Digital Imaging and Communications in Medicine (DICOM) Part 14: Grayscale Standard Display Function

Published by

National Electrical Manufacturers Association 1300 N. 17th Street Rosslyn, Virginia 22209 USA

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FOREWORD

This DICOM Standard was developed according to the procedures of the DICOM Standards Committee.

While other parts of the DICOM Standard specify how digital image data can be moved from system to system, it does not specify how the pixel values should be interpreted or displayed. PS 3.14 specifies a function that relates pixel values to displayed Luminance levels.

A digital signal from an image can be measured, characterized, transmitted, and reproduced objectively and accurately. However, the visual interpretation of that signal is dependent on the varied characteristics of the systems displaying that image. Currently, images produced by the same signal may have completely different visual appearance, information, and characteristics on different display devices.

In medical imaging, it is important that there be a visual consistency in how a given digital image appears, whether viewed, for example, on the display monitor of a workstation or as a film on a light-box. In the absence of any standard that regulates how these images are to be visually presented on any device, a digital image which has good diagnostic value when viewed on one device could look very different and have greatly reduced diagnostic value when viewed on another device. Accordingly, PS 3.14 was developed to provide an objective, quantitative mechanism for mapping digital image values into a given range of Luminance. An application that knows this relationship between digital values and display Luminance can produce better visual consistency in how that image appears on diverse display devices. The relationship that PS 3.14 defines between digital image values and displayed Luminance is based upon measurements and models of human perception over a wide range of Luminance, not upon the characteristics of any one image presentation device or of any one imaging modality. It is also not dependent upon user preferences, which can be more properly handled by other constructs such as the DICOM Presentation Lookup Table.

The DICOM Standard is structured as a multi-part document using the guidelines established in the following document:

- ISO/IEC Directives, 1989 Part 3 : Drafting and Presentation of International Standards

PS 3.1 should be used as the base reference for the current parts of this standard.

1. Scope and Field of Application

PS 3.14 specifies a standardized Display Function for display of grayscale images. It provides examples of methods for measuring the Characteristic Curve of a particular Display System for the purpose of either altering the Display System to match the Grayscale Standard Display Function, or for measuring the conformance of a Display System to the Grayscale Standard Display Function. Display Systems include such things as monitors with their associated driving electronics and printers producing films that are placed on light-boxes or alternators.

PS 3.14 is neither a performance nor an image display standard. PS 3.14 does not define which Luminance and/or Luminance Range or optical density range an image presentation device must provide. PS 3.14 does not define how the particular picture element values in a specific imaging modality are to be presented.

PS 3.14 does not specify functions for display of color images, as the specified function is limited to the display of grayscale images. Color Display Systems may be calibrated to the Grayscale Standard Display Function for the purpose of displaying grayscale images. Color images, whether associated with an ICC Profile or not, may be displayed on standardized grayscale displays, but there are no normative requirements for the display of the luminance information in a color image using the GSDF.

2. Normative References

The following standards contain provisions, which, through reference in this text, constitute provisions of this Standard. At the time of publication, the editions indicated were valid. All standards are subject to revision, and parties to agreements based on this Standard are encouraged to investigate the possibilities of applying the most recent editions of the standards indicated below.

ISO/IEC Directives, 1989 Part 3 - Drafting and presentation of International Standards.

3. Definitions

For the purposes of PS 3.14 the following definitions apply.

<u>Characteristic Curve</u>: The inherent Display Function of a Display System including the effects of ambient light. The Characteristic Curve describes Luminance versus DDL of an emissive display device, such as a CRT/display controller system, or Luminance of light reflected from a print medium, or Luminance derived from the measured optical density versus DDL of a hard-copy medium and the given Luminance of a light-box. The Characteristic Curve depends on operating parameters of the Display System.

Note: The Luminance generated by an emissive display may be measured with a photometer. Diffuse optical density of a hard-copy may be measured with a densitometer.

<u>Contrast Sensitivity</u> characterizes the sensitivity of the average human observer to Luminance changes of the Standard Target. Contrast Sensitivity is inversely proportional to Threshold Modulation.

<u>Contrast Threshold</u>: A function that plots the Just-Noticeable Difference divided by the Luminance across the Luminance Range.

<u>Digital Driving Level</u> (DDL): A digital value which given as input to a Display System produces a Luminance. The set of DDLs of a Display System is all the possible discrete values that can produce Luminance values on that Display System. The mapping of DDLs to Luminance values for a Display System produces the Characteristic Curve of that Display System. The actual output for a given DDL is specific to the Display System and is not corrected for the Grayscale Standard Display Function.

<u>Display Function</u>: A function that describes a defined grayscale rendition of a Display System, the mapping of the DDLs in a defined space to Luminance, including the effects of ambient light at a given state of adjustment of the Display System. Distinguished from Characteristic Curve, which is the inherent Display Function of a Display System.

<u>Display System</u>: A device or devices that accept DDLs to produce corresponding Luminance values. This includes emissive displays, transmissive hardcopy viewed on light boxes, and reflective hardcopy.

<u>Illuminance</u>: Light from the environment surrounding the Display System which illuminates the display medium. It contributes to the Luminance that is received by an observer from the image display. Ambient Light reduces the contrast in the image.

<u>Just-Noticeable Difference</u> (JND): The Luminance difference of a given target under given viewing conditions that the average human observer can just perceive.

<u>JND Index</u>: The input value to the Grayscale Standard Display Function, such that one step in JND Index results in a Luminance difference that is a Just-Noticeable Difference.

<u>Luminance</u> is the luminous intensity per unit area projected in a given direction. The Système Internationale unit (used in PS 3.14) is candela per square meter (cd/m²), which is sometimes called nit. Another unit often used is footlambert (fL). 1 fL = 3.426 cd/m^2 .

Luminance Range: The span of Luminance values of a Display System from a minimum Luminance to a maximum Luminance.

<u>P-Value</u>: A device independent value defined in a perceptually linear grayscale space. The output of the DICOM Presentation LUT is P-Values, ie. the pixel value after all DICOM defined grayscale transformations have been applied. P-Values are the input to a Standardized Display System.

<u>Grayscale Standard Display Function</u>: The mathematically defined mapping of an input JND index to Luminance values defined in PS 3.14.

<u>Standardized Display System</u>: A device or devices that produce Luminance values which are related to input P-Values by the Grayscale Standard Display Function. How this is performed is not defined, though it may be achieved by transformation of P-Values into DDLs accepted by a Display System.

Standard Luminance Level: Any one of the Standard Luminance levels in Table B-1.

<u>Standard Target</u>: A 2-deg x 2-deg square filled with a horizontal or vertical grating with sinusoidal modulation of 4 cycles per degree. The square is placed in a uniform background of a Luminance equal to the mean Luminance of the Target.

Note: The Standard Target is defined in terms of the subtended viewing angle, not in terms of the distance from the viewer to the target.

<u>Threshold Modulation</u>: The minimum Luminance modulation required by the average human observer to detect the Standard Target at a given mean Luminance level. The Threshold Modulation corresponds to the Just-Noticeable Difference in Luminance of the Standard Target.

4. Symbols and Abbreviations

The following symbols and abbreviations are used in PS 3.14.

ACR American College of Radiology	
ANSI American National Standards Institute	
CEN TC251 Comite' Europeen de Normalisation - Technical Committee 251 - Medical Informati	cs
DICOM Digital Imaging and Communications in Medicine	
HL7 Health Level 7	
IEEE Institute of Electrical and Electronics Engineers	
ISO International Standards Organization	
JIRA Japan Industries Association of Radiological Systems	
NEMA National Electrical Manufacturers Association	

5. Conventions

The following conventions are used in PS 3.14:

The terminology defined in Section 3 above is capitalized throughout PS 3.14.

6. Overview

PS 3.14 defines, mathematically, the Grayscale Standard Display Function of Standardized Display Systems. These systems may be printers producing hard-copies viewed on light-boxes or electronic Display Systems for soft-copies.

Hard-copies may consist of transmissive films as well as reflective prints. The image in these prints is represented by optical density variations in transmission or diffuse reflection. To an observer, every element of the image appears with a certain Luminance depending on the Illuminance and the optical density of the image element.

Soft-copies may be produced by emissive Display Systems (such as CRT monitors) or electronic light valves (such as light sources and liquid crystal displays).

For the purpose of PS 3.14, Display Systems take a Digital Driving Level and produce Luminance or optical density variations that represent the image. Predictable application of image transformations, such as the modality, value-of-interest, and presentation look-up tables specified in the DICOM standard, requires knowledge of the Characteristic Curve of the Display System. Standardizing the response function expected of the Display System simplifies the application of such image transformations across several different Display Systems such as encountered in a network environment.

PS 3.14 does not define when conformance with the Grayscale Standard Display Function is achieved or how to characterize the degree of conformance reached.

Note: A definition of conformance would require thorough evaluations of human visual system sensitivity to deviations of Display Functions from the Grayscale Standard Display Function for medical images.

Figures 6-1 and 6-2 show the context for the Grayscale Standard Display Function. The Grayscale Standard Display Function is part of the image presentation. There will be a number of other modifications to the image before the Grayscale Standard Display Function is applied. The image acquisition device will adjust the image as it is formed. Other elements may perform a "window and level" to select a part of the dynamic range of the image to be presented. Yet other elements can adjust the selected dynamic range in preparation for display. The Presentation LUT outputs P-Values (presentation values). These P-Values become the Digital Driving Levels for Standardized Display Systems. The Grayscale Standard Display Function maps P-Values to the log-luminance output of the Standardized Display System. How a Standardized Display System performs this mapping is implementation dependent.

The boundary between the DICOM model of the image acquisition and presentation chain, and the Standardized Display System, expressed in P-Values, is intended to be both device independent and conceptually (if not actually) perceptually linear. In other words, regardless of the capabilities of the Standardized Display System, the same range of P-Values will be presented isimilarlyî.

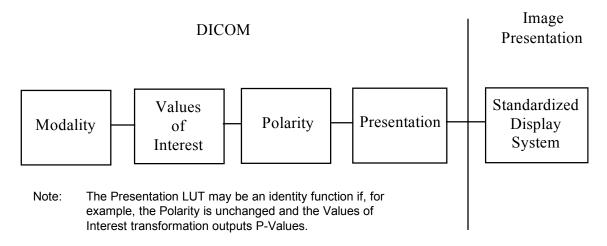


Figure 6-1. The Grayscale Standard Display Function is an element of the image presentation after several modifications to the image have been completed by other elements of the image acquisition and presentation chain.

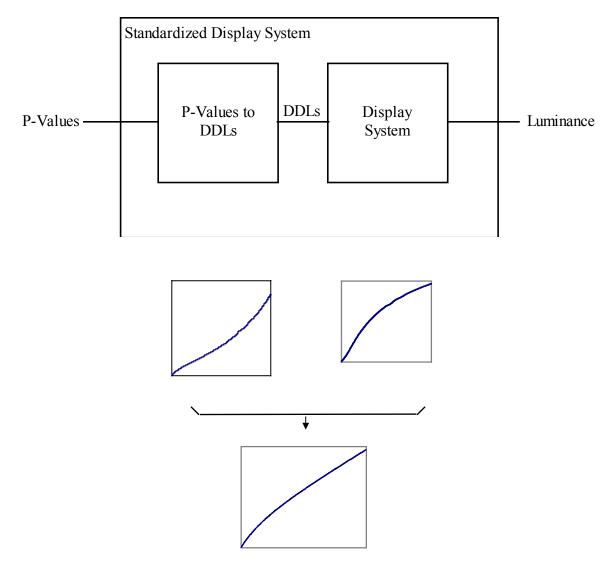


Figure 6-2. The conceptual model of a Standardized Display System maps P-Values to Luminance via an intermediate transformation to Digital Driving Levels of an unstandardized Display System.

The main objective of PS 3.14 is to define mathematically an appropriate Grayscale Standard Display Function for all image presentation systems. The purpose of defining this Grayscale Standard Display Function is to allow applications to know *a priori* how P-Values are transformed to viewed Luminance values by a Standardized Display System. In essence, defining the Grayscale Standard Display Function fixes the "units" for the P-Values output from the Presentation LUT and used as Digital Driving Levels to Standardized Display Systems.

A second objective of PS 3.14 is to select a Display Function which provides some level of similarity in grayscale perception or basic appearance for a given image between Display Systems of different Luminance and which facilitates good use of the available Digital Driving Levels of a Display System. While many different functions could serve the primary objective, this Grayscale Standard Display Function was chosen to meet the second objective. With such a function, P-Values are approximately linearly related to human perceptual response. Similarity does not guarantee equal information content. Display Systems with a wider Luminance Range and/or higher Luminance will be capable of presenting

more just-noticeable Luminance differences to an observer. Similarity also does not imply strict perceptual linearity, since perception is dependent on image content and on the viewer. In order to achieve strict perceptual linearity, applications would need to adjust the presentation of images to match user expectations through the other constructs defined in the DICOM Standard (eg. VOI and Presentation LUT). Without a defined Display Function, such adjustments on the wide variety of Display Systems encountered on a network would be difficult.

The choice of the function is based on several ideas that are discussed further in Annex A.

Annex B contains the Grayscale Standard Display Function in tabular form.

Informative Annex C provides an example procedure for comparing mathematically the shape of the actual Display Function with the Grayscale Standard Display Function and for quantifying how well the actual discrete Luminance intervals match those of the Grayscale Standard Display Function.

Display Systems often will have Characteristic Curves different from the Grayscale Standard Display Function. These devices may contain means for incorporating externally defined transformations that make the devices conform with the Grayscale Standard Display Function. PS 3.14 provides examples of test patterns for Display Systems with which their behavior can be measured and the approximation to the Grayscale Standard Display Function evaluated (see Informative Annex D.1, D.2, D.3).

7 The Grayscale Standard Display Function

As explained in greater detail in Annex A, the Grayscale Standard Display Function is based on human Contrast Sensitivity. Human Contrast Sensitivity is distinctly non-linear within the Luminance Range of the Grayscale Standard Display Function . The human eye is relatively less sensitive in the dark areas of an image than it is in the bright areas of an image. This variation in sensitivity makes it much easier to see small relative changes in Luminance in the bright areas of the image than in the dark areas of the image. A Display Function that adjusts the brightness such that equal changes in P-Values will result in the same level of perceptibility at all driving levels is "perceptually linearized". The Grayscale Standard Display Function incorporates the notion of perceptual linearization without making it an explicit objective of PS 3.14.

The employed data for Contrast Sensitivity are derived from Barten's model of the human visual system (Ref. 1, 2 and Annex B). Specifically, the Grayscale Standard Display Function refers to Contrast Sensitivity for the Standard Target consisting of a 2-deg x 2-deg square filled with a horizontal or vertical grating with sinusoidal modulation of 4 cycles per degree. The square is placed in a uniform background of Luminance equal to the mean Luminance L of the Target. The Contrast Sensitivity is defined by the Threshold Modulation at which the grating becomes just visible to the average human observer. The Luminance modulation represents the Just-Noticeable Difference (JND) for the Target at the Luminance L.

Note: The academic nature of the Standard Target is recognized. With the simple target, the essential objectives of PS 3.14 appear to be realizable. Only spurious results with more realistic targets in complex surroundings were known at the time of writing PS 3.14 and these were not assessed.

The Grayscale Standard Display Function is defined for the Luminance Range from 0.05 to 4000 cd/m². The minimum Luminance corresponds to the lowest practically useful Luminance of cathode-ray-tube (CRT) monitors and the maximum exceeds the unattenuated Luminance of very bright light-boxes used for interpreting X-Ray mammography. The Grayscale Standard Display Function explicitly includes the effects of the diffused ambient Illuminance.

Within the Luminance Range happen to fall 1023 JNDs (see Annex A).

7.1 GENERAL FORMULAS

The Grayscale Standard Display Function is defined by a mathematical interpolation of the 1023 Luminance levels derived from Barten's model. The Grayscale Standard Display Function allows us to calculate luminance, L, in candelas per square meter, as a function of the Just-Noticeable Difference (JND) Index, j:

$$\log_{10} L(j) = \frac{a + c \cdot Ln(j) + e \cdot (Ln(j))^2 + g \cdot (Ln(j))^3 + m \cdot (Ln(j))^4}{1 + b \cdot Ln(j) + d \cdot (Ln(j))^2 + f \cdot (Ln(j))^3 + h \cdot (Ln(j))^4 + k \cdot (Ln(j))^5}$$

with Ln referring to the natural logarithm, j the index (1 to 1023) of the Luminance levels L_j of the JNDs, and a = -1.3011877, b = -2.5840191E-2, c = 8.0242636E-2, d = -1.0320229E-1, e = 1.3646699E-1, f = 2.8745620E-2, g = -2.5468404E-2, h = -3.1978977E-3, k = 1.2992634E-4, m = 1.3635334E-3.

The logarithms to the base 10 of the Luminance L_j are very well interpolated by this function over the entire Luminance Range. The relative deviation of any log(Luminance)-value from the function is at most 0.3%, and the root-mean-square-error is 0.0003. The continuous representation of the Grayscale Standard Display Function permits a user to compute discrete JNDs for arbitrary start levels and over any desired Luminance Range.

Notes: 1. To apply the above formula to a device with a specific range of L values, it is convenient to also have the inverse of this relationship, which is given by:

$$j(L) = A + B \cdot Log_{10}(L) + C \cdot (Log_{10}(L))^2 + D \cdot (Log_{10}(L))^3 + E \cdot (Log_{10}(L))^4 + F \cdot (Log_{10}(L))^5 + G \cdot (Log_{10}(L))^6 + H \cdot (Log_{10}(L))^7 + I \cdot (Log_{10}(L))^8$$

where Log_{10} represents logarithm to the base 10, and A = 71.498068, B = 94.593053, C = 41.912053 D = 9.8247004, E = 0.28175407, F = -1.1878455, G = -0.18014349, H = 0.14710899, I = -0.017046845.

2. When incorporating the formulas for L(j) and j(L) into a computer program, the use of double precision is recommended.

3. Alternative methods may be used to calculate the JND Index values. One method is use a numerical algorithm such as the Van Vijngaarden-Dekker-Brent method described in *Numerical Recipes in C* (Cambridge University press, 1991). The value j may be calculated from L iteratively given the Grayscale Standard Display Function's formula for L(j). Another method would be to use the Grayscale Standard Display Function's tabulated values of j and L to calculate the j corresponding to an arbitrary L by linearly interpolating between the two nearest tabulated L, j pairs.

4. No specification is intended as to how these formulas are implemented. These could be implemented dynamically, by executing the equation directly, or through discrete values, such as a LUT, etc.

Annex B lists the Luminance levels computed with this equation for the 1023 integer JND Indices and Fig. 7-1 shows a plot of the Grayscale Standard Display Function. The exact value of the Luminance levels, of course, depends on the start level of 0.05 cd/m^2 .

The Characteristic Curve of a Display System represents the Luminance produced by a Display System as a function of DDL and the effect of ambient Illuminance. The Characteristic Curve is measured with Standard Test Patterns (see Annex D). In general, the Display Function describes, for example,

a) the Luminance (including ambient Illuminance) measured as a function of DDL for emissive displays such as a CRT-monitor/digital display controller system,

- b) the Luminance (including ambient Illuminance) as a function of DDL measured for a transmissive medium hung in front of a light-box after a printer produced an optical density, depending on DDL, on the medium,
- c) the Luminance (including ambient light) as a function of DDL measured for a diffusely reflective medium illuminated by a office lights after a printer produced a reflective density, depending on DDL, on the medium.

By internal or external means, the system may have been configured (or calibrated) such that the Characteristic Curve is consistent with the Grayscale Standard Display Function.

Some Display Systems adapt themselves to ambient light conditions. Such a system may conform to the Grayscale Standard Display Function for one level of ambient Illuminance only, unless it had the capability of adjusting its Display Function without user-intervention so that it remains in conformance with the Grayscale Standard Display Function.

7.2 TRANSMISSIVE HARDCOPY PRINTERS

For transmissive hardcopy printing, the relationship between luminance, L, and the printed optical density, D, is:

$$L = L_a + L_0 \cdot 10^{-D}$$

where:

 L_0 is the luminance of the light box with no film present, and

La is the luminance contribution due to ambient illuminance reflected off the film.

If film is to be printed with a density ranging from D_{min} to D_{max}, the final luminance will range between:

$$L_{\rm min} = L_a + L_0 \cdot 10^{-D_{\rm max}}$$
 , $L_{\rm max} = L_a + L_0 \cdot 10^{-D_{\rm min}}$

and the j values will correspondingly range from $j_{min} = j(L_{min})$ to $j_{max} = j(L_{max})$.

If this span of j values is represented by an N-bit P-Value, ranging from 0 for j_{min} to 2^N-1 for j_{max} , the j values will correspond to P-Values as follows:

$$j(p) = j_{\min} + \frac{p}{2^N - 1} \cdot (j_{\max} - j_{\min})$$

and the corresponding L values will be L(j(p)).

Finally, converting the L(j(p)) values to densities results in:

$$D(p) = -Log_{10}(\frac{L(j(p)) - L_a}{L_0}).$$

Note: Typical values for the parameters used in transmissive hardcopy printing are $L_0 = 2000 \text{ cd/m}^2$ $L_a = 10 \text{ cd/m}^2$.

7.3 REFLECTIVE HARDCOPY PRINTERS

For reflective hardcopy printing, the relationship between luminance, L, and the printed optical density, D, is:

$$L = L_0 \cdot 10^{-D}$$

where:

L₀ is the maximum luminance obtainable from diffuse reflection of the illumination that is present.

If film is to be printed with a density ranging from D_{min} to D_{max}, the final luminance will range between:

$$L_{\rm min} = L_0 \cdot 10^{-D_{\rm max}}$$
 , $L_{\rm max} = L_0 \cdot 10^{-D_{\rm min}}$

and the j values will correspondingly range from $j_{min} = j(L_{min})$ to $j_{max} = j(L_{max})$.

If this span of j values is represented by an N-bit P-Value, ranging from 0 for j_{min} to 2^N-1 for j_{max} , the j values will correspond to P-Values as follows:

$$j(p) = j_{\min} + \frac{p}{2^{N} - 1} \cdot (j_{\max} - j_{\min})$$

and the corresponding L values will be L(j(p)).

Finally, converting the L(j(p)) values to densities results in

$$D(p) = -Log_{10}(\frac{L(j(p))}{L_0}).$$

Note: Typical values for the parameters used in reflective hardcopy printing are $L_0 = 150 \text{ cd/m}^2$.

8 References

- 1) Barten, P.G.J., Physical model for the Contrast Sensitivity of the human eye. Proc. SPIE 1666, 57-72 (1992)
- 2) Barten, P.G.J., Spatio-temporal model for the Contrast Sensitivity of the human eye and its temporal aspects. Proc. SPIE 1913-01 (1993)

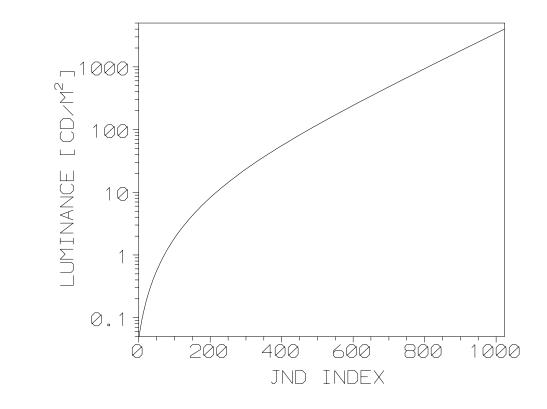


Fig. 7-1. The Grayscale Standard Display Function presented as logarithm-of-Luminance versus JND-Index

Annex A (INFORMATIVE) A DERIVATION OF THE GRAYSCALE STANDARD DISPLAY FUNCTION

A.1. RATIONALE FOR SELECTING THE GRAYSCALE STANDARD DISPLAY FUNCTION

In choosing the Grayscale Standard Display Function, it was considered mandatory to have only one continuous, monotonically behaving mathematical function for the entire Luminance Range of interest. Correspondingly, for simplicity of implementing the Grayscale Standard Display Function, it was felt to be useful to define it by only one table of data pairs. As a secondary objective, it was considered desirable that the Grayscale Standard Display Function provide similarity in grayscale rendition on Display Systems of different Luminance Range and that good use of the available DDLs of a Display System was facilitated.

Perceptual linearization was thought to be a useful concept for arriving at a Grayscale Standard Display Function for meeting the above secondary objectives; however, it is not considered an objective by itself. Apart from the fact that is probably an elusive goal to perceptually linearize all types of medical images under various viewing conditions by one mathematical function, medical images are mostly presented by application-specific Display Functions that assign contrast non-uniformly according to clinical needs.

Intuitively, one would assume that perceptually linearized images on different Display Systems will be judged to be similar. To achieve perceptual linearization, a model of the human visual system response was required and the Barten model [A1] was chosen.

Early experiments showed that an appealing degree of contrast equalization and similarity could be obtained with a Display Function derived from Bartenís model of human visual system response. The employed images were square patterns, the SMPTE pattern, and the Briggsí pattern [A2].

It was wished to relate DDLs of a Display System to some perceptually linear scale, primarily, to gain efficient utilization of the available input levels. If digitization levels lead to luminance or optical density levels that are perceptually indistinguishable, they are wasted. If they are too far apart, the observer may see contours. Hence, the concept of perceptual linearization was retained, not as a goal for the Grayscale Standard Display Function, but to obtain a concept for a measure of how well these objectives have been met.

Perceptual linearization is realizable, in a strict sense, only for rather simple images like square patterns or gratings in a uniform surrounding. Nevertheless, the concept of a perceptually linearized Display Function derived from experiments with simple test patterns has been successfully applied to complex images as described in the literature [A3-A8]. While it was clearly recognized that perceptual linearization can never be achieved for all details or spatial frequencies and object sizes at once, perceptual linearization for frequencies and object sizes near the peak of human Contrast Sensitivity seemed to do a ireasonable jobî also in complex images.

Limited (unpublished) experiments have indicated that perceptual linearization for a particular detail in a complex image with a wide Luminance Range and heterogeneous surround required Display Functions which are rather strongly bent in the dark regions of the image and that such Display Functions for a low-luminance and a high-luminance display system would not be part of a continuous, monotonic function. This experience may underly the considerations of the CIELab curve [A9] proposed by other standards groups.

Other experiments and observations with computed radiographs seemed to suggest that similarity could also be obtained between grayscale renditions on Display Systems of different Luminance when the same application-specific function is combined with log-linear Characteristic Curves of the Display Systems. Thus similarity, if not contrast equalization, could be gained by a straight, luminance-independent shape for the Display Function.

While it might have been equally sensible to choose the rather simple log-linear Display Function as a standard, this was not done for the following reason, among others.

For high-resolution Display Systems with high intrinsic video bandwidth, digitization resolution is limited to 8 or 10 bits because of technology and other constraints. The more a Grayscale Standard Display Function deviates from the Characteristic Curve of a Display System, the poorer the utilization of DDLs typically is from a perception point of view. The Characteristic Curve of CRT Display Systems has a convex curvature with respect to a log-linear straight line. It differs much less from Display Functions derived from human vision models and the concept of perceptual linearization than from a log-linear Display Function.

When using application-specific display processes which cause the resultant Display Function to deviate strongly from the Grayscale Standard Display Function, the function conceivably does not provide good similarity. In this case, other functions may yield better similarity.

In summary, a Display Function was derived from Bartenís model of the human visual system to gain a single continuous mathematical function which in its curvature falls between a log-linear response and a Display Function that may yield perceptual linearization in complex scenery with a wide luminance range within the image. Other models of human contrast sensitivity may potentially provide a better function, but were not evaluated. The notion of perceptual linearization was chosen to meet the secondary objectives of the Grayscale Standard Display Function, but not as an explicit goal of the Grayscale Standard Display Function itself. It is recognized that better functions may exist to meet these objectives. It is believed that almost any single mathematically defined Standard Function will greatly improve image presentations on Display Systems in communication networks.

A.2. DETAILS OF THE BARTEN MODEL

Barten's model considers neural noise, lateral inhibition, photon noise, external noise, limited integration capability, the optical modulation transfer function, orientation, and temporal filtering. Neuron noise represents the upper limit of Contrast Sensitivity at high spatial frequencies. Low spatial frequencies appear to be attenuated by lateral inhibition in the ganglion cells which seems to be caused by the subtraction of a spatially low-pass filtered signal from the original. Photon noise is defined by the fluctuations of the photon flux h, the pupil diameter d, and quantum detection efficiency η of the eye. At low light levels, the Contrast Sensitivity is proportional to the square-root of Luminance according to the de Vries-Rose law. The temporal integration capability in the model used here is simply represented by a time constant of T = 0.1 sec. Temporal filtering effects are not included. Next to the temporal integration capability, the eye also has limited spatial integration capability: There is a maximum angular size $X_F x$

 Y_E as well as a maximum number of cycles N_E over which the eye can integrate information in the presence of various noise sources. The optical modulation transfer function

$$M_{opt}(u) = e^{-\pi^2 \cdot \sigma^2 \cdot u^2}$$
 $\sigma = \sqrt{\sigma_0^2 + (C_{sph} \cdot d^3)^2}$ (A1)

(u, spatial frequency in c/deg) is derived from a Gaussian point-spread function including the optical properties of the eye-lens, stray light from the optical media, diffusion in the retina, and the discrete nature of the receptor elements as well as from the spherical aberration, C_{sph} , which is the main pupil-

diameter-dependent component. σ_0 is the value of σ at small pupil sizes. External noise may stem from Display System noise and image noise. Contrast sensitivity varies approximately sinusoidally with the orientation of the test pattern with equal maximum sensitivity at 0 and 90 deg and minimal sensitivity at 45 deg. The difference in Contrast Sensitivity is only present at high spatial frequencies. The effect is modeled by a variation in integration capability.

The combination of these effects yields the equation for contrast as a function of spatial frequency:

$$S(u) = \frac{1}{k} \sqrt{\frac{T}{2}} \frac{M_{opt}(u)}{\sqrt{\left(\frac{1}{\eta h I_{L}} + \frac{\Phi_{0}}{(1 - F(u))^{2}} + \Phi_{ext}(u)\right) \cdot \left(\frac{1}{X_{0}^{2}} + \frac{1}{X_{E}^{2}} + \left(\frac{u}{N_{E}}\right)^{2}\right)}}$$
(A2)

The effect of noise appears in the first parenthesis within the square-root as a noise contrast related to the variances of photon (first term), filtered neuron (second term), and external noise. The Illuminance, $I_{L} = \pi/4 \text{ d}^2 \text{L}$, of the eye is expressed in trolands [td], d is the pupil diameter in mm, and L the Luminance of the Target in cd/m². The pupil diameter is determined by the formula of de Groot and Gebhard:

$d = 4.6 - 2.8 \cdot \tanh(0.4 \cdot \log_{10}(0.625 \cdot L))$ (A3)

The term $(1 - F(u))^2 = 1 - \exp(-u^2/u_0^2)$ describes the low frequency attenuation of neuron noise due to lateral inhibition ($u_0 = 8 \text{ c/deg}$). Equation (A2) represents the simplified case of square targets, $X_0 = Y_0$ [deg]. Φ_{ext} is the contrast variance corresponding to external noise. k = 3.3, $\eta = 0.025$, $h = 357 \cdot 3600$ photons/td sec deg²; the contrast variance corresponding to the neuron noise $\Phi_0 = 3 \cdot 10^{-8} \text{ sec} \text{ deg}^2$, $X_E = 12 \text{ deg}$, $N_E = 15 \text{ cycles}$ (at 0 and 90 deg and $N_E = 7.5 \text{ cycles}$ at 45 deg for frequencies above 2 c/deg), $\sigma_0 = 0.0133 \text{ deg}$, $C_{sph} = 0.0001 \text{ deg/mm}^3$ [A1]. Equation (A2) provides a good fit of experimental data for $10^{-4} \le L \le 10^3 \text{ cd/m}^2$, $0.5 \le X_0 \le 60 \text{ deg}$, $0.2 \le u \le 50 \text{ c/deg}$.

After inserting all constants, Equation (A2) reduces to

$$S(L) = \frac{q_1 \cdot M_{opt}(L)}{\sqrt{\frac{q_2}{d^2 L} + q_3}}$$
(A4)

with $q_1 = 0.1183034375$, $q_2 = 3.962774805 \cdot 10^{-5}$, and $q_3 = 1.356243499 \cdot 10^{-7}$.

When viewed from 250 mm distance, the Standard Target has a size of about 8.7 mm x 8.7 mm and the spatial frequency of the grid equals about 0.92 line pairs per millimeter.

The Grayscale Standard Display Function is obtained by computing the Threshold Modulation \mathbf{S}_{j} as a function of mean grating Luminance and then stacking these values on top of each other. The mean Luminance of the next higher level is calculated by adding the peak-to-peak modulation to the mean Luminance \mathbf{L}_{i} of the previous level:

$$L_{j+1} = L_j. \frac{1 + S_j}{1 - S_j}$$
 (A5)

Thus, in PS 3.14, the peak-to-peak Threshold Modulation is called a just-noticeable Luminance difference.

When a Display System conforms with the Grayscale Standard Display Function, it is perceptually linearized when observing the Standard Target: If a Display System had infinitely fine digitization resolution, equal increments in P-Value would produce equally perceivable contrast steps and, under certain conditions, just-noticeable Luminance differences (displayed one at a time) for the Standard Target (the grating with sinusoidal modulation of 4 c/degree over a 2 degree x 2 degree area, embedded in a uniform background with a Luminance equal to the mean target Luminance).

The display of the Standard Target at different Luminance levels one at a time is an academic display situation. An image containing different Luminance levels with different targets and Luminance distributions at the same time is in general not perceptually linearized. It is once more emphasized that the concept of perceptual linearization of Display Systems for the Standard Target served as a logical means for deriving a continuous mathematical function and for meeting the secondary goals of the Grayscale Standard Display Function. The function may represent a compromise between perceptual linearization of complex images by strongly-bent Display Functions and gaining similarity of grayscale perception within an image on Display Systems of different Luminance by a log-linear Display Function.

The Characteristic Curve of the Display System is measured and represented by {Luminance, DDL}-pairs $L_m = F(D_m)$. A discrete transformation may be performed that maps the previously used DDLs, D_{input} , to D_{output} according to Equations (A6) and (A7) such that the available ensemble of discrete Luminance levels is used to approximate the Grayscale Standard Display Function L = G(j). The transformation is illustrated in Fig. A1. By such an operation, conformance with the Grayscale Standard Display Function may be reached.

$$D_{output} = s \cdot F^{-1}[G(j)]$$
(A6)

s is a scale factor for accommodating different input and output digitization resolutions.

The index j (which in general will be a non-integer number) of the Standard Luminance Levels is determined from the starting index j_0 of the Standard Luminance level at the minimum Luminance of the Display System (including ambient light), the number of Standard JNDs, N_{JND} , over the Luminance Range of the Display System, the digitization resolution DR, and the DDLs, D_{input} , of the Display System:

$$I = I_0 + \frac{N_{JND}}{DR} . D_{input}$$
 (A7)

A detailed example for executing such a transformation is given in Annex D.

A.3. REFERENCES

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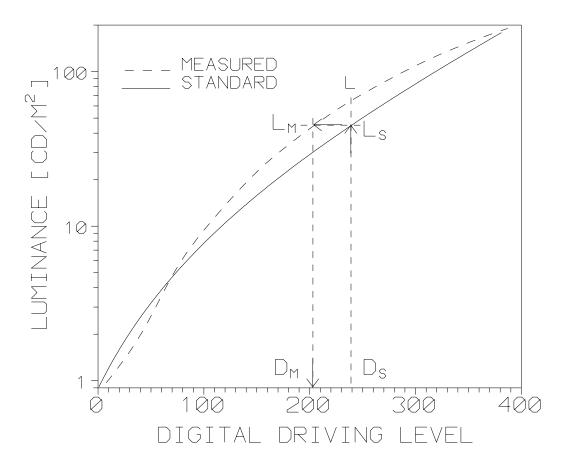


Fig. A-1. Illustration for determining the transform that changes the Characteristic Curve of a Display System to a Display Function that approximates the Grayscale Standard Display Function

Annex B (INFORMATIVE) TABLE OF THE GRAYSCALE STANDARD DISPLAY FUNCTION

The Grayscale Standard Display Function based on the Barten model was introduced in Section 7 and details are presented in Annex A above. This annex presents the Grayscale Standard Display Function as a table of values for Luminance as a function of the Just-Noticeable Difference Index for integer values of the Just-Noticeable Difference Index.

Grayscale Standard Display Function: Luminance versus JND Index										
JND	L[cd/m ²]	JND	L[cd/m ²]		JND	L[cd/m ²]		JND	L[cd/m ²]	
1	0.0500	2	0.0547		3	0.0594		4	0.0643	
5	0.0696	6	0.0750		7	0.0807		8	0.0866	
9	0.0927	10	0.0991		11	0.1056	ĺ	12	0.1124	
13	0.1194	14	0.1267		15	0.1342	ĺ	16	0.1419	
17	0.1498	18	0.1580		19	0.1664	ĺ	20	0.1750	
21	0.1839	22	0.1931		23	0.2025		24	0.2121	
25	0.2220	26	0.2321		27	0.2425	ĺ	28	0.2532	
29	0.2641	30	0.2752		31	0.2867	ĺ	32	0.2984	
33	0.3104	34	0.3226		35	0.3351	ĺ	36	0.3479	
37	0.3610	38	0.3744		39	0.3880	ĺ	40	0.4019	
41	0.4161	42	0.4306		43	0.4454	Ì	44	0.4605	
45	0.4759	46	0.4916		47	0.5076	ĺ	48	0.5239	
49	0.5405	50	0.5574		51	0.5746	ĺ	52	0.5921	
53	0.6100	54	0.6281		55	0.6466	ĺ	56	0.6654	
57	0.6846	58	0.7040		59	0.7238	ĺ	60	0.7440	
61	0.7644	62	0.7852		63	0.8064	ĺ	64	0.8278	
65	0.8497	66	0.8718		67	0.8944	ĺ	68	0.9172	
69	0.9405	70	0.9640		71	0.9880	ĺ	72	1.0123	
73	1.0370	74	1.0620		75	1.0874	ĺ	76	1.1132	
77	1.1394	78	1.1659		79	1.1928	Ì	80	1.2201	
81	1.2478	82	1.2759		83	1.3044	ĺ	84	1.3332	
85	1.3625	86	1.3921		87	1.4222	Ì	88	1.4527	
89	1.4835	90	1.5148		91	1.5465	Ì	92	1.5786	
93	1.6111	94	1.6441		95	1.6775		96	1.7113	
97	1.7455	98	1.7802		99	1.8153	ĺ	100	1.8508	
101	1.8868	102	1.9233		103	1.9601	ĺ	104	1.9975	
105	2.0352	106	2.0735		107	2.1122	ĺ	108	2.1514	
109	2.1910	110	2.2311		111	2.2717	ĺ	112	2.3127	
113	2.3543	114	2.3963		115	2.4388		116	2.4817	

 Table B-1

 Grayscale Standard Display Function: Luminance versus JND Index

· · · - ·							
117	2.5252	118	2.5692	119	2.6137	120	2.6587
121	2.7041	122	2.7501	123	2.7966	124	2.8436
125	2.8912	126	2.9392	127	2.9878	128	3.0369
129	3.0866	130	3.1367	131	3.1875	132	3.2387
133	3.2905	134	3.3429	135	3.3958	136	3.4493
137	3.5033	138	3.5579	139	3.6131	140	3.6688
141	3.7252	142	3.7820	143	3.8395	144	3.8976
145	3.9563	146	4.0155	147	4.0754	148	4.1358
149	4.1969	150	4.2586	151	4.3209	152	4.3838
153	4.4473	154	4.5115	155	4.5763	156	4.6417
157	4.7078	158	4.7745	159	4.8419	160	4.9099
161	4.9785	162	5.0479	163	5.1179	164	5.1886
165	5.2599	166	5.3319	167	5.4046	168	5.4780
169	5.5521	170	5.6269	171	5.7024	172	5.7786
173	5.8555	174	5.9331	175	6.0114	176	6.0905
177	6.1702	178	6.2508	179	6.3320	180	6.4140
181	6.4968	182	6.5803	183	6.6645	184	6.7496
185	6.8354	186	6.9219	187	7.0093	188	7.0974
189	7.1863	190	7.2760	191	7.3665	192	7.4578
193	7.5500	194	7.6429	195	7.7366	196	7.8312
197	7.9266	198	8.0229	199	8.1199	200	8.2179
201	8.3167	202	8.4163	203	8.5168	204	8.6182
205	8.7204	206	8.8235	207	8.9275	208	9.0324
209	9.1382	210	9.2449	211	9.3525	212	9.4611
213	9.5705	214	9.6809	215	9.7922	216	9.9044
217	10.0176	218	10.1318	219	10.2469	220	10.3629
221	10.4800	222	10.5980	223	10.7169	224	10.8369
225	10.9579	226	11.0799	227	11.2028	228	11.3268
229	11.4518	230	11.5779	231	11.7050	232	11.8331
233	11.9622	234	12.0925	235	12.2237	236	12.3561
237	12.4895	238	12.6240	239	12.7596	240	12.8963
241	13.0341	242	13.1730	243	13.3130	244	13.4542
245	13.5965	246	13.7399	247	13.8844	248	14.0302
249	14.1770	250	14.3251	251	14.4743	252	14.6247
253	14.7763	254	14.9291	255	15.0831	256	15.2384
257	15.3948	258	15.5525	259	15.7114	260	15.8716
261	16.0330	262	16.1957	263	16.3596	264	16.5249
265	16.6914	266	16.8592	267	17.0283	268	17.1987
269		070	17.5436	271	17.7180	272	17.8938
209	17.3705	270	17.5450	2/1	17.7100	212	17.0000

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301 23.6179 302 23.8383 303 24.0605 304 24.2842 305 24.5097 306 24.7368 307 24.9656 308 25.1961 309 25.4283 310 25.6622 311 25.8979 312 26.1353 313 26.3744 314 26.6153 315 26.8580 316 27.1025 317 27.3488 318 27.5969 319 27.8468 320 28.0985 321 28.3521 322 28.6075 323 28.8648 324 29.1240 325 29.3851 326 29.6481 327 29.9130 328 30.1793 333 31.5434 334 31.820 335 32.1027 336 32.3854 337 32.6702 338 32.9570 339 33.2459 340 33.5369 341 33.800 342 34.1251 343 34.4224 344 34.7219	293	21.9120	294	22.1197	295	22.3289	296	22.5398
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$\begin{array}{ c c c c c c c c c c c c c c c c c c c$	301	23.6179	302	23.8383	303	24.0605	304	24.2842
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317 27.3488 318 27.5969 319 27.8468 320 28.0985 321 28.3521 322 28.6075 323 28.8648 324 29.1240 325 29.3851 326 29.6481 327 29.9130 328 30.1798 329 30.4486 330 30.7193 331 30.9920 332 31.2667 333 31.5434 334 31.8220 335 32.1027 336 32.3854 337 32.6702 338 32.9570 339 33.2459 340 33.5369 341 33.8300 342 34.1251 343 34.4224 344 34.7219 345 35.0235 346 35.3272 347 35.6332 348 35.9413 349 36.2516 350 36.542 351 36.8790 352 37.1960 357 38.8155 358 39.1463 359 39.4795 360 39.8151	309	25.4283	310	25.6622	311	25.8979	312	26.1353
321 28.3521 322 28.6075 323 28.8648 324 29.1240 325 29.3851 326 29.6481 327 29.9130 328 30.1798 329 30.4486 330 30.7193 331 30.9920 332 31.2667 333 31.5434 334 31.8220 335 32.1027 336 32.3854 337 32.6702 338 32.9570 339 33.2459 340 33.5369 341 33.8300 342 34.1251 343 34.4224 344 34.7219 345 35.0235 346 35.3272 347 35.6332 348 35.9413 349 36.2516 350 36.642 351 36.8790 352 37.1960 357 38.8155 358 39.1463 359 39.4795 360 39.8151 361 40.1530 362 40.4933 363 40.8361 364 41.1813	313	26.3744	314	26.6153	315	26.8580	316	27.1025
32529.385132629.648132729.913032830.179832930.448633030.719333130.992033231.266733331.543433431.822033532.102733632.385433732.670233832.957033933.245934033.536934133.830034234.125134334.422434434.721934535.023534635.327234735.633234835.941334936.251635036.564235136.879035237.196035337.515335437.836935538.160835638.487035738.815535839.146335939.479536039.815136140.153036240.493336340.836136441.181336541.528936641.879036742.231636842.586636942.944237043.304337143.666937244.032137344.399837444.770237545.143137645.518738147.436538247.828138348.222538448.619738549.019638649.422438749.827938850.236339352.321339452.747039553.175739653.607439754.042139854.479839954.9205400 <t< td=""><td>317</td><td>27.3488</td><td>318</td><td>27.5969</td><td>319</td><td>27.8468</td><td>320</td><td>28.0985</td></t<>	317	27.3488	318	27.5969	319	27.8468	320	28.0985
32930.448633030.719333130.992033231.266733331.543433431.822033532.102733632.385433732.670233832.957033933.245934033.536934133.830034234.125134334.422434434.721934535.023534635.327234735.633234835.941334936.251635036.564235136.879035237.196035337.515335437.836935538.160835638.487035738.815535839.146335939.479536039.815136140.153036240.493336340.836136441.181336541.528936641.879036742.231636842.586636942.944237043.304337143.666937244.032137344.399837444.770237545.143137645.518738147.436538247.828138348.222538448.619738549.019638649.422438749.827938850.236339352.321339452.747039553.175739653.607439754.042139854.479839954.920540055.364340155.811240256.261140356.7142404 <t< td=""><td>321</td><td>28.3521</td><td>322</td><td>28.6075</td><td>323</td><td>28.8648</td><td>324</td><td>29.1240</td></t<>	321	28.3521	322	28.6075	323	28.8648	324	29.1240
33331.543433431.822033532.102733632.385433732.670233832.957033933.245934033.536934133.830034234.125134334.422434434.721934535.023534635.327234735.633234835.941334936.251635036.664235136.879035237.196035337.515335437.836935538.160835638.487035738.815535839.146335939.479536039.815136140.153036240.493336340.836136441.181336541.528936641.879036742.231636842.586636942.944237043.304337143.666937244.032137344.399837444.770237545.143137645.518737745.896937846.277837946.661338047.047538147.436538247.828138348.222538448.619738549.019638649.422438749.827938850.236339352.321339452.747039553.175739653.607439754.042139854.479839954.920540055.364340155.811240256.261140356.7142404 <t< td=""><td>325</td><td>29.3851</td><td>326</td><td>29.6481</td><td>327</td><td>29.9130</td><td>328</td><td>30.1798</td></t<>	325	29.3851	326	29.6481	327	29.9130	328	30.1798
33732.670233832.957033933.245934033.536934133.830034234.125134334.422434434.721934535.023534635.327234735.633234835.941334936.251635036.664235136.879035237.196035337.515335437.836935538.160835638.487035738.815535839.146335939.479536039.815136140.153036240.493336340.836136441.181336541.528936641.879036742.231636842.586636942.944237043.304337143.666937244.032137344.399837444.770237545.143137645.518737745.896937846.277837946.661338047.047538147.436538247.828138348.222538448.619738549.019638649.422438749.827938850.236339352.321339452.747039553.175739653.607439754.042139854.479839954.920540055.364340155.811240256.261140356.714240457.170440557.629840658.092340758.5581408 <t< td=""><td>329</td><td>30.4486</td><td>330</td><td>30.7193</td><td>331</td><td>30.9920</td><td>332</td><td>31.2667</td></t<>	329	30.4486	330	30.7193	331	30.9920	332	31.2667
34133.830034234.125134334.422434434.721934535.023534635.327234735.633234835.941334936.251635036.564235136.879035237.196035337.515335437.836935538.160835638.487035738.815535839.146335939.479536039.815136140.153036240.493336340.836136441.181336541.528936641.879036742.231636842.586636942.944237043.304337143.666937244.032137344.399837444.770237545.143137645.518737745.896937846.277837946.661338047.047538147.436538247.828138348.222538448.619738549.019638649.422438749.827938850.236339352.321339452.747039553.175739653.607439754.042139854.479839954.920540055.364340155.811240256.261140356.714240457.170440557.629840658.092340758.558140859.027040959.499241059.974741160.4534412 <t< td=""><td>333</td><td>31.5434</td><td>334</td><td>31.8220</td><td>335</td><td>32.1027</td><td>336</td><td>32.3854</td></t<>	333	31.5434	334	31.8220	335	32.1027	336	32.3854
34535.023534635.327234735.633234835.941334936.251635036.564235136.879035237.196035337.515335437.836935538.160835638.487035738.815535839.146335939.479536039.815136140.153036240.493336340.836136441.181336541.528936641.879036742.231636842.586636942.944237043.304337143.666937244.032137344.399837444.770237545.143137645.518737745.896937846.277837946.661338047.047538147.436538247.828138348.222538448.619738549.019638649.422438749.827938850.236339352.321339452.747039553.175739653.607439754.042139854.479839954.920540055.364340155.811240256.261140356.714240457.170440557.629840658.092340758.558140859.027040959.499241059.974741160.453441260.935441361.420841863.898041964.4037420 <t< td=""><td>337</td><td>32.6702</td><td>338</td><td>32.9570</td><td>339</td><td>33.2459</td><td>340</td><td>33.5369</td></t<>	337	32.6702	338	32.9570	339	33.2459	340	33.5369
34936.251635036.564235136.879035237.196035337.515335437.836935538.160835638.487035738.815535839.146335939.479536039.815136140.153036240.493336340.836136441.181336541.528936641.879036742.231636842.586636942.944237043.304337143.666937244.032137344.399837444.770237545.143137645.518737745.896937846.277837946.661338047.047538147.436538247.828138348.222538448.619738549.019638649.422438749.827938850.236339352.321339452.747039553.175739653.607439754.042139854.479839954.920540055.364340155.811240256.261140356.714240457.170440557.629840658.092340758.558140859.027040959.499241059.974741160.453441260.935441361.420841863.898041964.403742064.912942165.425642265.941842366.4615424 <t< td=""><td>341</td><td>33.8300</td><td>342</td><td>34.1251</td><td>343</td><td>34.4224</td><td>344</td><td>34.7219</td></t<>	341	33.8300	342	34.1251	343	34.4224	344	34.7219
35337.515335437.836935538.160835638.487035738.815535839.146335939.479536039.815136140.153036240.493336340.836136441.181336541.528936641.879036742.231636842.586636942.944237043.304337143.666937244.032137344.399837444.770237545.143137645.518737745.896937846.277837946.661338047.047538147.436538247.828138348.222538448.619738549.019638649.422438749.827938850.236339352.321339452.747039553.175739653.607439754.042139854.479839954.920540055.364340155.811240256.261140356.714240457.170440557.629840658.092340758.558140859.027040959.499241059.974741160.453441260.935441361.420841461.909441562.401541662.896941763.395841863.898041964.403742064.912942165.425642265.941842366.4615424 <t< td=""><td>345</td><td>35.0235</td><td>346</td><td>35.3272</td><td>347</td><td>35.6332</td><td>348</td><td>35.9413</td></t<>	345	35.0235	346	35.3272	347	35.6332	348	35.9413
35738.815535839.146335939.479536039.815136140.153036240.493336340.836136441.181336541.528936641.879036742.231636842.586636942.944237043.304337143.666937244.032137344.399837444.770237545.143137645.518737745.896937846.277837946.661338047.047538147.436538247.828138348.222538448.619738549.019638649.422438749.827938850.236338950.647539051.061639151.478639251.898539352.321339452.747039553.175739653.607439754.042139854.479839954.920540055.364340155.811240256.261140356.714240457.170440557.629840658.092340758.558140859.027040959.499241059.974741160.453441260.935441361.420841461.909441562.401541662.896941763.395841863.898041964.403742064.912942165.425642265.941842366.4615424 <t< td=""><td>349</td><td>36.2516</td><td>350</td><td>36.5642</td><td>351</td><td>36.8790</td><td>352</td><td>37.1960</td></t<>	349	36.2516	350	36.5642	351	36.8790	352	37.1960
36140.153036240.493336340.836136441.181336541.528936641.879036742.231636842.586636942.944237043.304337143.666937244.032137344.399837444.770237545.143137645.518737745.896937846.277837946.661338047.047538147.436538247.828138348.222538448.619738549.019638649.422438749.827938850.236338950.647539051.061639151.478639251.898539352.321339452.747039553.175739653.607439754.042139854.479839954.920540055.364340155.811240256.261140356.714240457.170440557.629840658.092340758.558140859.027040959.499241059.974741160.453441260.935441361.420841863.898041964.403742064.912942165.425642265.941842366.461542466.984842567.511742668.042242768.576342869.114042969.655543070.200643170.7495432 <t< td=""><td>353</td><td>37.5153</td><td>354</td><td>37.8369</td><td>355</td><td>38.1608</td><td>356</td><td>38.4870</td></t<>	353	37.5153	354	37.8369	355	38.1608	356	38.4870
36541.528936641.879036742.231636842.586636942.944237043.304337143.666937244.032137344.399837444.770237545.143137645.518737745.896937846.277837946.661338047.047538147.436538247.828138348.222538448.619738549.019638649.422438749.827938850.236338950.647539051.061639151.478639251.898539352.321339452.747039553.175739653.607439754.042139854.479839954.920540055.364340155.811240256.261140356.714240457.170440557.629840658.092340758.558140859.027040959.499241059.974741160.453441260.935441361.420841863.898041964.403742064.912942165.425642265.941842366.461542466.984842567.511742668.042242768.576342869.114042969.655543070.200643170.749543271.3021	357	38.8155	358	39.1463	359	39.4795	360	39.8151
36942.944237043.304337143.666937244.032137344.399837444.770237545.143137645.518737745.896937846.277837946.661338047.047538147.436538247.828138348.222538448.619738549.019638649.422438749.827938850.236338950.647539051.061639151.478639251.898539352.321339452.747039553.175739653.607439754.042139854.479839954.920540055.364340155.811240256.261140356.714240457.170440557.629840658.092340758.558140859.027040959.499241059.974741160.453441260.935441361.420841461.909441562.401541662.896941763.395841863.898041964.403742064.912942165.425642265.941842366.461542466.984842567.511742668.042242768.576343271.302142969.655543070.200643170.749543271.3021	361	40.1530	362	40.4933	363	40.8361	364	41.1813
37344.399837444.770237545.143137645.518737745.896937846.277837946.661338047.047538147.436538247.828138348.222538448.619738549.019638649.422438749.827938850.236338950.647539051.061639151.478639251.898539352.321339452.747039553.175739653.607439754.042139854.479839954.920540055.364340155.811240256.261140356.714240457.170440557.629840658.092340758.558140859.027040959.499241059.974741160.453441260.935441361.420841461.909441562.401541662.896941763.395841863.898041964.403742064.912942165.425642265.941842366.461542466.984842567.511742668.042242768.576342869.114042969.655543070.200643170.749543271.3021	365	41.5289	366	41.8790	367	42.2316	368	42.5866
37745.896937846.277837946.661338047.047538147.436538247.828138348.222538448.619738549.019638649.422438749.827938850.236338950.647539051.061639151.478639251.898539352.321339452.747039553.175739653.607439754.042139854.479839954.920540055.364340155.811240256.261140356.714240457.170440557.629840658.092340758.558140859.027040959.499241059.974741160.453441260.935441361.420841461.909441562.401541662.896941763.395841863.898041964.403742064.912942165.425642265.941842366.461542466.984842567.511742668.042242768.576342869.114042969.655543070.200643170.749543271.3021	369	42.9442	370	43.3043	371	43.6669	372	44.0321
38147.436538247.828138348.222538448.619738549.019638649.422438749.827938850.236338950.647539051.061639151.478639251.898539352.321339452.747039553.175739653.607439754.042139854.479839954.920540055.364340155.811240256.261140356.714240457.170440557.629840658.092340758.558140859.027040959.499241059.974741160.453441260.935441361.420841461.909441562.401541662.896941763.395841863.898041964.403742064.912942165.425642265.941842366.461542466.984842567.511742668.042242768.576342869.114042969.655543070.200643170.749543271.3021	373	44.3998	374	44.7702	375	45.1431	376	45.5187
38549.019638649.422438749.827938850.236338950.647539051.061639151.478639251.898539352.321339452.747039553.175739653.607439754.042139854.479839954.920540055.364340155.811240256.261140356.714240457.170440557.629840658.092340758.558140859.027040959.499241059.974741160.453441260.935441361.420841461.909441562.401541662.896941763.395841863.898041964.403742064.912942165.425642265.941842366.461542466.984842567.511742668.042242768.576342869.114042969.655543070.200643170.749543271.3021	377	45.8969	378	46.2778	379	46.6613	380	47.0475
38950.647539051.061639151.478639251.898539352.321339452.747039553.175739653.607439754.042139854.479839954.920540055.364340155.811240256.261140356.714240457.170440557.629840658.092340758.558140859.027040959.499241059.974741160.453441260.935441361.420841461.909441562.401541662.896941763.395841863.898041964.403742064.912942165.425642265.941842366.461542466.984842567.511742668.042242768.576342869.114042969.655543070.200643170.749543271.3021	381	47.4365	382	47.8281	383	48.2225	384	48.6197
39352.321339452.747039553.175739653.607439754.042139854.479839954.920540055.364340155.811240256.261140356.714240457.170440557.629840658.092340758.558140859.027040959.499241059.974741160.453441260.935441361.420841461.909441562.401541662.896941763.395841863.898041964.403742064.912942165.425642265.941842366.461542466.984842567.511742668.042242768.576342869.114042969.655543070.200643170.749543271.3021	385	49.0196	386	49.4224	387	49.8279	388	50.2363
39754.042139854.479839954.920540055.364340155.811240256.261140356.714240457.170440557.629840658.092340758.558140859.027040959.499241059.974741160.453441260.935441361.420841461.909441562.401541662.896941763.395841863.898041964.403742064.912942165.425642265.941842366.461542466.984842567.511742668.042242768.576342869.114042969.655543070.200643170.749543271.3021	389	50.6475	390	51.0616	391	51.4786	392	51.8985
40155.811240256.261140356.714240457.170440557.629840658.092340758.558140859.027040959.499241059.974741160.453441260.935441361.420841461.909441562.401541662.896941763.395841863.898041964.403742064.912942165.425642265.941842366.461542466.984842567.511742668.042242768.576342869.114042969.655543070.200643170.749543271.3021	393	52.3213	394	52.7470	395	53.1757	396	53.6074
40155.811240256.261140356.714240457.170440557.629840658.092340758.558140859.027040959.499241059.974741160.453441260.935441361.420841461.909441562.401541662.896941763.395841863.898041964.403742064.912942165.425642265.941842366.461542466.984842567.511742668.042242768.576342869.114042969.655543070.200643170.749543271.3021	397	54.0421	398	54.4798	399	54.9205	400	55.3643
40557.629840658.092340758.558140859.027040959.499241059.974741160.453441260.935441361.420841461.909441562.401541662.896941763.395841863.898041964.403742064.912942165.425642265.941842366.461542466.984842567.511742668.042242768.576342869.114042969.655543070.200643170.749543271.3021								
40959.499241059.974741160.453441260.935441361.420841461.909441562.401541662.896941763.395841863.898041964.403742064.912942165.425642265.941842366.461542466.984842567.511742668.042242768.576342869.114042969.655543070.200643170.749543271.3021	-				407		408	
41361.420841461.909441562.401541662.896941763.395841863.898041964.403742064.912942165.425642265.941842366.461542466.984842567.511742668.042242768.576342869.114042969.655543070.200643170.749543271.3021							412	
41763.395841863.898041964.403742064.912942165.425642265.941842366.461542466.984842567.511742668.042242768.576342869.114042969.655543070.200643170.749543271.3021	413							
42165.425642265.941842366.461542466.984842567.511742668.042242768.576342869.114042969.655543070.200643170.749543271.3021	-							
42567.511742668.042242768.576342869.114042969.655543070.200643170.749543271.3021	-							
429 69.6555 430 70.2006 431 70.7495 432 71.3021								
	433	71.8585	434	72.4187	435	72.9827	436	73.5505

	1		1	1	1	r	1
437	74.1222	438	74.6978	439	75.2773	440	75.8608
441	76.4482	442	77.0396	443	77.6351	444	78.2346
445	78.8381	446	79.4458	447	80.0576	448	80.6735
449	81.2936	450	81.9179	451	82.5464	452	83.1792
453	83.8163	454	84.4577	455	85.1034	456	85.7535
457	86.4079	458	87.0668	459	87.7302	460	88.3980
461	89.0703	462	89.7472	463	90.4286	464	91.1147
465	91.8053	466	92.5006	467	93.2006	468	93.9053
469	94.6147	470	95.3289	471	96.0480	472	96.7718
473	97.5005	474	98.2341	475	98.9726	476	99.7161
477	100.4646	478	101.2181	479	101.9767	480	102.7403
481	103.5091	482	104.2830	483	105.0621	484	105.8464
485	106.6359	486	107.4308	487	108.2309	488	109.0364
489	109.8473	490	110.6637	491	111.4854	492	112.3127
493	113.1455	494	113.9838	495	114.8278	496	115.6773
497	116.5326	498	117.3935	499	118.2602	500	119.1326
501	120.0109	502	120.8950	503	121.7850	504	122.6809
505	123.5828	506	124.4907	507	125.4047	508	126.3247
509	127.2508	510	128.1831	511	129.1215	512	130.0662
513	131.0172	514	131.9745	515	132.9381	516	133.9082
517	134.8847	518	135.8676	519	136.8571	520	137.8531
521	138.8557	522	139.8650	523	140.8810	524	141.9037
525	142.9331	526	143.9694	527	145.0125	528	146.0625
529	147.1195	530	148.1835	531	149.2545	532	150.3326
533	151.4178	534	152.5101	535	153.6097	536	154.7166
537	155.8307	538	156.9523	539	158.0812	540	159.2175
541	160.3614	542	161.5128	543	162.6718	544	163.8384
545	165.0128	546	166.1948	547	167.3847	548	168.5824
549	169.7880	550	171.0015	551	172.2230	552	173.4526
553	174.6902	554	175.9360	555	177.1900	556	178.4522
557	179.7227	558	181.0016	559	182.2889	560	183.5846
561	184.8889	562	186.2017	563	187.5232	564	188.8533
565	190.1921	566	191.5398	567	192.8963	568	194.2617
569	195.6360	570	197.0194	571	198.4119	572	199.8134
573	201.2242	574	202.6442	575	204.0735	576	205.5122
577	206.9603	578	208.4179	579	209.8851	580	211.3618
581	212.8482	582	214.3444	583	215.8503	584	217.3661
585	218.8919	586	220.4276	587	221.9733	588	223.5292
589	225.0952	590	226.6715	591	228.2581	592	229.8550
593	231.4624	594	233.0803	595	234.7088	596	236.3479

601 2 605 2	237.9977 244.7054	598	239.6583	599	241.3297	600	243.0120
605 2	244.7054						
		602	246.4097	603	248.1252	604	249.8519
609 2	251.5899	606	253.3392	607	255.0999	608	256.8721
	258.6559	610	260.4512	611	262.2583	612	264.0772
613 2	265.9079	614	267.7506	615	269.6052	616	271.4720
617 2	273.3509	618	275.2420	619	277.1455	620	279.0614
621 2	280.9897	622	282.9306	623	284.8841	624	286.8504
625 2	288.8294	626	290.8213	627	292.8262	628	294.8442
629 2	296.8752	630	298.9195	631	300.9770	632	303.0480
633 3	305.1324	634	307.2304	635	309.3420	636	311.4673
637 3	313.6065	638	315.7595	639	317.9266	640	320.1077
641 3	322.3030	642	324.5126	643	326.7365	644	328.9749
645 3	331.2278	646	333.4953	647	335.7776	648	338.0747
649 3	340.3867	650	342.7137	651	345.0558	652	347.4131
653 3	349.7858	654	352.1738	655	354.5773	656	356.9964
657 3	359.4312	658	361.8818	659	364.3483	660	366.8308
661 3	369.3294	662	371.8442	663	374.3754	664	376.9229
665 3	379.4869	666	382.0676	667	384.6650	668	387.2793
669 3	389.9105	670	392.5587	671	395.2241	672	397.9068
673 4	400.6069	674	403.3245	675	406.0596	676	408.8125
677 4	411.5833	678	414.3719	679	417.1787	680	420.0036
681 4	122.8468	682	425.7085	683	428.5886	684	431.4875
685 4	434.4051	686	437.3415	687	440.2970	688	443.2717
689 4	446.2655	690	449.2788	691	452.3116	692	455.3640
693 4	458.4361	694	461.5282	695	464.6402	696	467.7724
697 4	470.9249	698	474.0977	699	477.2911	700	480.5052
701 4	483.7400	702	486.9958	703	490.2726	704	493.5706
705 4	496.8900	706	500.2308	707	503.5932	708	506.9774
709 5	510.3835	710	513.8116	711	517.2619	712	520.7344
713 5	524.2294	714	527.7471	715	531.2874	716	534.8507
717 5	538.4370	718	542.0465	719	545.6793	720	549.3356
721 5	553.0155	722	556.7192	723	560.4469	724	564.1986
725 5	567.9746	726	571.7750	727	575.6000	728	579.4497
729 5	583.3242	730	587.2238	731	591.1486	732	595.0988
733 5	599.0744	734	603.0758	735	607.1030	736	611.1563
737 6	615.2357	738	619.3415	739	623.4738	740	627.6328
741 6	631.8187	742	636.0316	743	640.2717	744	644.5392
745 6	648.8343	746	653.1571	747	657.5079	748	661.8867
749 6	66.2939	750	670.7295	751	675.1937	752	679.6868
753 6	684.2089	754	688.7602	755	693.3409	756	697.9512

				-			
757	702.5913	758	707.2613	759	711.9615	760	716.6921
761	721.4531	762	726.2450	763	731.0678	764	735.9217
765	740.8070	766	745.7238	767	750.6723	768	755.6529
769	760.6655	770	765.7106	771	770.7882	772	775.8986
773	781.0420	774	786.2187	775	791.4287	776	796.6724
777	801.9500	778	807.2616	779	812.6075	780	817.9880
781	823.4031	782	828.8533	783	834.3386	784	839.8594
785	845.4158	786	851.0081	787	856.6365	788	862.3012
789	868.0025	790	873.7407	791	879.5158	792	885.3283
793	891.1783	794	897.0661	795	902.9919	796	908.9559
797	914.9585	798	920.9998	799	927.0801	800	933.1997
801	939.3588	802	945.5577	803	951.7966	804	958.0758
805	964.3956	806	970.7561	807	977.1578	808	983.6008
809	990.0853	810	996.6118	811	1003.1800	812	1009.7910
813	1016.4450	814	1023.1420	815	1029.8820	816	1036.6650
817	1043.4930	818	1050.3640	819	1057.2800	820	1064.2400
821	1071.2460	822	1078.2960	823	1085.3920	824	1092.5340
825	1099.7220	826	1106.9570	827	1114.2380	828	1121.5670
829	1128.9420	830	1136.3660	831	1143.8370	832	1151.3570
833	1158.9250	834	1166.5420	835	1174.2080	836	1181.9240
837	1189.6890	838	1197.5050	839	1205.3710	840	1213.2890
841	1221.2570	842	1229.2770	843	1237.3480	844	1245.4720
845	1253.6480	846	1261.8770	847	1270.1600	848	1278.4950
849	1286.8850	850	1295.3290	851	1303.8270	852	1312.3810
853	1320.9900	854	1329.6540	855	1338.3740	856	1347.1510
857	1355.9840	858	1364.8750	859	1373.8230	860	1382.8290
861	1391.8930	862	1401.0160	863	1410.1970	864	1419.4380
865	1428.7390	866	1438.1000	867	1447.5220	868	1457.0040
869	1466.5480	870	1476.1530	871	1485.8210	872	1495.5510
873	1505.3440	874	1515.2010	875	1525.1210	876	1535.1050
877	1545.1540	878	1555.2680	879	1565.4470	880	1575.6930
881	1586.0040	882	1596.3820	883	1606.8280	884	1617.3410
885	1627.9220	886	1638.5710	887	1649.2900	888	1660.0780
889	1670.9350	890	1681.8630	891	1692.8620	892	1703.9310
893	1715.0730	894	1726.2860	895	1737.5730	896	1748.9320
897	1760.3650	898	1771.8720	899	1783.4530	900	1795.1090
901	1806.8410	902	1818.6490	903	1830.5330	904	1842.4940
905	1854.5330	906	1866.6500	907	1878.8450	908	1891.1190
909	1903.4730	910	1915.9060	911	1928.4200	912	1941.0160
913	1953.6930	914	1966.4520	915	1979.2940	916	1992.2190

917	2005.2270	91	8	2018.3200	919	2031.4980	920	2044.7620
921	2058.1110	92	2	2071.5470	923	2085.0700	924	2098.6800
925	2112.3790	92	6	2126.1670	927	2140.0440	928	2154.0110
929	2168.0690	93	0	2182.2170	931	2196.4580	932	2210.7910
933	2225.2170	93	4	2239.7360	935	2254.3500	936	2269.0580
937	2283.8620	93	8	2298.7620	939	2313.7590	940	2328.8530
941	2344.0450	94	2	2359.3350	943	2374.7250	944	2390.2140
945	2405.8040	94	6	2421.4960	947	2437.2890	948	2453.1850
949	2469.1840	95	0	2485.2860	951	2501.4940	952	2517.8060
953	2534.2250	95	4	2550.7500	955	2567.3820	956	2584.1230
957	2600.9720	95	8	2617.9310	959	2634.9990	960	2652.1790
961	2669.4710	96	2	2686.8740	963	2704.3910	964	2722.0220
965	2739.7670	96	6	2757.6270	967	2775.6040	968	2793.6970
969	2811.9080	97	0	2830.2380	971	2848.6870	972	2867.2550
973	2885.9440	97	4	2904.7550	975	2923.6880	976	2942.7450
977	2961.9250	97	8	2981.2300	979	3000.6600	980	3020.2170
981	3039.9020	98	2	3059.7140	983	3079.6550	984	3099.7260
985	3119.9270	98	6	3140.2600	987	3160.7260	988	3181.3240
989	3202.0570	99	0	3222.9240	991	3243.9280	992	3265.0680
993	3286.3460	99	4	3307.7620	995	3329.3180	996	3351.0140
997	3372.8520	99	8	3394.8310	999	3416.9540	1000	3439.2210
1001	3461.6330	100)2	3484.1910	1003	3506.8970	1004	3529.7500
1005	3552.7520	100)6	3575.9030	1007	3599.2060	1008	3622.6610
1009	3646.2680	10	10	3670.0300	1011	3693.9460	1012	3718.0180
1013	3742.2480	10	14	3766.6350	1015	3791.1810	1016	3815.8880
1017	3840.7550	10	18	3865.7850	1019	3890.9780	1020	3916.3350
1021	3941.8580	102	22	3967.5470	1023	3993.4040		

Annex C (INFORMATIVE) MEASURING THE ACCURACY WITH WHICH A DISPLAY SYSTEM MATCHES THE GRAYSCALE STANDARD DISPLAY FUNCTION

C.1. GENERAL CONSIDERATIONS REGARDING CONFORMANCE AND METRICS

To demonstrate conformance with the Grayscale Standard Display Function is a much more complex task than, for example, validating the responses of a totally digital system to DICOM messages.

Display systems ultimately produce analog output, either directly as Luminances or indirectly as optical densities. For some Display Systems, this analog output can be affected by various imperfections in addition to whatever imperfections exist in the Display System's Display Function which is to be validated. For example, there may be spatial non-uniformities in the final presented image (e.g., arising from film, printing, or processing non-uniformities in the case of a hardcopy printer) which are measurable but are at low spatial frequencies which do not ordinarily pose an image quality problem in diagnostic radiology.

It is worth noting that CRTs and light-boxes also introduce their own spatial non-uniformities. These nonuniformities are outside the scope of the Grayscale Standard Display Function and the measurement procedures described here. But because of them, even a test image which is perfectly presented in terms of the Grayscale Standard Display Function will be less than perfectly perceived on a real CRT or a real light-box.

Furthermore, the question "How close (to the Grayscale Standard Display Function) is close enough?" is currently unanswered, since the answer depends on psychophysical studies not yet done to determine what difference in Display Function is "just noticeable" when two nearly identical image presentations (e.g., two nearly identical films placed on equivalent side-by-side light-boxes) are presented to an observer.

Furthermore, the evaluation of a given Display System could be based either on visual tests (e.g., assessing the perceived contrast of many low-contrast targets in one or more test images) or by quantitative analysis based on measured data obtained from instruments (e.g., photometers or densitometers).

Even the quantitative approach could be addressed in different ways. One could, for example, simply superimpose plots of measured and theoretical analog output (i.e., Luminance or optical density) vs. P-Value, perhaps along with "error bars" indicating the expected uncertainty (non-repeatable variations) in the measured output. As a mathematically more elegant alternative, all the measured data points could be used as input to a statistical mathematical analysis which could attempt to determine the underlying Display Function of the Display System, yielding one or more quantitative values (metrics) which define how well the Display System conforms with the Grayscale Standard Display Function.

In what follows in this and the following annexes, an example of the latter type of metric analysis is used, in which measured data is analyzed using a "FIT" test which is intended to validate the shape of the Characteristic Curve and a "LUM" test which is intended to show the degree of scatter from the ideal Grayscale Standard Display Function. This approach has been applied, for example, to quantitatively demonstrate how improvements were successfully made to the Display Function of certain Display Systems.

Before proceeding with the description of the methodology of this specific metric approach, it should be noted that it is offered as one possible approach, not necessarily as the most appropriate approach for evaluating all Display Systems. In particular, the following notes should be considered before selecting or interpreting results from any particular metric approach.

1) There may be practical issues which limit the number of P-Values which can be meaningfully used in the analysis. For example, it may be practical to measure all 256 possible Luminances from a fixed position on the screen of an 8-bit video monitor, but it may be impractical to meaningfully measure all

4096 densities theoretically printable by a 12-bit film printer. One reason for the impracticality is the limited accuracy of densitometers (or even film digitizers). A second reason is that the film density measurements, unlike the CRT photometer measurements, are obtained from different locations on the display area, so any spatial non-uniformity which is present in the film affects the hardcopy measurement. Current hardcopy printers and densitometers both have absolute optical density accuracy limitations which are significantly worse than the change which would be caused by a change in just the least significant bit of a 12-bit P-Value. In general, selecting a larger number of P-Values allows, in principle, more localized aberrations from the Grayscale Standard Display Function to be "caught", but the signal-to-noise ratio (or significance) of each of these will be decreased.

2) If the measurement data for a particular Display System has significant "noise" (as indicated by limited repeatability in the data when multiple sets of measurements are taken), it may be desirable to apply a statistical analysis technique which goes beyond the "FIT" and "LUM" metric by explicitly utilizing the known standard deviations in the input data set, along with the data itself, to prevent the fitting technique from over-reacting to noise. See, for example, the section "General Linear Least Squares" in Reference C1 and the chapter "Least-Squares Fit to a Polynomial" in Reference C2. If measurement noise is not explicitly taken into account in the analysis, the metric's returned root-mean-square error of the data points relative to the fit could be misleadingly high, since it would include the combined effect of errors due to incorrectness in the Display Function and errors due to measurement noise.

3) If possible, the sensitivity and specificity of the metric being considered should be checked against visual tests. For example, a digital test pattern with many low-contrast steps at many ambient Luminances could be printed on a "laboratory standard" Grayscale Standard Display Function printer and also printed on a printer being evaluated. The resultant films could then be placed side-by-side on light-boxes for comparison by a human observer. A good metric technique should detect as sensitively and repeatably as the human observer the existence of deviations (of any shape) from the Grayscale Standard Display Function. For example, if a Display System has a Characteristic Curve which, for even a very short interval of DDL values, is too contrasty, too flat, or (worse yet) non-monotonic, the metric should be able to detect and respond to that anomaly as strongly as the human observer does.

4) Finally, in addition to the experimentally encountered non-repeatabilities in the data from a Display System, there may be reason to consider additional possible causes of variations. For example, varying the ordering of P-Values in a test pattern (temporally for CRTs, spatially for printers) might affect the results. For printers, switching to different media might affect the results. A higher confidence can be placed in the results obtained from any metric if the results are stable in the presence of any or all such changes.

C.2. METHODOLOGY

Step (1)

The Characteristic Curve of the test Display System should be determined with as many measurements as practical (see Sections D.1, D.2, and D.3). Using the Grayscale Standard Display Function, the fractional number of JNDs are calculated for each Luminance interval between equally spaced P-Value steps. The JNDs/Luminance interval may be calculated directly, or iteratively. For example, if only a few JNDs belong to every Luminance interval, a linear interpolation may be performed. After transformation of the grayscale response of the Display System, the Luminance Levels for every P-Value are Li and the corresponding Standard Luminance Levels are Lj; dj specifies the JNDs /Luminance-Interval on the Grayscale Standard Display Function for the given number of P-Values. Then, the JNDs/Luminance interval for the transformed Display Function are

$$r = d_j (L_{i+1} - L_i)(L_{j+1} + L_j) / ((L_{i+1} + L_i)(L_{j+1} - L_j))$$
(C1)

Additionally, an iterative method can be used to calculate the number of JNDs per Luminance interval, requiring only the Grayscale Standard Display Function that defines a JND step in Luminance given a Luminance value. This is done by simply counting the number of complete JND steps in the Luminance interval, and then the remaining fractional step. Start at the Luminance low end of the interval, and calculate from the Grayscale Standard Display Function the Luminance step required for one JND step.

Then continue stepping from the low Luminance value to the high Luminance value in single JND steps, until the Luminance value of the upper end of the Luminance Range is passed. Calculate the fraction portion of one JND that this last step represents. the total number of completed integer JND steps plus the fractional portion of the last uncompleted step is the fractional number of JND steps in the Luminance interval.

Plot the number of JNDs per Luminance interval (vertical axis) versus the index of the Luminance interval (horizontal axis). This curve is referred to as the *Luminance intervals vs JNDs* curve. An example of a plot of Luminance intervals vs JNDs is shown in figure C-1. The plot is matched very well by a horizontal line when a linear regression is applied.

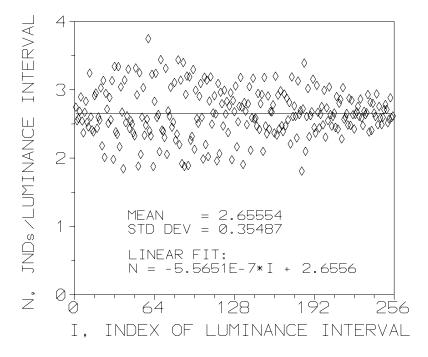


Figure C-1. Illustration for the LUM and FIT conformance measures

The JNDs/Luminance interval data are evaluated by two statistical measures [C4]. The first assesses the global match of the test Display Function with the Grayscale Standard Display Function. The second measure locally analyses the approximation of the Grayscale Standard Display Function to the test Display Function.

Step (2)

Two related measures of a regression analysis are applied after normal multiple linear regression assumptions are verified for the data [C3]. The first measure, named the *FIT* test, attempts to match the Luminance-Intervals-vs-JNDs curve of the test Luminance distribution with different order polynomial fits. The Grayscale Standard Display Function is characterized by exactly one JND per Luminance interval over the entire Luminance Range. Therefore, ideally, the data of JNDs/Luminance intervals vs index of the Luminance interval are best fit by a horizontal line of a constant number of JNDs/Luminance interval, indicating that both the local and global means of JNDs/Luminance interval are constant over the given Luminance Range. If the curve is better matched by a higher-order curve, the distribution is not closely approximating the Grayscale Standard Display Function. The regression analysis should test comparisons through third-order curves.

The second measure, the Luminance uniformity metric (LUM), analyzes whether the size of Luminance steps are uniform in perceptual size (i.e. JNDs) across the Luminance Range. This is measured by the Root Mean Square Error (RMSE) of the curve fit by a horizontal line of the JNDs/Luminance interval. The smaller the RMSE of the JNDs/Luminance interval, the more closely the test Display Function approximates the Grayscale Standard Display Function on a microscopic scale.

Both the FIT and LUM measures can be conveniently calculated on standard statistical packages.

Assuming the test Luminance distribution passes the FIT test, then the measure of quality of the distribution is determined by the single quantitative measurement (LUM) of the standard deviation of the JNDs/Luminance interval from their mean. Clinical practice is expected to determine the tolerances for the FIT and LUM values.

An important factor in reaching a close approximation of a test Display Function to the Grayscale Standard Display Function is the number of discrete output levels of the Display System. For instance, the LUM measure can be improved by using only a subset of the available DDLs while maintaining the full available output digitization resolution at the cost of decreasing contrast resolution.

While the LUM is influenced by the choice of the number of discrete output gray levels in the Grayscale Standard Display Function, the appropriate number of output levels is determined by the clinical application, including possible gray scale image processing that may occur independently of the Grayscale Standard Display Function standardization. Thus, PS 3.14 does not prescribe a certain number of gray levels of output. However, in general, the larger the number of distinguishable gray levels available, the higher the possible image quality because the contrast resolution is increased. It is recommended that the number of necessary output driving levels for the transformed Display Function be determined prior to standardization of the Display System (based on clinical applications of the Display System), so that this information can be used when calculating the transformation in order to avoid using gray scale distributions with fewer output levels than needed.

C.3. REFERENCES

- [C1] Press, William H, et al., Numerical Recipes in C, Cambridge University Press, 1988, Section "General Linear Least Squares"
- [C2] Bevington, Phillip R., Data Reduction and Error Analysis for the Physical Sciences, McGraw-Hill, 1969, the chapter "Least-Squares Fit to a Polynomial".
- [C3] Kleinbaum DG, Kupper LL, Muller KE, Applied Regression Analysis and Other Multivariable Methods, Duxbury Press, 2nd Edition, pp 45-49, 1987.

[C4] Hemminger, B., Muller, K., "Performance Metric for evaluating conformance of medical image displays with the ACR/NEMA display function standard", SPIE Medical Imaging 1997, editor Yongmin Kim, vol 3031-25, 1997.

Annex D (INFORMATIVE) ILLUSTRATIONS FOR ACHIEVING CONFORMANCE WITH THE GRAYSCALE STANDARD DISPLAY FUNCTION

The following sections illustrate how conformance with the Grayscale Standard Display Function may be achieved for emissive (soft-copy) Display Systems as well as systems producing image presentations (hard-copies) on transmissive and reflective media. Each section contains four sub-sections on 1) a procedure for measuring the system Characteristic Curve, 2) the application of the Grayscale Standard Display Function to the Luminance Range of the Display System, 3) the implementation of the Grayscale Standard Display Function, and 4) the application of the conformance metrics as proposed in Annex C.

It is emphasized that there are different ways to configure a Display System or to change its performance so that it conforms to the Grayscale Standard Display Function. In fact, conceivably, a Display System may calibrate itself automatically to maintain conformance with the Standard. Hence, the following three illustrations are truly only examples.

Luminance of any Display System, hard-copy or softcopy, may be measured with a photometer. The photometer should have the following characteristics:

- be accurate to within 3% or less of the absolute Luminance level across its full range of operation;
- have a relative accuracy of at least two times the least significant digit at any Luminance level in its range of operation;
- maintain this accuracy at Luminance levels that are one-tenth of the minimum measured Luminance of the Display System;
- have an acceptance angle that is small enough to incorporate only the measurement field without overlapping the surrounding background.
- Note: The photometer may be of the type that attaches directly to the display face (with a suction cup) or of the type that is held away from the display face. If of the latter type, the photometer should be well baffled to exclude extraneous light sources, including light from the background area of the test pattern.

For a film Display System the photometer may be appropriately used to measure the background Illuminance and the Luminance of the light-box on which the film will be displayed. The Luminance characteristics of the film Display System may be measured directly with the photometer or indirectly using measured optical density of the film and the values for the measured background Illuminance and the light-box Luminance.

D.1 EMISSIVE DISPLAY SYSTEMS

D.1.1 Measuring the system Characteristic Curve

Before the characteristic Luminance response of the emissive Display System is measured, it is allowed to warm up as recommended by the manufacturer and is adjusted such that it conforms to the manufacturer's performance specifications. In particular, adjustment procedures for setting the black and white levels of the display should be obtained from the Display System manufacturer. The goal is to maximize the dynamic Luminance Range of the display without introducing artifacts, resulting in the highest possible number of Just-Noticeable Differences (JNDs).

Note: A simple test that the system is set up properly can be performed by viewing the 5% and 95% squares in the SMPTE pattern. The perceived contrast between the 5% square and its 0% surrounding should be equal to the perceived contrast between the 95% square and a white square.

Measurement of the Characteristic Curve of the Display System may be accomplished using a test pattern (Fig. D.1-1) consisting of:

a square measurement field comprising 10% of the total number of pixels displayed by the system positioned in the center of the display;

- a full-screen uniform background of 20% of maximum Luminance surrounding the target.
- Note: With a measurement field of 10% of the total number of displayed pixels and a surrounding set to 20% of maximum Luminance, internal light scatter in the monitor causes the Luminance Range to be typically comparable to that found in radiographs, such as a thorax radiograph, when displayed on the CRT monitor.

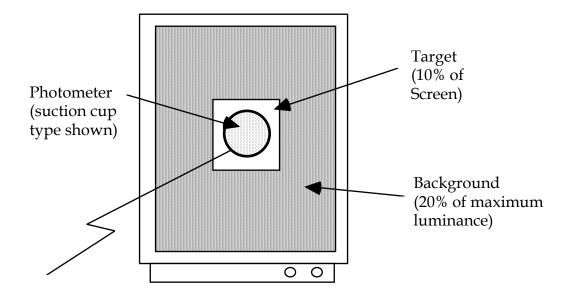


Figure D.1-1. The test pattern will be a variable intensity square in the center of a low Luminance background area.

Notes: 1. For example, on a 5-megapixel Display System with a matrix of 2048 by 2560 pixels, the target would be a square with 724 pixels on each side.

2. Ideally, the test pattern should fill the entire screen. Under certain windowed operating environments, it may be difficult to eliminate certain user-interface objects from the display, in particular, menu bars at the top of the screen. In this case, the background should fill as much of the screen as possible.

The Characteristic Curve of the Display System may be determined by

- turning off all ambient lighting (necessary only when a suction cup photometer is used or when a handheld photometer casts a shadow on the display screen);
- displaying the above test pattern;
- setting the DDL for the measurement field to a sequence of different values, starting with 0 and increasing at each step until the maximum DDL is reached;
- using a photometer to measure and record the Luminance of the measurement field at each command value.

As discussed in Annex C, the number and distribution of DDLs at which measurements are taken must be sufficient to accurately model the Characteristic Curve of the Display System over the entire Luminance Range.

Notes: 1. If a handheld photometer is used, it should be placed at a distance from the display screen so that Luminance is measured in the center of the measurement field, without overlapping the surrounding background. This distance can be calculated using the acceptance angle specification provided by the photometer manufacturer.

2. The exact number and distribution of DDLs should be based both on the characteristics of the Display System and on the mathematical technique used to interpolate the Characteristic Curve of the system. It is recommended that at least 64 different command values be used in the procedure.

3. Successive Luminance measurements should be spaced in time such that the Display System always reaches a steady state. It may be particularly important to allow the system to settle before taking the initial measurement at DDL 0.

As stated in the normative section, the effect of ambient light on the apparent Characteristic Curve must always be included when configuring a Display System to conform with the Grayscale Standard Display Function.

If a handheld photometer that does not cast a shadow on the display screen is used to measure the Characteristic Curve, then the Luminance produced by the display plus the effect of ambient light may be measured simultaneously.

When a suction cup photometer is used to take the Luminance measurements or when a handheld photometer casts a shadow on the display screen, all ambient lighting should be turned off while measuring the Characteristic Curve. The effect of ambient light is determined separately: The Display System is turned off, the ambient light is turned on, and the Luminance produced by scattering of ambient light at the display screen is measured by placing the photometer at a distance from the display screen so that its acceptance angle includes a major portion of the screen and that the measurement is not affected by direct illumination from areas outside the display screen. The Luminance related to ambient light is added to the previously measured Luminance levels produced by the Display System to determine the effective Characteristic Curve of the system.

Note: Changes in ambient lighting conditions may require recalibration of the display subsystem in order to maintain conformance to this standard.

In the following, an example for measurements and transformation of a Display Function is presented. The Display System for this example is a CRT monitor with display controller. It is assumed that the display controller allows a transformation of the DDLs with 8-bit input precision and 10-bit output precision.

The Luminance is measured with a photometer with a narrow (1⁰) acceptance angle. The ambient light level was adjusted as low as possible. No localized highlights were visible.

- The maximum Luminance was measured when setting the DDL for the measurement field to the value that yielded the highest Luminance and the DDL of the surrounding to the middle DDL range. From this measurement, the Luminance - 20% of the maximum Luminance - for the surrounding of the measurement field was calculated.
- 2) The ambient light was turned off. With the photometer centered on the measurement field of the test pattern of Fig. D.1-1, the Luminance was measured when varying the input level D_m in increments of

1 from 0 to 255. The transformation operator of the hypothetical display controller linearly mapped 8 bits on the input to 10 bits on the output. The measured data represent the Characteristic Curve $L = F(D_m)$ for the given operating conditions and this test pattern.

3) Next, the CRT was turned off and the ambient light turned on. The photometer was placed on the center axis of the CRT sufficiently far away so that it did not cast a shadow on the CRT face and its

aperture intercepted light scattered from a major portion of the CRT face. The measured Luminance of 0.3 cd/m^2 produced by the ambient light on the CRT face was added to the measured Luminance values of the Characteristic Curve without ambient light. The result is listed in Table D.1-1 and plotted in Fig. D.1-2.

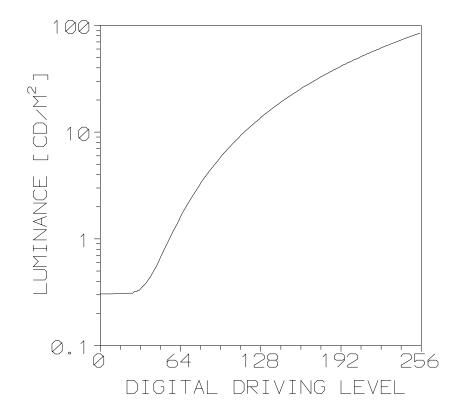


Figure D.1-2. Measured Characteristic Curve with Ambient Light of an emissive Display System

DDL Luminance DDL Luminance DDL Luminance 0 0.305 1 0.305 2 0.305 3 0.305 4 0.305 5 0.305 6 0.305 7 0.305 8 0.307 13 0.307 14 0.307 15 0.307 16 0.307 21 0.307 18 0.307 19 0.307 20 0.307 21 0.307 22 0.310 23 0.310 24 0.310 25 0.310 26 0.310 27 0.320 32 0.340 33 0.350 34 0.360 33 0.552 36 0.380 37 0.392 38 0.410 39 0.424 44 0.544 45 0.562 46 0.594 47 0.626 57 1.08 58 1.168 59 1.232 66 1.	Measured Characteristic Curve plus Ambient Light									
$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$	DDL	Luminance	DDL	Luminance		DDL	Luminance		DDL	Luminance
$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$	0	0.305	1	0.305		2	0.305		3	0.305
$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$	4	0.305	5	0.305		6	0.305		7	0.305
$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$	8	0.305	9	0.305		10	0.305		11	0.307
$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$	12	0.307	13	0.307		14	0.307		15	0.307
$\begin{array}{ c c c c c c c c c c c c c c c c c c c$	16	0.307	17	0.307		18	0.307		19	0.307
$\begin{array}{ c c c c c c c c c c c c c c c c c c c$	20	0.307	21	0.307		22	0.310		23	0.310
$\begin{array}{ c c c c c c c c c c c c c c c c c c c$	24	0.310	25	0.310		26	0.310		27	0.320
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	28	0.320	29	0.320		30	0.330		31	0.330
40 0.442 41 0.464 42 0.486 43 0.512 44 0.534 45 0.562 46 0.594 47 0.626 48 0.674 49 0.710 50 0.750 51 0.796 52 0.842 53 0.888 54 0.938 55 0.994 56 1.048 57 1.108 58 1.168 59 1.232 60 1.294 61 1.366 62 1.438 63 1.512 64 1.620 65 1.702 66 1.788 67 1.876 68 1.960 69 2.056 70 2.154 71 2.248 72 2.350 73 2.456 74 2.564 75 2.670 76 2.790 77 2.908 78 3.022 79 3.146 80 3.328 81 3.460 82 3.584 83 3.732 84 3.870 85 4.006 86 4.156 87 4.310 92 5.104 93 5.268 94 5.444 95 5.630 96 5.864 97 6.050 98 6.238 99 6.438 100 6.610 101 6.820 102 7.024 103 7.224 104 7.428 105 7.644 106 7.872 107 8.066 112 1.340	32	0.340	33	0.350		34	0.360		35	0.370
44 0.534 45 0.562 46 0.594 47 0.626 48 0.674 49 0.710 50 0.750 51 0.796 52 0.842 53 0.888 54 0.938 55 0.994 56 1.048 57 1.108 58 1.168 63 1.512 60 1.294 61 1.366 62 1.438 63 1.512 64 1.620 65 1.702 66 1.788 67 1.876 68 1.960 69 2.056 70 2.154 71 2.248 72 2.350 73 2.456 74 2.564 75 2.670 76 2.790 77 2.908 78 3.022 79 3.146 80 3.328 81 3.460 82 3.584 83 3.732 84 3.870 85 4.006 86 4.156 87 4.310 88 4.456 89 4.608 90 4.766 91 4.944 92 5.104 93 5.268 94 5.444 95 5.630 96 5.864 97 6.050 98 6.238 99 6.438 100 6.610 101 6.820 102 7.024 103 7.224 104 7.428 105 7.644 106 7.872 107 8.066 112 9.30 <	36	0.380	37	0.392		38	0.410		39	0.424
48 0.674 49 0.710 50 0.750 51 0.796 52 0.842 53 0.888 54 0.938 55 0.994 56 1.048 57 1.108 58 1.168 63 1.232 60 1.294 61 1.366 62 1.438 63 1.512 64 1.620 655 1.702 666 1.788 67 1.876 68 1.960 69 2.056 70 2.154 71 2.248 72 2.350 73 2.456 74 2.564 75 2.670 76 2.790 77 2.908 78 3.022 79 3.146 80 3.328 81 3.460 86 4.156 87 4.310 84 3.870 85 4.006 86 4.156 87 4.310 88 4.456 89 4.608 90 4.766 91 4.944 92 5.104 93 5.268 94 5.444 95 5.630 96 5.864 97 6.050 98 6.238 99 6.438 100 6.610 101 6.820 102 7.024 103 7.224 104 7.428 105 7.644 106 7.872 111 8.982 112 9.30 113 9.574 114 9.796 115 10.600 116	40	0.442	41	0.464		42	0.486		43	0.512
52 0.842 53 0.888 54 0.938 55 0.994 56 1.048 57 1.108 58 1.168 59 1.232 60 1.294 61 1.366 62 1.438 63 1.512 64 1.620 65 1.702 66 1.788 67 1.876 68 1.960 69 2.056 70 2.154 71 2.248 72 2.350 73 2.456 74 2.564 75 2.670 76 2.790 77 2.908 78 3.022 79 3.146 80 3.328 81 3.460 82 3.584 83 3.732 84 3.870 85 4.006 86 4.156 87 4.310 88 4.456 89 4.608 90 4.766 91 4.944 92 5.104 93 5.268 94 5.444 95 5.630 96 5.864 97 6.050 98 6.238 99 6.438 100 6.610 101 6.820 102 7.024 103 7.224 104 7.428 105 7.644 106 7.872 111 8.982 112 9.330 113 9.574 114 9.796 115 10.600 116 10.314 117 10.560 138 16.920 131 14.640 132 <td>44</td> <td>0.534</td> <td>45</td> <td>0.562</td> <td></td> <td>46</td> <td>0.594</td> <td></td> <td>47</td> <td>0.626</td>	44	0.534	45	0.562		46	0.594		47	0.626
56 1.048 57 1.108 58 1.168 59 1.232 60 1.294 61 1.366 62 1.438 63 1.512 64 1.620 65 1.702 66 1.788 67 1.876 68 1.960 69 2.056 70 2.154 71 2.248 72 2.350 73 2.456 74 2.564 75 2.670 76 2.790 77 2.908 78 3.022 79 3.146 80 3.328 81 3.460 82 3.584 83 3.732 84 3.870 85 4.006 86 4.156 87 4.310 88 4.456 89 4.608 90 4.766 91 4.944 92 5.104 93 5.268 94 5.444 95 5.630 96 5.864 97 6.050 98 6.238 99 6.438 100 6.610 101 6.820 102 7.024 103 7.224 104 7.428 105 7.644 106 7.872 111 8.982 112 9.330 113 9.574 114 9.796 115 10.060 116 10.314 117 10.560 136 13.020 127 13.300 124 12.460 125 12.700 126 13.020 127 13.300	48	0.674	49	0.710		50	0.750		51	0.796
60 1.294 61 1.366 62 1.438 63 1.512 64 1.620 65 1.702 66 1.788 67 1.876 68 1.960 69 2.056 70 2.154 71 2.248 72 2.350 73 2.456 74 2.564 75 2.670 76 2.790 77 2.908 78 3.022 79 3.146 80 3.328 81 3.460 82 3.584 83 3.732 84 3.870 85 4.006 86 4.156 87 4.310 88 4.456 89 4.608 90 4.766 91 4.944 92 5.104 93 5.268 94 5.444 95 5.630 96 5.864 97 6.050 98 6.238 99 6.438 100 6.610 101 6.820 102 7.024 103 7.224 104 7.428 105 7.644 106 7.872 107 8.066 118 10.820 111 8.982 1114 9.796 115 10.060 116 10.314 117 10.560 118 10.820 127 13.300 124 12.460 125 12.700 126 13.020 127 13.300 124 12.460 137 16.560 138 16.920 139 17.220 <td>52</td> <td>0.842</td> <td>53</td> <td>0.888</td> <td></td> <td>54</td> <td>0.938</td> <td></td> <td>55</td> <td>0.994</td>	52	0.842	53	0.888		54	0.938		55	0.994
64 1.620 65 1.702 66 1.788 67 1.876 68 1.960 69 2.056 70 2.154 71 2.248 72 2.350 73 2.456 74 2.564 75 2.670 76 2.790 77 2.908 78 3.022 79 3.146 80 3.328 81 3.460 82 3.584 83 3.732 84 3.870 85 4.006 86 4.156 87 4.310 88 4.456 89 4.608 90 4.766 91 4.944 92 5.104 93 5.268 94 5.444 95 5.630 96 5.864 97 6.050 98 6.238 99 6.438 100 6.610 101 6.820 102 7.024 103 7.224 104 7.428 105 7.644 106 7.872 107 8.066 118 10.300 113 9.574 114 9.796 115 10.060 116 10.314 117 10.560 136 13.020 127 13.300 128 13.720 129 14.020 130 14.360 131 14.640 132 14.940 133 15.300 134 15.600 135 15.900 136 16.240 137 16.560 138 16.920 143 18.640 <	56	1.048	57	1.108		58	1.168		59	1.232
$\begin{array}{ c c c c c c c c c c c c c c c c c c c$	60	1.294	61	1.366		62	1.438		63	1.512
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	64	1.620	65	1.702		66	1.788		67	1.876
$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$	68	1.960	69	2.056		70	2.154		71	2.248
80 3.328 81 3.460 82 3.584 83 3.732 84 3.870 85 4.006 86 4.156 87 4.310 88 4.456 89 4.608 90 4.766 91 4.944 92 5.104 93 5.268 94 5.444 95 5.630 96 5.864 97 6.050 98 6.238 99 6.438 100 6.610 101 6.820 102 7.024 103 7.224 104 7.428 105 7.644 106 7.872 107 8.066 108 8.298 109 8.528 110 8.752 111 8.982 112 9.330 113 9.574 114 9.796 115 10.060 124 12.460 125 12.700 126 13.020 127 13.300 128 13.720 129 14.020 <t< td=""><td>72</td><td>2.350</td><td>73</td><td>2.456</td><td></td><td>74</td><td>2.564</td><td></td><td>75</td><td>2.670</td></t<>	72	2.350	73	2.456		74	2.564		75	2.670
843.870854.006864.156874.310884.456894.608904.766914.944925.104935.268945.444955.630965.864976.050986.238996.4381006.6101016.8201027.0241037.2241047.4281057.6441067.8721078.0661088.2981098.5281108.7521118.9821129.3301139.5741149.79611510.06011610.31411710.56011810.82011911.08012011.34012111.62012211.88012312.18012412.46012512.70013014.36013114.64013214.94013315.30013415.60013515.90013616.24013716.56013816.92013917.22014017.60014117.94014619.80014720.260	76	2.790	77	2.908		78	3.022		79	3.146
884.456894.608904.766914.944925.104935.268945.444955.630965.864976.050986.238996.4381006.6101016.8201027.0241037.2241047.4281057.6441067.8721078.0661088.2981098.5281108.7521118.9821129.3301139.5741149.79611510.06011610.31411710.56012211.88012312.18012011.34012111.62012211.80012312.18012412.46012512.70012613.02013114.64013214.94013315.30013415.60013515.90013616.24013716.56013816.92013917.22014017.60014117.94014619.80014720.260	80	3.328	81	3.460		82	3.584		83	3.732
925.104935.268945.444955.630965.864976.050986.238996.4381006.6101016.8201027.0241037.2241047.4281057.6441067.8721078.0661088.2981098.5281108.7521118.9821129.3301139.5741149.79611510.06011610.31411710.56011810.82011911.08012011.34012111.62012211.88012312.18012412.46012512.70012613.02012713.30012813.72012914.02013415.60013515.90013616.24013716.56013816.92013917.22014017.60014117.94014619.80014720.260	84	3.870	85	4.006		86	4.156		87	4.310
965.864976.050986.238996.4381006.6101016.8201027.0241037.2241047.4281057.6441067.8721078.0661088.2981098.5281108.7521118.9821129.3301139.5741149.79611510.06011610.31411710.56011810.82011911.08012011.34012111.62012211.88012312.18012412.46012512.70012613.02012713.30012813.72012914.02013014.36013114.64013214.94013315.30013415.60013515.90013616.24014117.94014218.24014318.64014419.12014519.46014619.80014720.260	88	4.456	89	4.608		90	4.766		91	4.944
1006.6101016.8201027.0241037.2241047.4281057.6441067.8721078.0661088.2981098.5281108.7521118.9821129.3301139.5741149.79611510.06011610.31411710.56011810.82011911.08012011.34012111.62012211.88012312.18012412.46012512.70012613.02012713.30012813.72012914.02013014.36013114.64013214.94013315.30013415.60013515.90013616.24014117.94014218.24014318.64014419.12014519.46014619.80014720.260	92	5.104	93	5.268		94	5.444		95	5.630
1047.4281057.6441067.8721078.0661088.2981098.5281108.7521118.9821129.3301139.5741149.79611510.06011610.31411710.56011810.82011911.08012011.34012111.62012211.88012312.18012412.46012512.70012613.02012713.30012813.72012914.02013014.36013114.64013214.94013315.30013415.60013515.90014017.60014117.94014218.24014318.64014419.12014519.46014619.80014720.260	96	5.864	97	6.050		98	6.238		99	6.438
1088.2981098.5281108.7521118.9821129.3301139.5741149.79611510.06011610.31411710.56011810.82011911.08012011.34012111.62012211.88012312.18012412.46012512.70012613.02012713.30012813.72012914.02013014.36013114.64013214.94013315.30013415.60013515.90013616.24014117.94014218.24014318.64014419.12014519.46014619.80014720.260	100	6.610	101	6.820		102	7.024		103	7.224
1129.3301139.5741149.79611510.06011610.31411710.56011810.82011911.08012011.34012111.62012211.88012312.18012412.46012512.70012613.02012713.30012813.72012914.02013014.36013114.64013214.94013315.30013415.60013515.90013616.24013716.56013816.92013917.22014017.60014117.94014218.24014318.64014419.12014519.46014619.80014720.260	104	7.428	105	7.644		106	7.872		107	8.066
11610.31411710.56011810.82011911.08012011.34012111.62012211.88012312.18012412.46012512.70012613.02012713.30012813.72012914.02013014.36013114.64013214.94013315.30013415.60013515.90013616.24013716.56013816.92013917.22014017.60014117.94014218.24014318.64014419.12014519.46014619.80014720.260	108	8.298	109	8.528		110	8.752		111	8.982
12011.34012111.62012211.88012312.18012412.46012512.70012613.02012713.30012813.72012914.02013014.36013114.64013214.94013315.30013415.60013515.90013616.24013716.56013816.92013917.22014017.60014117.94014218.24014318.64014419.12014519.46014619.80014720.260	112	9.330	113	9.574		114	9.796		115	10.060
12412.46012512.70012613.02012713.30012813.72012914.02013014.36013114.64013214.94013315.30013415.60013515.90013616.24013716.56013816.92013917.22014017.60014117.94014218.24014318.64014419.12014519.46014619.80014720.260	116	10.314	117	10.560		118	10.820		119	11.080
12813.72012914.02013014.36013114.64013214.94013315.30013415.60013515.90013616.24013716.56013816.92013917.22014017.60014117.94014218.24014318.64014419.12014519.46014619.80014720.260	120	11.340	121	11.620		122	11.880		123	12.180
13214.94013315.30013415.60013515.90013616.24013716.56013816.92013917.22014017.60014117.94014218.24014318.64014419.12014519.46014619.80014720.260	124	12.460	125	12.700		126	13.020		127	13.300
13616.24013716.56013816.92013917.22014017.60014117.94014218.24014318.64014419.12014519.46014619.80014720.260	128	13.720	129	14.020		130	14.360		131	14.640
14017.60014117.94014218.24014318.64014419.12014519.46014619.80014720.260	132	14.940	133	15.300		134	15.600		135	15.900
144 19.120 145 19.460 146 19.800 147 20.260	136	16.240	137	16.560		138	16.920		139	17.220
	140	17.600	141	17.940]	142	18.240		143	18.640
148 20.560 149 20.920 150 21.360 151 21.760	144	19.120	145	19.460		146	19.800		147	20.260
	148	20.560	149	20.920		150	21.360		151	21.760

 Table D.1-1

 Measured Characteristic Curve plus Ambient Light

152	22.060	153	22.520	154	22.960	155	23.300
156	23.700	157	24.080	158	24.600	159	24.980
160	25.520	161	26.040	162	26.480	163	26.700
164	27.380	165	27.620	166	28.040	167	28.580
168	28.980	169	29.400	170	29.840	171	30.540
172	30.800	173	31.380	174	31.880	175	32.400
176	33.060	177	33.400	178	34.040	179	34.400
180	34.840	181	35.360	182	35.900	183	36.400
184	37.060	185	37.400	186	38.300	187	38.420
188	39.160	189	39.760	190	39.980	191	40.840
192	41.540	193	41.900	194	42.800	195	43.060
196	43.620	197	44.520	198	44.620	199	45.500
200	46.100	201	46.380	202	47.400	203	47.600
204	48.320	205	49.060	206	49.380	207	50.320
208	50.920	209	51.600	210	52.420	211	52.680
212	53.520	213	54.220	214	54.620	215	55.420
216	56.100	217	56.600	218	57.400	219	57.820
220	58.660	221	59.320	222	59.800	223	60.720
224	61.520	225	62.240	226	63.040	227	63.480
228	64.460	229	65.020	230	65.500	231	66.500
232	66.960	233	67.840	234	68.600	235	68.980
236	70.040	237	70.520	238	71.420	239	72.180
240	72.900	241	73.980	242	74.580	243	75.320
244	76.200	245	76.540	246	77.720	247	78.220
248	79.200	249	79.880	250	80.420	251	81.560
252	81.960	253	83.140	254	83.720	255	84.340

D.1.2 Application of the Standard Formula

The section of the Grayscale Standard Display Function for the Luminance Range of the CRT monitor Display System is shown in Figure D.1-3. Minimum and maximum Luminance levels correspond to JND indices of $JND_{min} = 32.54$ and $JND_{max} = 453.85$, respectively. Thus, there are theoretically about 420 just-noticeable Luminance differences for the Standard Target (see Normative Section 6). Obviously, with 8-bit input digitization resolution, at best 256 noticeable Luminance increments can be realized.

D.1.3 Implementation of the Standard

The measured Characteristic Curve is interpolated for the available output levels D_{output} , in this case, yielding 1024 Luminance levels $L_{I,m}$. The Grayscale Standard Display Function is also interpolated between JND_{min} and JND_{max} ((JND = [JND_{max} - JND_{min}]/1023 = [453.85 - 32.54]/1023) yielding 1024 Standard Luminance levels $L_{I,STD}$. Interpolations can be performed by a variety of techniques. Here, a cubic spline technique was employed.

For every $L_{I,STD}$, the closest $L_{J,m}$ is determined. The data pair I,J defines the transformation between D_{input} and D_{output} (Table D.1-2) by which the Luminance response of the Display System is made to approximate the Grayscale Standard Display Function.

Incut	Outrout		able for Cal					Incut	Outrout
Input	Output	Input	Output	-	out	Output	-	Input	Output
0	0	1	118	2		131	-	3	140
4	148	5	153	6		160	-	7	164
8	169	9	173	1		178	-	11	182
12	185	13	189	1		191	_	15	194
16	198	17	201	1		204	F	19	207
20	210	21	214	2		217	-	23	219
24	222	25	225	2	6	228	Ļ	27	231
28	234	29	237	3	0	240		31	243
32	245	33	248	34	4	251		35	253
36	255	37	257	3	8	260		39	263
40	265	41	268	42	2	271		43	274
44	276	45	279	4	6	282		47	284
48	287	49	290	5	0	292		51	295
52	298	53	301	5	4	303		55	306
56	308	57	311	5	8	314		59	317
60	319	61	320	6	2	323	Γ	63	326
64	329	65	331	6	6	334		67	336
68	339	69	342	7	0	345		71	347
72	350	73	353	74	4	356	Γ	75	359
76	361	77	364	7	8	367	Γ	79	370
80	372	81	375	8	2	378	Ē	83	381
84	383	85	385	8	6	388	Ē	87	391
88	393	89	396	9	0	399		91	402
92	405	93	407	94	4	410	ſ	95	413
96	416	97	419	98	8	422	f	99	425
100	428	101	431	10)2	434	f	103	437
104	440	105	443	10		445	ŀ	107	448
108	450	109	452	11	0	456	ľ	111	459
112	462	113	465	11	4	468	f	115	471
116	474	117	477	11	8	480	f	119	483
120	486	121	490	12		492	ŀ	123	495
124	499	125	502	12	26	505	ľ	127	509
128	511	129	513	13	30	516	ľ	131	519

Table D.1-2 Look-Up Table for Calibrating Display System

132	522	133	526	134	529	135	532
136	535	137	539	138	542	139	545
140	549	141	552	142	555	143	559
144	562	145	565	146	569	147	572
148	575	149	578	150	581	151	585
152	588	153	591	154	595	155	599
156	602	157	605	158	609	159	613
160	616	161	619	162	623	163	627
164	631	165	633	166	637	167	640
168	643	169	646	170	650	171	655
172	657	173	663	174	666	175	669
176	674	177	678	178	682	179	684
180	688	181	693	182	696	183	700
184	703	185	706	186	711	187	714
188	719	189	723	190	727	191	731
192	735	193	738	194	743	195	745
196	752	197	754	198	758	199	764
200	766	201	769	202	775	203	777
204	783	205	787	206	789	207	796
208	799	209	805	210	808	211	811
212	818	213	821	214	827	215	830
216	834	217	838	218	841	219	848
220	851	221	856	222	861	223	864
224	870	225	874	226	880	227	883
228	889	229	893	230	897	231	901
232	905	233	911	234	915	235	922
236	925	237	931	238	935	239	941
240	945	241	951	242	955	243	960
244	964	245	969	246	975	247	979
248	985	249	991	250	995	251	1002
252	1006	253	1012	254	1016	255	1023

D.1.4 Measures of Conformance

The FIT and the LUM metrics proposed in Annex C are applied to determine the macroscopic and microscopic approximation of the $L_{J,m}$ to the $L_{I,STD}$. Figure D.1-3 shows the perceptually linearized Display Function superimposed on the Grayscale Standard Display Function and Figure D.1-4 summarizes the results of the two metrics. A good global fit was achieved as demonstrated by the nearly horizontal-line fit as best fit obtained with the FIT metric. The RMSE is acceptable. All 255 P-Value intervals lead to JNDs on the transformed Display Function for the Standard Target.

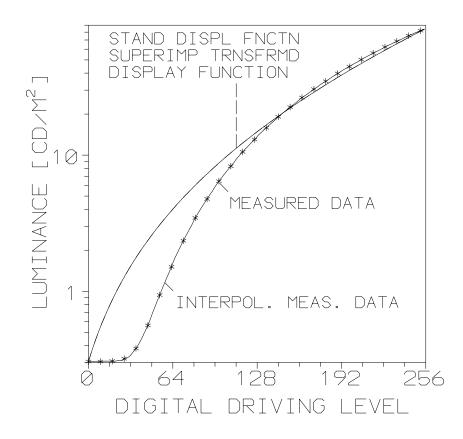


Figure D.1-3. Measured and interpolated Characteristic Curve, Grayscale Standard Display Function and transformed Display Function of an emissive Display System. The transformed Display Function for this Display System matches the Grayscale Standard Display Function and the two curves are superimposed and indistinguishable.

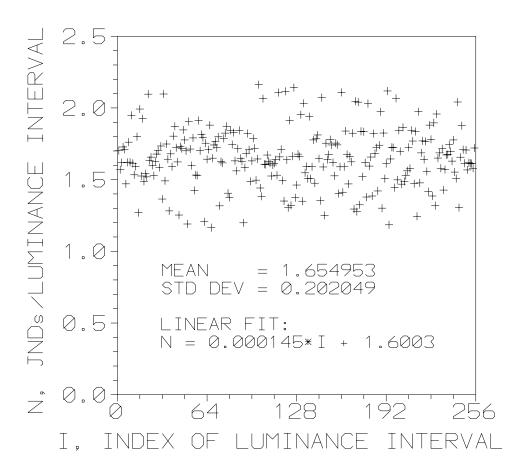


Figure D.1-4. LUM and FIT measures of conformance for a the transformed Display Function of an emissive Display System

D.2 TRANSPARENT HARDCOPY DEVICES

D.2.1 Measuring the system Characteristic Curve

A transparent hardcopy device is exemplified by a laser printer (including processor) which prints (exposes and processes) one or more images on a sheet of transparent film (typically a 14" x 17" film). This film is eventually placed over a high Luminance light-box in a darkened room for viewing.

The Characteristic Curve for such a transparent hardcopy device is obtained by printing a test image consisting of a pattern of n bars, each bar having a specific numeric value (DDL). The optical density of each printed bar is then measured, using a transmission densitometer, for each of the printed bars.

To accurately define a printer's Characteristic Curve, it is desirable that n be as large as possible (to capture as many points as possible on the Characteristic Curve). However, the limitations on absolute quantitative repeatability imposed by the printer, processor, or media technologies may dictate that a much smaller value of n be used (to prevent a conformance metric which is sensitive to differences from becoming unstable and meaningless, as the density differences between adjacent bars become "in the noise" as the number of bars becomes large).

One example of a test image is a pattern of 32 approximately equal-height bars, spanning the usable printable region of the film, having 32 approximately equi-spaced DDLs as follows:

Density	-	
 Density	-	
 Density		
Density	<u> </u>	
Density		
Density	Step	5
Density	Step	6
Density	Step	7
Density	Step	8
Density	Step	9
Density	Step	10
 Density	Step	11
Density	Step	12
Density	Step	13
Density	Step	14
Density	Step	15
Density	Step	16
Density	Step	17
Density	Step	18
Density		
Density	Step	20
Density	Step	21
Density	Step	22
Density	Step	23
Density	Step	24
Density		
Density	Step	26
Density	Step	27
Density		
 Density	Step	29
Density	-	
Density		

Figure D.2-1. Layout of a Test Pattern for Transparent Hardcopy Media

To define a test pattern with n DDLs for a printer with an N-bit input, the DDL of step # i can be set to

DDL_i =
$$(2^{N}-1) \frac{i}{n-1}$$
 (D.2-1)

rounded to the nearest integer.

The tabulated values of DDL_i and the corresponding measured optical densities OD_i constitute a Characteristic Curve of the printer.

D.2.2 Application of the Grayscale Standard Display Function

The films which are produced by transparent hardcopy printers are often brought to a variety of locations, where they may be viewed on different light-boxes and under a variety of viewing conditions. Accordingly, the approach of PS 3.14 is to define, for hardcopy transparent printers, what densities (rather than Luminances) should be produced, and to provide here a method of applying the Grayscale Standard Display Function to the transparent hardcopy case, based on parameters which are typical of the expected range of light-box Luminances and other viewing parameters.

The specific parameters which are used in the following example are as follows:

 L_0 (Luminance of light-box with no film present): 2000 cd/m²

 L_a (ambient room light reflected by film): 10 cd/m²

D_{min} (minimum optical density obtainable on film): 0.20

D_{max} (maximum optical density desirable on film): 3.00.

The process of constructing a table of desired OD values from the Grayscale Standard Display Function begins with defining the Luminance Range and the corresponding range of the Just-Noticeable Difference Index, j. The minimum and maximum Luminance values are given respectively by

$L_{min} = L_a + L_0 10^{-Dmax} = 12.0 \text{ cd/m}^2$	(D.2-2)
$L_{max} = L_a + L_o 10^{-Dmin} = 1271.9 \text{ cd/m}^2$	(D.2-3)

Next, calculate the corresponding Just-Noticeable Difference Index values, j_{min} and j_{max}. For the current example, we obtain

This gives us the range of j-values which the printer should cover. The printer should map its minimum input (P-Value = 0) to j_{min} and the corresponding L_{min} . It should map its maximum input (P-Value = 2^{N-1} where N is the number of input bits) to j_{max} and the corresponding L_{max} . At any intermediate input it should map its input proportionately:

$$j (PV) = j_{min} + (j_{max}-j_{min}) \frac{PV}{2^{N}-1}$$
 (D.2-6)

and target values for the Luminance given by the Standard's formula: L(j(P-Value)). This "targeting" consists of producing an optical density OD for this P-Value which will give the desired Luminance L(j(P-Value)) under the conditions of L₀ and L_a previously defined. The required density can thus be calculated as follows: .

$$OD(PV) = -\log_{10} \frac{(L(j(PV)) - L_a)}{L_0}$$
 (D.2-7)

D.2.3 Implementation of the Grayscale Standard Display Function

Carrying this example into the even more specific case of a printer with an 8-bit input leads to the following table, which defines the OD's to be generated for each of the 256 possible P-Values.

P-Value	Optical Density (OD)	P-Value	e Optical Density (OD)	P-Value	Optical Density (OD)	P-Value	Optical Density (OD)
0	3.000	1	2.936	2	2.880	3	2.828
4	2.782	5	2.739	6	2.700	7	2.662
8	2.628	9	2.595	10	2.564	11	2.534
12	2.506	13	2.479	14	2.454	15	2.429

 Table D.2-1 Optical Densities for Each P-Value for an 8-Bit Printer

16	2.405	17	2.382	18	2.360	19	2.338
20	2.403	21	2.302	22	2.300	23	2.258
20	2.239	25	2.237	26	2.203	23	2.185
28	2.168	29	2.152	30	2.135	31	2.100
32	2.100	33	2.088	34	2.073	35	2.058
36	2.043	37	2.000	38	2.013	39	2.000
40	1.986	41	1.973	42	1.959	43	1.946
44	1.933	45	1.920	46	1.907	47	1.894
48	1.882	49	1.870	50	1.857	51	1.845
52	1.833	53	1.821	54	1.807	55	1.798
56	1.787	57	1.775	58	1.764	59	1.753
60	1.742	61	1.731	62	1.720	63	1.709
64	1.698	65	1.688	66	1.677	67	1.667
68	1.656	69	1.646	70	1.636	71	1.626
72	1.616	73	1.605	74	1.595	75	1.586
72	1.576	73	1.566	74	1.556	79	1.547
80	1.537	81	1.500	82	1.518	83	1.508
84	1.499	85	1.490	86	1.480	87	1.471
88	1.462	89	1.453	90	1.444	91	1.434
92	1.425	93	1.416	94	1.407	95	1.398
96	1.390	97	1.381	98	1.372	99	1.363
100	1.354	101	1.346	102	1.337	103	1.328
100	1.320	101	1.311	102	1.303	100	1.294
101	1.286	109	1.277	110	1.269	111	1.260
112	1.252	113	1.244	114	1.235	115	1.227
116	1.219	117	1.211	118	1.202	119	1.194
120	1.186	121	1.178	122	1.170	123	1.162
124	1.154	125	1.146	126	1.138	127	1.130
128	1.122	129	1.114	130	1.106	131	1.098
132	1.090	133	1.082	134	1.074	135	1.066
136	1.058	137	1.051	138	1.043	139	1.035
140	1.027	141	1.020	142	1.012	143	1.004
144	0.996	145	0.989	146	0.981	147	0.973
148	0.966	149	0.958	150	0.951	151	0.943
152	0.935	153	0.928	154	0.920	155	0.913
156	0.905	157	0.898	158	0.890	159	0.883
160	0.875	161	0.868	162	0.860	163	0.853
164	0.845	165	0.838	166	0.831	167	0.823
168	0.816	169	0.808	170	0.801	171	0.794
172	0.786	173	0.779	174	0.772	175	0.764

176	0.757	177	0.750	178	0.742	179	0.735
180	0.728	181	0.721	182	0.713	183	0.706
184	0.699	185	0.692	186	0.684	187	0.677
188	0.670	189	0.663	190	0.656	191	0.648
192	0.641	193	0.634	194	0.627	195	0.620
196	0.613	197	0.606	198	0.598	199	0.591
200	0.584	201	0.577	202	0.570	203	0.563
204	0.556	205	0.549	206	0.542	207	0.534
208	0.527	209	0.520	210	0.513	211	0.506
212	0.499	213	0.492	214	0.485	215	0.478
216	0.471	217	0.464	218	0.457	219	0.450
220	0.443	221	0.436	222	0.429	223	0.422
224	0.415	225	0.408	226	0.401	227	0.394
228	0.387	229	0.380	230	0.373	231	0.366
232	0.359	233	0.352	234	0.345	235	0.338
236	0.331	237	0.324	238	0.317	239	0.311
240	0.304	241	0.297	242	0.290	243	0.283
244	0.276	245	0.269	246	0.262	247	0.255
248	0.248	249	0.241	250	0.234	251	0.228
252	0.221	253	0.214	254	0.207	255	0.200

Plotting these values gives the curve of Figure D.2-3.

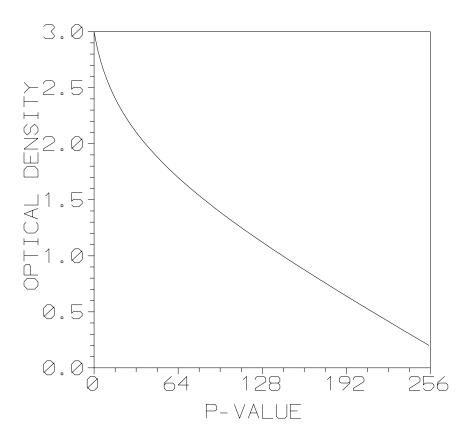


Figure D.2-3. Plot of OD vs P-Value for an 8-Bit Printer

D.2.4. Measures of Conformance

As an example, a bar pattern with 32 optical densities was printed on transmissive media (film). Beforehand, the printer had been set up to print over a density range from 0.2 (Dmin) to 3.0 (Dmax) and had been pre-configured by the manufacturer to use the Grayscale Standard Display Function, converted by the manufacturer into the table of target density values vs. P-Values described earlier.

The test pattern which was used for this was an 8-bit image consisting essentially of 32 horizontal bars. The 32 P-Values used for the bars were as follows: 0, 8, 16, 25, 33, 41, 49, 58, 66, 74, 82, 90,99, 107, 115, 123, 132, 140, 148, 156, 165, 173, 181, 189, 197, 206, 214,222, 230, 239, 247, 255.

For a given film, the 32 bars' optical densities were measured (near the middle of the film), converted to Luminances (using the standard parameters of light-box Luminance and reflected ambient light described earlier), and converted to Just-Noticeable Difference Indices by mathematically computing j(L) from L(j), where L(j) is the Grayscale Standard Display Function of Luminance L as a function of the Just-Noticeable Difference Index j. For each of the 31 intervals between consecutive measured values, a calculated value of "JNDs per increment in P-Values" was obtained by dividing the difference in Just-Noticeable Difference Index by the difference in P-Values for that interval. (In these calculations, density, L, and j are all floating-point variables. No rounding to integer values is done, so no truncation error is introduced.)

In this example, the film's data could be reasonably well fit by a horizontal straight line. That is, the calculated "JNDs per increment in P-Values was essentially constant at 2.4. A mathematical fit yielded a

slight non-zero slope (specifically, dropping from 2.5 to 2.3 as the P-Value went from 0 to 255), but the 0.2 total difference was considerably smaller than the noise which was present in the 31 individual values of "JNDs per increment in P-Value" so is of doubtful significance. (The "noise" referred to here consists of the random, non-repeatable variations which are seen if a new set of measured data (e.g., from a second print of the same test pattern) is compared with a previous set of measurements.)

No visual tests were done to see if a slope that small could be detected by a human observer in side-byside film comparisons.

Incidentally, if one considers just the 32 original absolute measured densities (rather than differential values based on small differences), one finds, in this case, quite reasonable agreement between the target and measured optical densities (within the manufacturer's norms for density accuracy, at a given density). But if one uses any metric which is based on differential information over small intervals, the results must be considered more cautiously, since they can be strongly affected by (and may be dominated by) various imperfections which are independent of a device's "true" (or averaged over many cases) characteristic behavior.

D.3 REFLECTIVE DISPLAY SYSTEMS

This last example illustrates how conformance with the Grayscale Standard Display Function may be achieved for a thermal-dye-transfer paper printer/office-light system. The thermal-dye-transfer printer produces black-and-white grayscale prints on a semi-glossy 8-inch x 10-inch heavy-gauge paper. The print is illuminated uniformly by fluorescent lamps so that the minimum reflective density produces a

Luminance of 150 cd/m^2 . The hypothetical transformation operator is assumed to have equal input and output digitization resolution of 8 bits.

D.3.1. Measuring the system Characteristic Curve

A print with a 64-step grayscale tablet was printed for DDLs 4, 8, 12, ...,248, 252, 255. The reflection optical densities (from 0.08 to 2.80) were measured with a densitometer. The Luminance levels corresponding to the measured optical densities and illumination conditions are plotted in Fig. D.3-1.

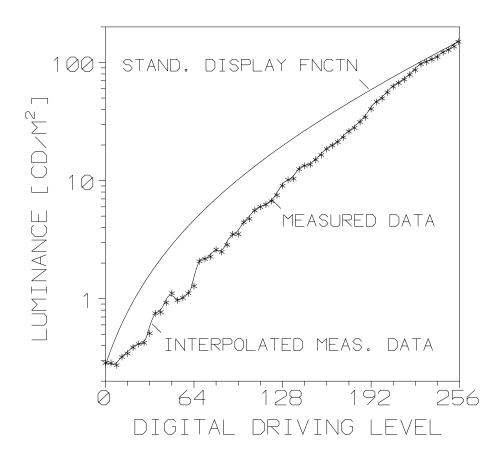


Figure D.3-1 Measured and interpolated Characteristic Curve and Grayscale Standard Display Function for a printer producing reflective hard-copies

D.3.2. Application of the Grayscale Standard Display Function

This last example illustrates how conformance with the Grayscale Standard Display Function may be achieved for a thermal-dye-transfer paper printer/office-light system. The thermal-dye-transfer printer produces black-and-white grayscale prints on a semi-glossy 8-inch x 10-inch heavy-gauge paper. The print is illuminated uniformly by fluorescent lamps so that the minimum reflective density produces a

Luminance of 150 cd/m^2 . The hypothetical transformation operator is assumed to have equal input and output digitization resolution of 8 bits.

D.3.3 Implementation of the Grayscale Standard Display Function

The measured Characteristic Curve is interpolated for the available DDLs yielding 256 Luminance levels $L_{I,m}$. The Grayscale Standard Display Function is also interpolated between JND_{min} and JND_{max} (*DJND* = [*JND_{max}* - *JND_{min}*]/255) yielding 256 Standard Luminance levels $L_{I,STD}$.

For every $L_{I,STD}$, the closest $L_{J,m}$ is determined. The data pair I,J defines the transformation between D_{input} and D_{output} (Table D.3-1 and Fig. D.3-2) by which the Luminance response of the Display System is made to approximates the Grayscale Standard Display Function.

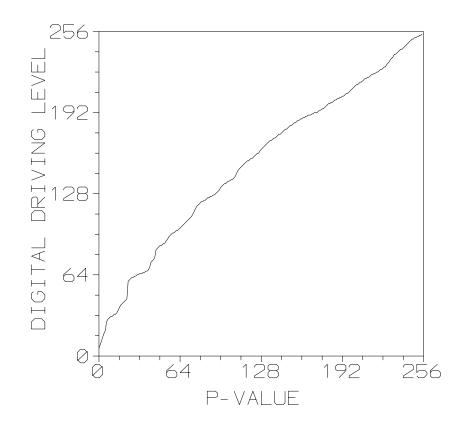


Figure D.3-2 Transformation for modifying the Characteristic Curve of the printer to a Display Function that approximates the Grayscale Standard Display Function

P-Value DDL P-Val 0 6 1 4 18 5 8 30 9 12 33 13 16 38 17 20 43 21 24 60 25 28 63 29 32 65 33 36 66 37 40 70 41 44 78 45 48 87 49 52 89 53 56 95 57 60 98 61 64 101 65 68 105 69 72 109 73 76 116 77 80 121 81	9 20 31 33 40 44 61 63 65 65 67 74		P-Value 2 6 10 14 18 22 26 30 34	DDL 12 27 31 34 41 45 62 64	P-Value 3 7 11 15 19 23 27 21	DDL 15 29 32 36 42 59 62
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	20 31 33 40 44 61 63 65 67 74		6 10 14 18 22 26 30	27 31 34 41 45 62	7 11 15 19 23 27	29 32 36 42 59
8 30 9 12 33 13 16 38 17 20 43 21 24 60 25 28 63 29 32 65 33 36 66 37 40 70 41 44 78 45 48 87 49 52 89 53 56 95 57 60 98 61 64 101 65 68 105 69 72 109 73 76 116 77 80 121 81	31 33 40 44 61 63 65 67 74		10 14 18 22 26 30	31 34 41 45 62	11 15 19 23 27	32 36 42 59
12 33 13 16 38 17 20 43 21 24 60 25 28 63 29 32 65 33 36 66 37 40 70 41 44 78 45 48 87 49 52 89 53 56 95 57 60 98 61 64 101 65 68 105 69 72 109 73 76 116 77 80 121 81	33 40 44 61 63 65 67 74	· · · · · · · · · · · · · · · · · · ·	14 18 22 26 30	34 41 45 62	15 19 23 27	36 42 59
16 38 17 20 43 21 24 60 25 28 63 29 32 65 33 36 66 37 40 70 41 44 78 45 48 87 49 52 89 53 56 95 57 60 98 61 64 101 65 68 105 69 72 109 73 76 116 77 80 121 81	40 44 61 63 65 67 74)	18 22 26 30	41 45 62	19 23 27	42 59
20 43 21 24 60 25 28 63 29 32 65 33 36 66 37 40 70 41 44 78 45 48 87 49 52 89 53 56 95 57 60 98 61 64 101 65 68 105 69 72 109 73 76 116 77 80 121 81	44 61 63 65 67 74	· · · · · · · · · · · · · · · · · · ·	22 26 30	45 62	23 27	59
24 60 25 28 63 29 32 65 33 36 66 37 40 70 41 44 78 45 48 87 49 52 89 53 56 95 57 60 98 61 64 101 65 68 105 69 72 109 73 76 116 77 80 121 81	61 63 65 67 74	5	26 30	62	27	
28 63 29 32 65 33 36 66 37 40 70 41 44 78 45 48 87 49 52 89 53 56 95 57 60 98 61 64 101 65 68 105 69 72 109 73 76 116 77 80 121 81	63 65 67 74	; ;	30			62
32 65 33 36 66 37 40 70 41 44 78 45 48 87 49 52 89 53 56 95 57 60 98 61 64 101 65 68 105 69 72 109 73 76 116 77 80 121 81	65 67 74	;		64	04	
36 66 37 40 70 41 44 78 45 48 87 49 52 89 53 56 95 57 60 98 61 64 101 65 68 105 69 72 109 73 76 116 77 80 121 81	67 74		34		31	64
40 70 41 44 78 45 48 87 49 52 89 53 56 95 57 60 98 61 64 101 65 68 105 69 72 109 73 76 116 77 80 121 81	74	,	U -1	65	35	66
44 78 45 48 87 49 52 89 53 56 95 57 60 98 61 64 101 65 68 105 69 72 109 73 76 116 77 80 121 81			38	67	39	68
48 87 49 52 89 53 56 95 57 60 98 61 64 101 65 68 105 69 72 109 73 76 116 77 80 121 81	84		42	75	43	76
52 89 53 56 95 57 60 98 61 64 101 65 68 105 69 72 109 73 76 116 77 80 121 81			46	85	47	86
56 95 57 60 98 61 64 101 65 68 105 69 72 109 73 76 116 77 80 121 81	87	,	50	88	51	89
60986164101656810569721097376116778012181	91		54	92	55	94
64101656810569721097376116778012181	96	i	58	97	59	97
68 105 69 72 109 73 76 116 77 80 121 81	99)	62	99	63	100
72 109 73 76 116 77 80 121 81	102	2	66	103	67	104
76 116 77 80 121 81	106	6	70	107	71	108
80 121 81	110	C	74	112	75	114
	118	3	78	119	79	120
0.4 400 05	122	2	82	122	83	123
84 123 85	124	4	86	125	87	125
88 126 89	126	6	90	127	91	127
92 128 93	129	9	94	130	95	131
96 133 97	134	4	98	135	99	136
100 136 101	137	7	102	138	103	138
104 139 105	5 139	9	106	140	107	141
108 143 109) 145	5	110	147	111	148
112 149 113	3 150	C	114	151	115	152
116 153 117	' 154	4	118	154	119	155
120 156 121	156	6	122	157	123	158
124 159 125	5 160	C	126	160	127	162
128 163 129	164	4	130	165	131	166
132 167 133	168	3	134	169	135	170
136 170 137	' 17 1	1	138	172	139	172
140 173 141	174	4	142	175	143	175
144 176 145	5 177	7	146	178	147	179
148 179 149		C	150	181	151	182

 Table D.3-1

 Look-Up Table for Calibrating Reflection Hardcopy System

152	182	153	183	154	184	ſ	155	184
156	185	157	186	158	186	ſ	159	187
160	187	161	188	162	188	Ī	163	189
164	189	165	190	166	190	ſ	167	190
168	191	169	191	170	192	Ī	171	192
172	192	173	193	174	194	Ī	175	194
176	195	177	195	178	196	Ī	179	197
180	198	181	199	182	199	ľ	183	200
184	200	185	201	186	202	Ī	187	202
188	203	189	203	190	204	ľ	191	204
192	205	193	205	194	206	ľ	195	207
196	207	197	208	198	209	ľ	199	210
200	211	201	212	202	213	ľ	203	214
204	214	205	215	206	216	ľ	207	216
208	217	209	218	210	219	Ī	211	219
212	220	213	220	214	221	Ī	215	222
216	222	217	223	218	223	ľ	219	224
220	224	221	225	222	226	ľ	223	226
224	227	225	228	226	228	ľ	227	230
228	231	229	232	230	234	ľ	231	235
232	236	233	238	234	238	ľ	235	239
236	240	237	241	238	242	Ī	239	242
240	243	241	244	242	245	Ī	243	246
244	247	245	248	246	249	Ī	247	250
248	250	249	251	250	251	Ī	251	252
252	252	253	253	254	253		255	254

D.3.4 Measures of Conformance

The FIT and LUM metrics as proposed in Annex C are applied to determine the macroscopic and microscopic approximation of the $L_{J,m}$ to the $L_{I,STD}$. Fig. D.3-3 shows the perceptually linearized Display Function superimposed on the Grayscale Standard Display Function and Figure D.3-4 summarizes the results of the two metrics. FIT provides as best fit of the JNDs/Luminance interval a straight line almost perfectly parallel to the horizontal axis indicating good global fit of the transformed Display Function with the Grayscale Standard Display Function. The RMSE computed by LUM is relatively large indicating more pronounced local deviations from the Grayscale Standard Display Function as, for example, with the soft-copy Display System illustrated in Section D.1. At least in part, the larger RMSE is due to the fact that the input and output digitization resolution for the transform are equal. The transformation table (Table D.3-1) and Fig. D.3-2 show that several P-Values lead to the same Luminance levels on the transformed Display Function. In fact, only 205 of the 255 Luminance intervals lead to JNDs for the Standard Target.

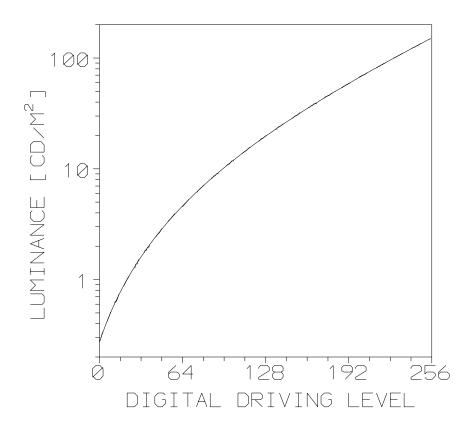


Figure D.3-3. Transformed Display Function and superimposed Grayscale Standard Display Function for a reflective hard-copy Display System. The transformed Display Function for this Display System matches the Grayscale Standard Display Function and the two curves are superimposed and indistinguishable.

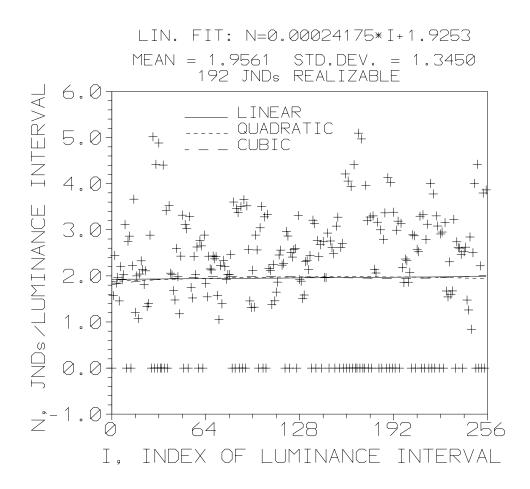


Figure D.3-4 Measures of conformance for a reflective hard-copy Display System with equal input and output digitization resolution of 8 bits

Annex E (INFORMATIVE) REALIZABLE JND RANGE OF A DISPLAY UNDER AMBIENT LIGHT

Dynamic range is an often used measures of the information content that can be presented by a Display System. However, there are many definitions of dynamic range, and most such definitions do not take into account real world conditions that affect the actual amount of information which can be conveyed by a gray scale pixel. For example, Poynton [E1] refers to the *contrast ratio* of a gray scale display device as the ratio of display intensity between the brightest white and the darkest black of the particular display device in question. However, this definition of dynamic range applies to ideal viewing conditions. Real world conditions such as veiling glare, noise, spatial frequency content of the image, power supply saturation, and ambient lighting in a cathode ray tube (CRT) based viewing situation can degrade the measured dynamic range of the system significantly [E2, E3]. Because of all of these variables dynamic range is an ill-defined concept for a Display System.

Note: *Veiling Glare* is the phenomenon wherein internal light reflections inside the CRT creates a "background lighting" thus reducing the contrast range of the CRT device.

The methods used to determine the degree to which the Display Function of a Display System approximates the Grayscale Standard Display Function can also be used to define two measures which might better characterize the potential capabilities of a Display System to convey information content. Two measures, the theoretically achievable JNDs and the realized JNDs, are useful for comparing Display Systems [E4].

The number of theoretically achievable JNDs is simply the number of JNDs predicted by the visual model given the Luminance Range of the Display System used. The number of theoretically achievable JNDs of a Display System may be found from Table B-1 of Annex B by counting the number of JNDs in the table which fall between the measured minimum and maximum Luminance of the Display System.

This number of JNDs may not actually be achievable due to resolution limitations of other portions of the Display System, in particular, the quantization resolution given by the finite number of bits per pixel driving the Display System. For example, Table B-1 of Annex B may show that a particular Display System is capable of delivering 352 JNDs. However, if only 8 bits per pixel are presented to the Display System, the number of JNDs achievable cannot exceed $2^8 = 256$ JNDs because of the quantizing effect. In actual fact, the number of JNDs realized in a Display System will always be smaller than or equal to the lower of the theoretically achievable JNDs and the quantization limit. This is because some of the quantized values input to the display may not line up with the input value required to achieve the next JND.

The more useful number of realized JNDs, describes how many JNDs are actually achieved given the specifics of the Display System (i.e. the number of gray levels of contrast resolution and the distribution of Luminance values). This definition gives a measure of the information which can actually be conveyed by the system to a human observer, in essence, an informational dynamic range. This number is calculated beginning at the minimum Luminance of the Display System, and then stepping one JND in Luminance from the current Luminance value, and choosing the smallest increment in DDL value that achieves a step at least that large. Repeating this through all the available DDLs will produce a sequence of steps, all at least 1 JND apart, and the length of this sequence of steps is then the number of realizable JNDs of the Display System.

The methods of PS 3.14 cannot precisely duplicate all of the real world sources of degradation in a Display System. However, this uniform method of determining the realizable number of JNDs should give a measure of the actual performance of a particular Display System which would be experienced by a human observer when using the Display System in a real world situation such as the viewing of radiological images in medicine.

References

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- [E4] Hemminger, B., Muller, K., "Performance Metric for evaluating conformance of medical image displays with the ACR/NEMA display function standard", SPIE Medical Imaging 1997, editor Yongmin Kim, vol 3031-25, 1997.