Radionuclide Decay
Curie/Becquerel

For a sample with N nuclei of a radioactive nuclide, the rate of decay in disintegrations per second is,

$$Q = -\lambda N = dN/dt$$

where Q is the amount of radioactive activity and λ (1/sec) is the decay constant.

The SI unit for activity (Q) is the Becquerel (Bq) which is equivalent to 1.0 disintegrations per second. The traditional unit of activity in the Curie (Ci) where 1.0 Curie is equal to $3.7x10^{10}$ disintegrations/second or 37 GBq. For radionuclides used in nuclear imaging, activities are often in the range from 1 to 20 mCi (40 to 800 MBq).

This simple differential equation has a solution of

$$N = N_o e^{-\lambda t}$$

and therefore;

$$Q = Q_o e^{-\lambda t}$$

The time for a radioactive sample of Q_o activity to decay to an activity equal to 1/2 of Q_o is easily shown to be;

$$T_{\frac{1}{2}} = \frac{ln(2)}{\lambda} = \frac{.693}{\lambda}$$

Table 7-1
Some Reactor-produced Radionuclides Used in Nuclear Medicine and Radiotracer Kinetics

Radionuclide	Decay Mode	Production Reaction	Natural Abundance of Target Isotope (%)	σ_c (b)*
^{14}C	β^-	$^{14}N(n,p)^{14}C$	99.6	1.81
^{24}Na	(β^-,γ)	$^{23}Na(n,\gamma)^{24}Na$	100	0.53
^{32}P	β^-	$^{31}P(n,\gamma)^{32}P$	100	0.19
		$^{32}S(n,p)^{32}P$	95.0	—
^{35}S	β^-	$^{35}Cl(n,p)^{35}S$	75.5	—
^{42}K	(β^-,γ)	$^{41}K(n,\gamma)^{42}K$	6.8	1.2
^{51}Cr	(EC,γ)	$^{50}Cr(n,\gamma)^{51}Cr$	4.3	17
^{59}Fe	(β^-,γ)	$^{58}Fe(n,\gamma)^{59}Fe$	0.3	1.1
^{75}Se	(EC,γ)	$^{74}Se(n,\gamma)^{75}Se$	0.9	30
^{125}I	(EC,γ)	$^{124}Xe(n,\gamma)^{125}Xe \xrightarrow{EC} {}^{125}I$	0.1	110
^{131}I	(β^-,γ)	$^{130}Te(n,\gamma)^{131}Te \xrightarrow{\beta^-} {}^{131}I$	34.5	0.24

*Thermal neutron capture cross-section, in barns, for (n,γ) reactions (see Section D.1). Values from ref. 1.

Table 7-2
Some Cyclotron-produced Radionuclides Used in Nuclear Medicine

Product	Decay Mode	Common Production Reaction	Natural Abundance of Target Isotope (%)
^{11}C	β^+	$^{10}B(d,n)^{11}C$	19.7
		$^{11}B(p,n)^{11}C$	80.3
^{13}N	β^+	$^{12}C(d,n)^{13}N$	98.9
^{15}O	β^+	$^{14}N(d,n)^{15}O$	99.6
^{18}F	β^+,EC	$^{20}Ne(d,\alpha)^{18}F$	90.9
^{22}Na	β^+,EC	$^{23}Na(p,2n)^{22}Na$	100
^{43}K	(β^-,γ)	$^{40}Ar(\alpha,p)^{43}K$	99.6
^{67}Ga	(EC,γ)	$^{68}Zn(p,2n)^{67}Ga$	18.6
^{111}In	(EC,γ)	$^{109}Ag(\alpha,2n)^{111}In$	48.7
		$^{111}Cd(p,n)^{111}In$	12.8
^{123}I	(EC,γ)	$^{122}Te(d,n)^{123}I$	2.5
		$^{124}Te(p,3n)^{123}I$	4.6
^{201}Tl	(EC,γ)	$^{201}Hg(d,2n)^{201}Tl$	13.2

Table 7-3
Some Radionuclide Generators Used in Nuclear Medicine

Daughter†	Decay Mode	$T_{1/2}$	Parent	$T_{1/2}$
^{68}Ga	β^+,EC	68 min	^{68}Ge	275 days
^{82}Rb	β^+,EC	1.3 min	^{82}Sr	25 days
^{87m}Sr	IT	2.8 hours	^{87}Y	80 hours
^{99m}Tc	IT	6 hours	^{99}Mo	66 hours
^{113m}In	IT	100 min	^{113}Sn	120 days

†Generator product.

Fig. 1.

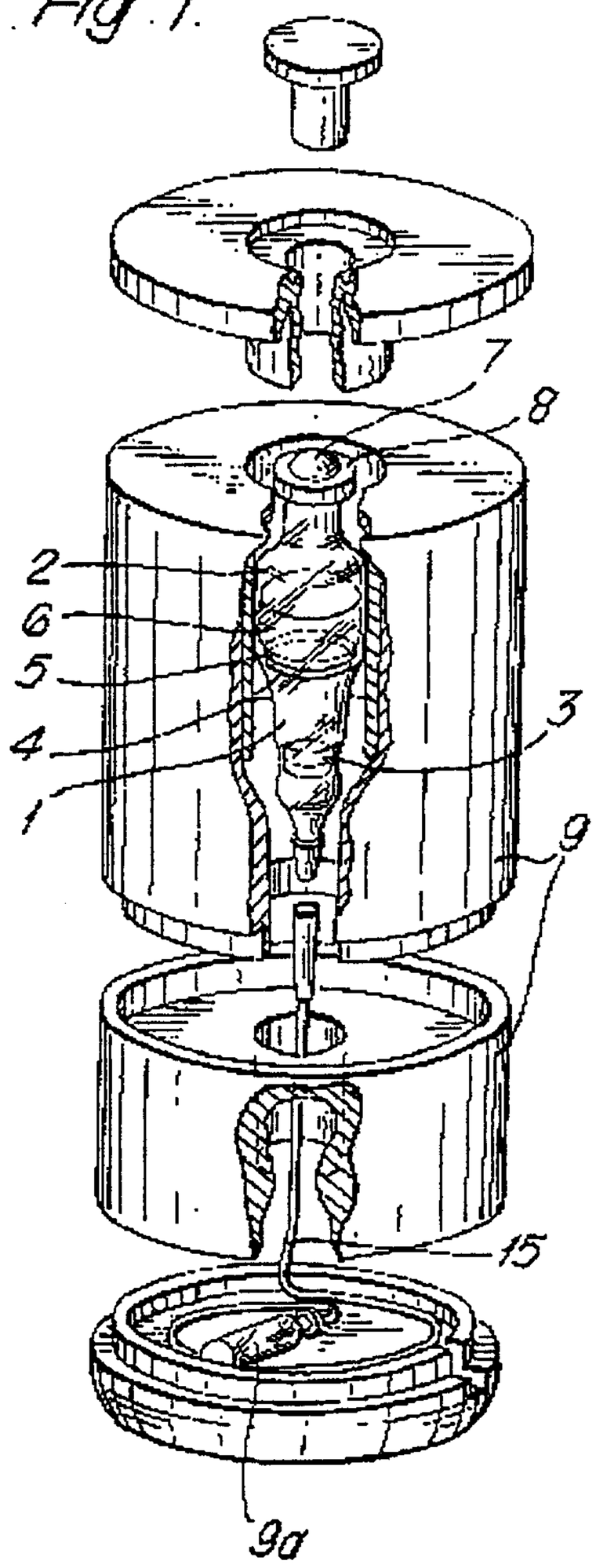
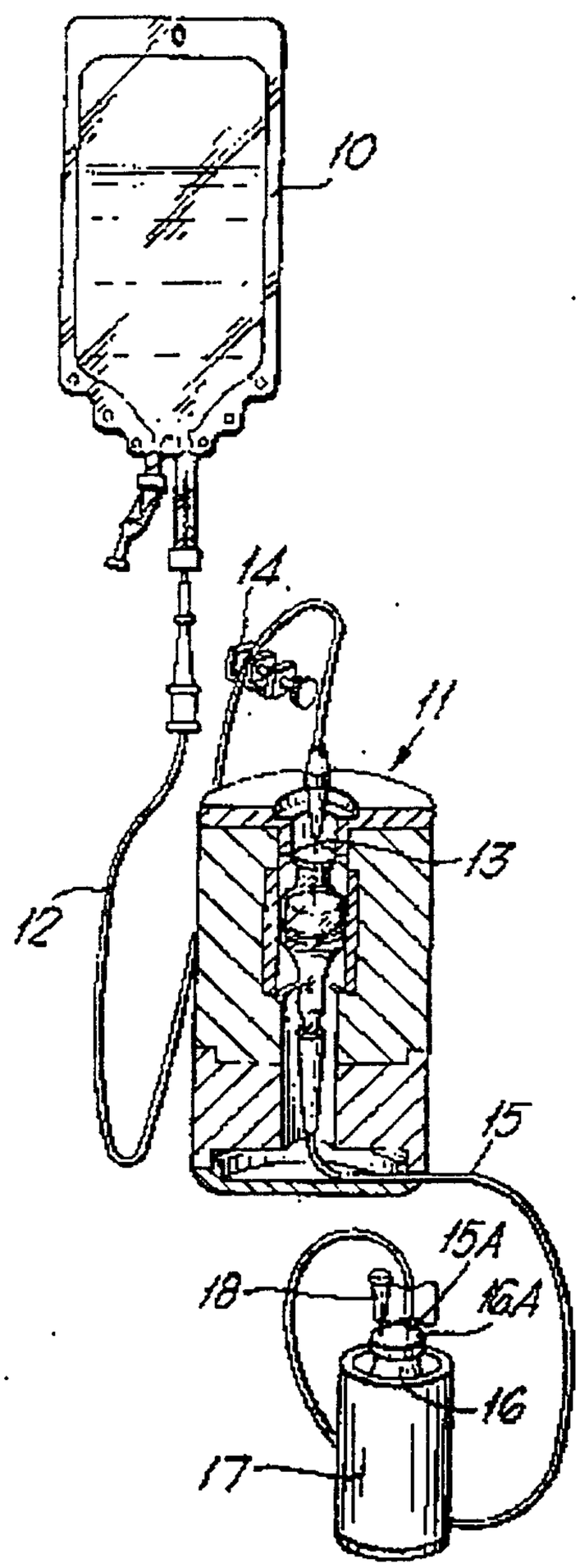


Fig. 2.



2. *Radioactive-series Decay. Growth of a Daughter Product*

In a number of cases a radioactive nuclide A decays into a nuclide B which is also radioactive. Let the initial part of such a series be represented by

$$A \xrightarrow{\lambda_A} B \xrightarrow{\lambda_B} C$$

where λ_A is the decay constant of atoms of type A and λ_B is the decay constant of atoms of type B, and where the symbols A and B represent the number of atoms of each type which are present at any time t. The limiting case in which B is stable is represented then by $\lambda_B = 0$.

a. The General Differential Equation for a Daughter Product. At any time t, the activity of A is $A\lambda_A$ and the activity of B is $B\lambda_B$. The rate of change dB/dt, in the number of atoms of type B, is then equal to the supply of new atoms of type B due to the decay of A, diminished by the rate of loss of B through its own decay, or

$$\frac{dB}{dt} = A\lambda_A - B\lambda_B \tag{2.1}$$

If the only source of atoms of type A is from an initial supply $A = A_0$ at $t = 0$, then

$$A = A_0 e^{-\lambda_A t}$$

and, with these initial conditions on A, Eq. (2.1) becomes

$$\frac{dB}{dt} = A_0\lambda_A e^{-\lambda_A t} - B\lambda_B \tag{2.2}$$

From this differential equation, we wish to obtain an explicit solution for B as a function of time. We proceed, as usual, by a seasoned guess that the general solution will be of the form

$$B = A_0(h_A e^{-\lambda_A t} + h_B e^{-\lambda_B t}) \tag{2.3}$$

In order to evaluate the coefficients h_A and h_B, we substitute B and dB/dt from Eq. (2.3) into Eq. (2.2) and collect terms, obtaining

$$e^{-\lambda_A t}(-h_A\lambda_A - \lambda_A + h_A\lambda_B) = 0 \tag{2.4}$$

If this is to be valid for all values of t, the parentheses must equal zero, and therefore we have

$$h_A = \frac{\lambda_A}{\lambda_B - \lambda_A} \tag{2.5}$$

The coefficient h_B depends on the value of B at $t = 0$. For the important special case in which $B = 0$ at $t = 0$, we have at once from Eq. (2.3)

$$h_A + h_B = 0 \tag{2.6}$$

Hence $h_B = -h_A$, and we have for the *amount* of B

$$B = A_0 \frac{\lambda_A}{\lambda_B - \lambda_A}(e^{-\lambda_A t} - e^{-\lambda_B t}) \tag{2.7}$$

	Half life min	Energy max. MeV	mean MeV	Range max. mm	mean mm
82-Rb	1.25	3.36	1.52	14.11	4.70
68-Ga	68.00	1.90	0.84	8.20	2.73
15-O	2.04	1.73	0.74	7.30	2.43
13-N	9.97	1.20	0.49	5.10	1.70
11-C	20.48	0.96	0.39	4.10	1.37
18-F	109.74	0.63	0.25	2.40	0.80