NERS/BIOE 481

Radiographic Image Formation
Signal, Contrast and Noise

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Transmissive Image Formation
Projections

The projection vector, p

In radiation transmission imaging, the object will modulate the intensity of radiation traveling along various vectors from the source. For a particular projection of radiation from the source, we use the variable t to represent the distance along the vector. The vector itself is described by the source position and the emission angle (ϕ, θ). For simplicity we will denote the projection vector as simply p.

Radiation transmission through uniform layers

The radiation emitted from the focal spot of a source can be expressed as $\phi(E, p, 0)$ in units of photons/keV/sec/mm^2 (or alternatively as ../sr rather than ../mm^2). The function is differential in x-ray energy E and dependent on the direction of the projection p. At some distance from the source the strength will also be dependent on the thickness of the object, T, for the projection p. For a single slab of homogeneous material, we have seen that:

$$\phi(E, p, T) = \phi(E, p, 0)e^{-\mu(E)T}$$

If we consider an object consisting of N slabs of thickness Δt The i-th slab will attenuate the radiation incident on it by $e^{-\mu_i(E)\Delta t}$. Thus the radiation transmitted through all slabs will be:

$$\phi(E, p, N) = \phi(E, p, 0)e^{-\sum_{i=0}^{N} \mu_i \Delta t}$$

Radiation transmission through nonuniform objects

In a more generalized object, the attenuation coefficient may vary as a continuous function along the projection as a function of t, $\mu(E,t)$, and the object thickness may be different for different projections, T_p In this case, the summation above can be written as an integral over the distance along the length of the projection:

$$\phi(E,p,T_p) = \phi(E,p,0)e^{-\int_0^{T_p} \mu(E,t)dt}$$

$\phi(E,p,T_p)$ is the photon fluence rate after traversing the object thickness T typically in units of photons/sr/sec or photons/mm^2/sec at the position of a detector.

The natural log of the relative transmission is frequently referred to a the projection of the linear attenuation coefficient and takes the form of the Radon transform:

$$P_p = -\ln \frac{\phi(E,p,T_p)}{\phi(E,p,0)} = \int_0^{T_p} \mu(E,t)dt$$

For computed tomography systems, the signal is measured electronically and immediately converted to a projection value, P_p, using stored calibration values for the measured fluence rate in the absence of an object.

Transmissive Image Formation
Ideal Detection

For radiation transmission imaging, every projection denoted by p is incident on the surface of a detector at some horizontal and vertical position of the recorded image field. Again for simplicity we will simply refered to the position on the detector where the projection p is detected as p_d.

Ideal Photon Counting Detector

Image formation can be understood in the context of an ideal imaging detector which records every photon incident on it's surface. An ideal photon counting detector will accumulate a record of the number of photons incident on the detector surface. The detected signal for an ideal photon counting detector, S_c, can thus be written as:

$$S_c(p_d) = A_d t_{sec} \int_0^{E_{max}} \phi(E, p, T_d) dE$$

or

$$S_c(p_d) = A_d t_{sec} \int_0^{E_{max}} \phi(E, p, 0) e^{-\int_0^{T_d} \mu(E,t)dt} dE$$

where t_{sec} is the exposure time in seconds, $\phi(E, p, T_d)$ is the photon fluence rate at the detector in photons/mm^2/sec, and T_d is the total thickness along the projection implied by p_d. At the point of detection, the detector has some effective area over which it will accumulate counts which is designated as A_d.

Transmissive Image Formation
Ideal Detection

Ideal Energy Integrating Detector

All most all detectors used for x-ray transmissive imaging record a signal proportional to the amount of energy deposited in the detector by the radiation beam. The signal from an ideal energy integrating detection, S_e, can thus be described by an integral over the energy fluence $\psi(E) = E\phi(E)$:

$$S_e(p_d) = A_d t_{sec} \int_0^{E_{max}} E\phi(E, p, T_d) dE$$

or

$$S_e(p_d) = A_d t_{sec} \int_0^{E_{max}} E\phi(E, p, 0) e^{-\int_0^{T_d} \mu(E,t) dt} dE$$

This is simply the first moment of the differential energy spectrum for the radiation flux incident on the detector, $\phi(E, p_d, T_d)$.

Signal / Noise

Terminology

If $\phi_m(E)$ is the normalized fluence rate incident on an imaging detector in units of photons/mA-s/mm^2, then the fluence rate is;

$$\phi(E) = \phi_m(E) \times mA$$

in units of photons/mm^2/s and the fluence in units of photons/mm^2 is;

$$\Phi(E) = \phi(E) \times s$$

The symbols chosen for fluence (Φ) and for fluence rate (ϕ) are chosen to be consistent with ICRU notations.

$\phi_m(E)$ is here understood to be the fluence at the detector surface for a particular projection, p, and object thickness, S_p (i.e. $\phi(E)$ is equivalent to $\phi(E, p, T_p)$ defined previously). Both $\phi(E)$ and $\Phi(E)$ therefore have an implied dependence on p and it's associated line integral describing attenuation in the object.

Monoenergetic Radiation

Consider a single position on the surface of a radiation imaging detector for which the detector response comes from an effective area of A_d. For a discrete detector with an array of small elements, this would be the individual element (i.e. pixel) area. If the source of radiation is all of energy E with a fluence at the detector surface of Φ_E photons/mm^2, then an ideal energy integrating detector will record $A_d\Phi_E$ number of photons with energy E and the signal in units of energy will be;

$$S_e = E(A_d\Phi_E)$$

For a counting detector, the variance of the recorded number is equal to the number. For this energy integrating detector, E is a constant and the variance of the signal comes from the variance in the recorded number of photons. A propogation of error analysis thus predicts the signal error to be;

$$\sigma_{s_e}^2 = E^2(A_d\Phi_E)$$

The signal to noise ratio (SNR) is thus seen to be independent of the incident energy;

$$\frac{S_e}{\sigma_{s_e}} = (A_d\Phi_E)^{1/2}$$

and the SNR squared is equal to the recorded number of photons recorded by the detector;

$$\left(\frac{S_e}{\sigma_{s_e}}\right)^2 = A_d\Phi_E$$

For actual detectors recording a spectrum of radiation, the actual SNR^2 is often related to the equivalent number of monoenergetic photons that would produce the same SNR with an ideal detector.

$$\Phi_E \equiv Q_{eq}$$

Polyenergetic Radiation (discrete spectra)

Consider first a discrete spectrum of radiation where the fluence incident on the detector at energy E_i is Φ_i and the signal is;

$$S_e \approx A_d \sum_i E_i \Phi_i$$

A propogation of error analysis indicates that each discrete component of variance, $E_i^2(A_d\Phi_i)$, will add to form the total variance. Since A_d is constant, we can express the relationship for signal variance as;

$$\sigma_{s_e}^2 \approx A_d \sum_i E_i^2 \Phi_i$$

Note that for an individual detector element, the above discrete summation is simply equal to the sum of the squared energy deposited in the detector by each photon that strikes the element, $\sum_n e_n^2$. This is used to estimate signal variance when using Monte Carlo simulations which analyze the interaction of each of a large number of photons.

Polyenergetic Radiation (continuous spectra)

When the fluence spectrum (i.e. the differential energy fluence spectrum) is known as a continuous function, $\Phi(E)$, the integral relationships for the signal and variance are;

$$S_e = A_d \int_0^{kV_p} E\Phi(E)dE$$

$$\sigma_{s_e}^2 = A_d \int_0^{kV_p} E^2\Phi(E)dE$$

and the SNR^2 is;

$$\left(\frac{S_e}{\sigma_{s_e}}\right)^2 = A_d \frac{\left(\int_0^{kV_p} E\Phi(E)dE\right)^2}{\int_0^{kV_p} E^2\Phi(E)dE}$$

The terms involving the first and second energy moments of the fluence spectrum represent the equivalent number of monoenergetic quantua;

$$Q_{eq} \equiv \frac{\left(\int_0^{kV_p} E\Phi(E)dE\right)^2}{\int_0^{kV_p} E^2\Phi(E)dE}$$

$$\left(\frac{S_e}{\sigma_{s_e}}\right)^2 = A_d Q_{eq}$$

The noise equivalent number of quanta is often referred to as the NEQ or symbolically as either Q_{eq} or Φ_{eq}.

Definitions

Consider an imaging detector which records a relatively uniform fluence in most positions on the surface except for that associated with a small object. The object is frequently a small defect or abnormality representing a target of interest. We define the difference between the signal in the position where the target projection is located and the signal in background regions as the contrast;

$$C = S_t - S_b$$

The relative contrast is defined with respect to the background signal;

$$C_r = \frac{S_t - S_b}{S_b}$$

The contrast to noise ratio, CNR, is thus seen to be simply the product of the relative contrast and the SNR.

$$\frac{C}{\sigma_s} = \frac{S_t - S_b}{\sigma_s} = C_r \left(\frac{S_b}{\sigma_s} \right)$$

The CNR is an important variable influencing the ability to detect a small target in an image containing noise.

Monoenergetic Radiation
Contrast for a small target

The relatifve contrast produced by a target object comes from a difference between the linear attenuation coefficient of the target material, μ_t, and the surrounding material producing the background signal, μ_b. Consider a object with uniform total thickness, t, and composition except for a small target object of thickness δt. For monoenergetic radiation, the relationships for the target signal, S_t, and the background signal, S_b, can be written in terms of the fluence incident on the entire object as;

$$S_b = EA_d\Phi_o e^{-\mu_b t}$$
$$S_t = EA_d\Phi_o e^{-\mu_b(t-\delta t)} e^{-\mu_t \delta t}$$

For S_t, the arguements of the exponents can be conveniently rearranged as;

$$S_t = EA_d\Phi_o e^{-\mu_b t} e^{-(\mu_t-\mu_b)\delta t}$$

and the target signal expressed as an exponential factor modifying S_b;

$$S_t = S_b e^{-(\mu_t-\mu_b)\delta t}$$

The relative contrast is thus;

$$C_r = \frac{S_t - S_b}{S_b} = e^{-(\mu_t-\mu_b)\delta t} - 1$$

which, since the target thickness is small, can be simplified by using a first order Taylor series expansion for the exponent (i.e. $e^\epsilon \approx 1 - \epsilon$ for $\epsilon \ll 1$);

$$C_r \approx -\Delta\mu\delta t$$

Recall that the attenuation coefficients are strong function of incident energy, therefore the change in the value of the linear attenuation coefficients for the background material and the target, $\Delta\mu = \mu_t - \mu_b$, is in fact a function of energy. An increase in contrast is generally observed for a decrease in energy due to increased photoelectric absorption. For the case of a void, $\Delta\mu$ is equal to $-\mu_b$ and C_r will be positive corresponding to an increase in signal at the position of the target.

Monoenergetic Radiation
Noise for a small target

The signal noise variance in the background can similarly be expressed in terms of the fluence incident to the object;

$$\sigma_{s_b}^2 = E^2 \left(A_d \Phi_o e^{-\mu_b t}\right)$$

$$\sigma_{s_b} = E \left(A_d \Phi_o\right)^{1/2} e^{-\frac{\mu_b t}{2}}$$

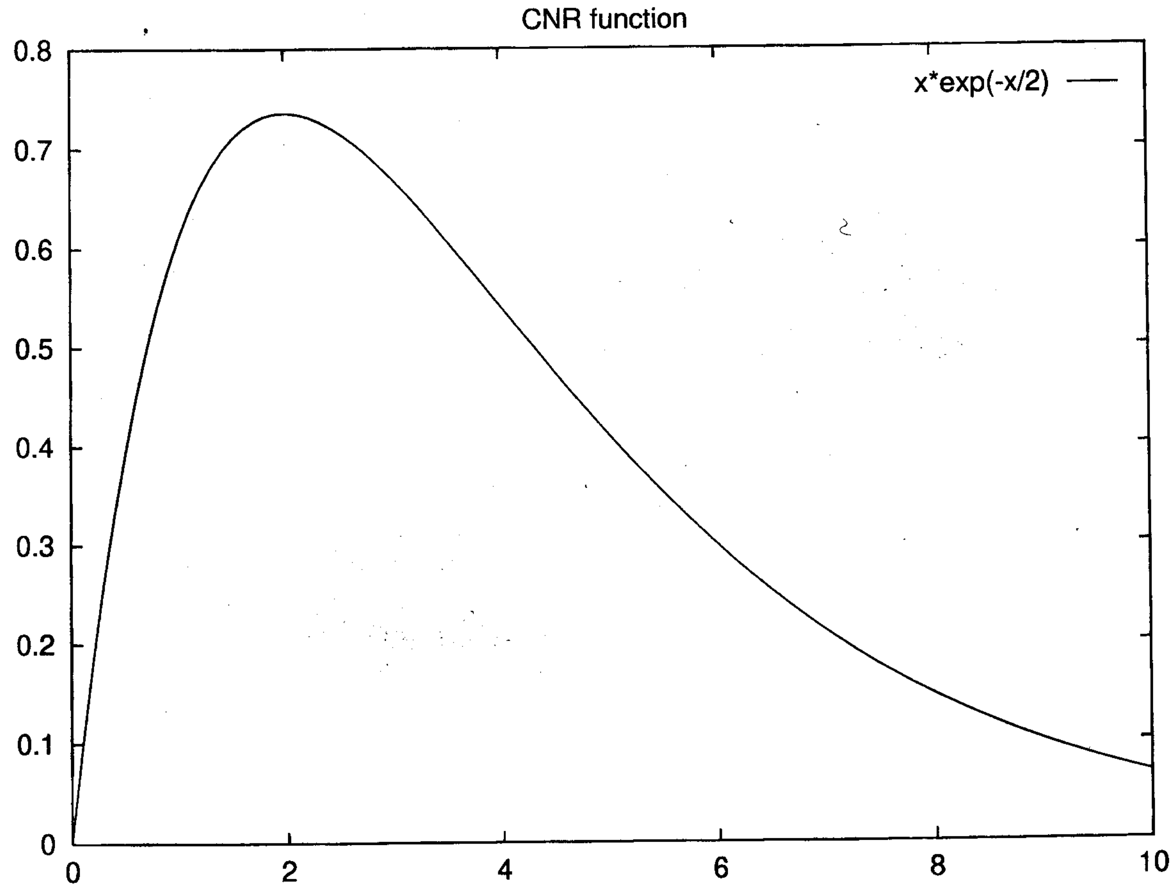
The SNR is thus;

$$\frac{S_b}{\sigma_{s_b}} = \left(A_d \Phi_o\right)^{1/2} e^{-\frac{\mu_b t}{2}}$$

and the CNR, C_r times SNR, is;

$$\frac{C}{\sigma_s} = -\Delta\mu\delta t \left(A_d \Phi_o\right)^{1/2} e^{-\frac{\mu_b t}{2}}$$

For a particular radiographic inspection task, a photon energy is sought which produces values of μ_b and $\Delta\mu$ that maximize CNR.


CNR function
x*exp(-x/2)
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
2
4
6
8
10

Polyenergetic Radiation

For polyenergetic radiation spectra and objects of arbitrary size, the signal in the background and target regions are described by integrals of the projection line, dt, and of energy, dE;

$$S_b = A_d \int_0^{kV_p} E\Phi_o(E)e^{-\int \mu_b(E,t)dt} dE$$

$$S_t = A_d \int_0^{kV_p} E\Phi_o(E)e^{-\int \mu_t(E,t)dt} dE$$

The CNR expressed in terms of noise equivalent quanta is;

$$\frac{C}{\sigma_s} = C_r \left(A_d \Phi_{eq}(T)\right)^{1/2} \quad , \quad T = kV_p$$

where C_r is determined by the integral expressions for S_b and S_t and the noise equivalent quanta is given by the ratio of the first and second moments of the flux incident on the detector;

$$\Phi_{eq}(T) = \frac{\left[\int_0^T E\Phi_o(E)e^{-\int \mu_b(E,s)ds} dE\right]^2}{\int_0^T E^2\Phi_o(E)e^{-\int \mu_b(E,s)ds} dE}$$

Optimization of the CNR for polyenergetic radiational spectra usually involves numeric simulation methods to perform the integrations.