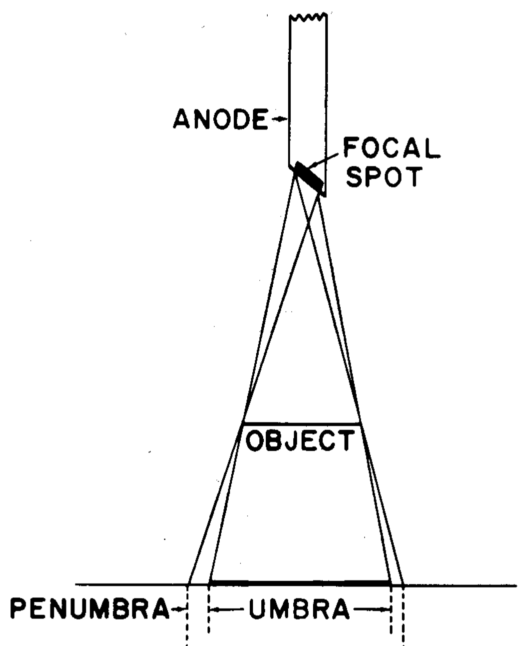

Figure 12–10 Penumbra (geometric unsharpness)

an angled rotating anode; focal spot size has been exaggerated. The region of complete image is called the **umbra.** These terms are used in astronomy to describe a solar eclipse, in which the umbra is the area of complete shadow within which a spectator can see no portion of the sun's disc, and the penumbra is the zone of partial shadow between the umbra and the full light. Note that, as shown in Figure 12–10, the width of the penumbra is less on the anode side than on the cathode side of the x-ray tube. This effect can be used to achieve maximum sharpness by placing the object of greatest interest toward the anode side of the x-ray tube.

The width of blurring caused by penumbra can be calculated. Figure 12–11 shows the penumbra (P) caused by focal spot F (dimensions are exaggerated for illustration). Note that two similar triangles are again formed, with the apex of each triangle at the object (labeled point X), the base of the upper triangle being the width of the focal spot and the base of the lower triangle being the width of the penumbra.

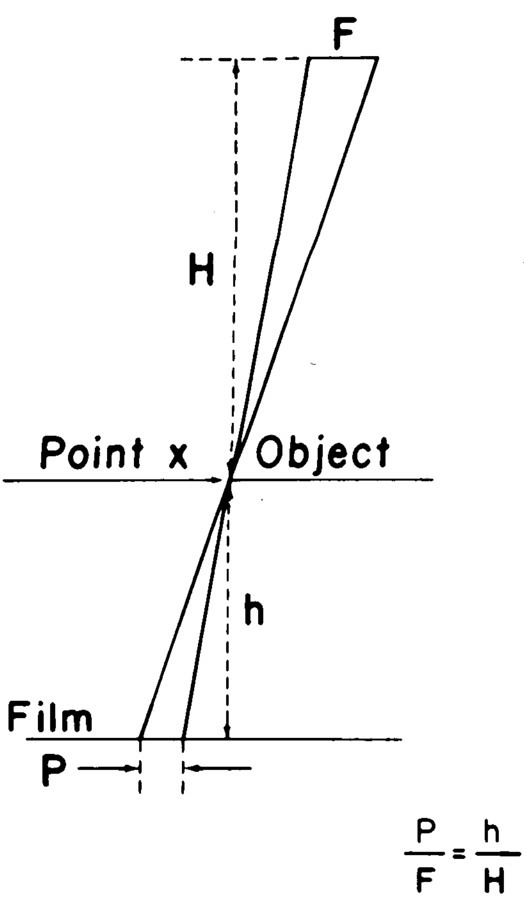

Figure 12–11 Calculation of the width of the zone of geometric unsharpness (penumbra)

Because the sides and altitudes (H and h) of similar triangles are proportional, a simple proportion can be established between focal spot size (F), penumbra (P), focus-object distance (H), and object-film distance (h):

$$\frac{P}{F} = \frac{h}{H}$$

Using a 2-mm focal spot and a focus-film distance of 40 inches, calculate the penumbra if the object is (1) 4 inches above the film and (2) 10 inches above the film. In case (1) focus-object distance (H) = 40 − 4 = 36 inches:

$$\frac{P}{2} = \frac{4}{36}$$
$$P = \frac{(4)(2)}{36}$$
$$P = 0.22 \text{ mm}$$

In case (2) H = 40 − 10 = 30 inches:

$$\frac{P}{2} = \frac{10}{30}$$
$$P = \frac{(2)(10)}{30}$$
$$P = 0.67 \text{ mm}$$

In case (2), if object-film distance remains 10 inches, but focus-film distance is increased to 72 inches, what will be the penumbra?

$$\frac{P}{2} = \frac{10}{62}$$
$$P = \frac{(2)(10)}{62}$$
$$P = 0.32 \text{ mm}$$

The importance of focal spot size may be appreciated by substituting a focal spot size of 1 mm in each of the preceding examples.

The width of the penumbra will be half of that caused by a 2-mm focal spot. These situations illustrate the factors that will decrease penumbra:

1. Put the object as close to the film as possible (make h small).
2. Use as large a focus-object distance as possible (make H large).
3. Use as small a focal spot as possible.

Keeping the object as close to the film as possible, and using a long focus-film distance, also decreases magnification. The use of the magnification technique, which we will discuss in Chapter 20, causes an increase in geometric unsharpness.

MOTION UNSHARPNESS

This term is used to describe image unsharpness caused by motion of the examined object during the exposure. Object motion will produce the same type of image unsharpness as penumbra. Object motion may be minimized by immobilizing the patient or by using a short exposure time. Similar motion unsharpness will result if the focal spot moves during the exposure. X-ray tube motion is much less important than object motion, unless considerable magnification is present. (The relationship between motion and magnification will be discussed in more detail in Chapter 20.)

ABSORPTION UNSHARPNESS

Absorption unsharpness is caused by the way in which x rays are absorbed in the subject. This type of unsharpness arises from the gradual change in x-ray absorption across the boundary, or edge, of an object. To illustrate the meaning of absorption unsharpness, let us again assume that x rays originate from a point source (F). Figure 12–12 shows a truncated cone, cube, and sphere, which all have the same thickness and are assumed to be made of the same material. The cone will show little absorption unsharpness because its edges are parallel to the diverging beam, and its edge will be sharply defined on the film. It

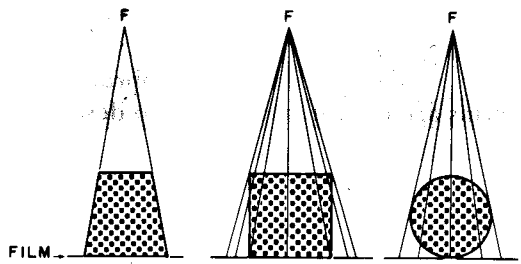

Figure 12–12 The origin of absorption unsharpness

is apparent in Figure 12–12 that the absorption of x rays by the cube will vary along the outer edge of its upper surface, with fewer x rays being absorbed along the sides, and more in the region of the lower corners. With the sphere, absorption unsharpness occurs across the entire image, with maximum x-ray absorption occurring only in the center.

Absorption unsharpness produces a poorly defined margin in the image of most solid objects because there is a gradual change in film density along the image edge. The cone shows high contrast (density difference) along its edge because there is a well-defined line on the film at which x-ray exposure changes from high (no absorption by the object) to low (high absorption by the object). In the case of the cube and the sphere, however, there is no abrupt change in the film exposure, only a gradual change, the magnitude of which depends on the shape of the object.

The effect of absorption unsharpness is particularly important when the accurate measurement of small round or oval structures is necessary, as in coronary angiography. Because absorption unsharpness is caused by the shape of the object being examined, it will occur no matter how exacting the conditions of generating and recording the radiographic image. Perhaps the term "subject unsharpness" would be more accurate.

INVERSE SQUARE LAW

X rays obey the physical laws of light. There is a well-known law of light propagation that states that the intensity of light falling on a flat surface from a point source is inversely proportional to the square of the distance from the point source. The principle is illustrated in Figure 12–13.

The number of x-ray photons emitted at the anode remains constant. At a distance of 1 foot, the diverging x-ray beam covers an area (A) represented by the square with each side of dimension x, or an area of $x \cdot x = x^2$. At 2 feet the diverging beam covers a square (B) in which each side is now twice as long as it was at 1 foot. The area covered by the beam at 2 feet is therefore $2x \cdot 2x = 4x^2$, which is four times the area at 1 foot. Because the intensity of the beam originating at the anode is constant, the intensity falling on square A must spread out over an area four times as large by the time it reaches square B. For example, assume that an x-ray tube has an output such that the intensity 1 foot from the tube is 144 units per square inch. At 2 feet, the 144 units are now divided between 4 square inches, or 36 units per square inch. Likewise, the intensity per square inch at 3 feet is

$$\frac{144}{3^2} = \frac{144}{9} = 16 \text{ units/in.}^2$$

and so forth. At 4 feet the intensity will be $(1/4)^2$ or $1/16$ that at 1 foot.

A practical example will illustrate use of the inverse square law. Assume that an exposure of 100 mAs (100 mA for 1 sec) is needed for a film of the abdomen using a 40-inch focus-film distance. Employing portable equipment, the maximum focus-film distance that can be obtained at the bedside is 30 inches. What mAs must be used to maintain the same radiographic density as that obtained at a 40-inch distance (kVp remains constant)? Because the intensities (mAs) of the x-ray beam at 40 and 30 inches are proportional to the squares of these distances,

$$\frac{100 \text{ mAs}}{X \text{ mAs}} = \frac{40^2}{30^2} = \frac{1600}{900}$$

$$X = \frac{(100)\,(900)}{1600} = 56.25 \text{ mAs}$$

The distance from the x-ray tube to the film should be kept as large as practical to minimize the geometric unsharpness caused by penumbra. This greater distance will increase the exposure (mAs) needed to maintain proper film density. Older techniques called for a 36-inch focus-film distance but, with x-ray tubes capable of increased output, a 40-inch distance is now routine. The increased exposure required in going from 36 to 40 inches is

$$\frac{40^2}{36^2} = \frac{1600}{1296} = 1.23$$

or a 23% increase in mAs. As the output of x-ray tubes increases, techniques using up to 60-inch focus-film distance may become routine.

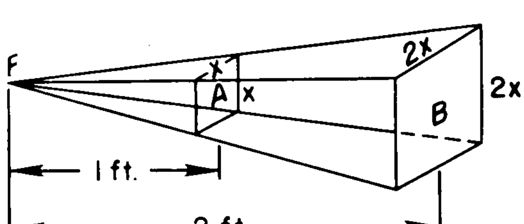

Figure 12–13 The inverse square law

SUMMARY

Geometric factors that influence the quality of the radiographic image include magnification, distortion, penumbra, and motion. Absorption unsharpness produces a similar effect. Factors that will decrease unsharpness caused by

MAGNIFICATION

1. small object-film distance
2. large focal spot-film distance

PENUMBRA

1. small focal-spot size
2. small object-film distance
3. large focal spot-film distance

MOTION

1. short exposure time
2. maximum possible limitation of actual object motion

To minimize distortion, the object of interest should be kept parallel to the film and near the central portion of the x-ray beam, and magnification should be minimized.

The intensity of the x-ray beam varies inversely as the square of the distance from the x-ray tube.

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