Spatial Response Functions
Point Spread Functions and Line Spread Functions

The resolution of a system may be measured or computed by considered the response of the system to radiation input along a single projection vector. If this vector is associated with the position $(0, 0)$ on the radiation imaging detector, then the **point response function** , $p(x, y)$, describes the manner in which the recorded signal from the detector blurs the input from a point source. While a source of radiation which is associated with a single vector is straightforward for analystic or computational models of a system, it is difficult to experimentally create such an input signal. A linear input to the system can be obtained from a slit phantom and has been commonly used to test radiation imaging systems.

For a linear imaging system, the **line spread function** is simply the integral of many differential elements of the line source each of which behave as point sources:

$$l(x') = \int_{-\infty}^{+\infty} p(x', -y)dy$$

The variable x' is the distance from the line where the line spread function is evaluated and the variable y is the distance along the line for a line oriented parallel to the y axis. For a differential element at a positive y position, the point spread function is evaluated with a negative y arguement indicating that $l(x')$ is evaluated as the signal along the x axis.

Since the order of integration is not relevant, this equation can be rewritten as:

$$l(x') = \int_{+\infty}^{-\infty} p(x', y')dy'$$

where $y' = -y$. Thus the line spread function may also be interpreted as the projection integral of the point spread function along vertical vectors located at various x' positions.

As written, this line spread function represents the response to a vertical line and thus the horizontal resolution response. Equivalent expressions can be written to describe vertical resolution or the response to a line at any angle in the (x, y) plane of the detector.

Spatial Response Functions
Edge Spread Functions

While slit test devices have been frequently used to measure the line spread function, a very narrow slit transmits very little radiation and must be precisely aligned so that the slit opening is parallel to the radiation vectors. A very sharp edge can be easier to fabricate and align tha a narrow slit and the open portion of the edge contains high radiation flux. The response to an edge is thus a resolution measurement that can be easy to perform.

In a similar fashion to the derivation of the line spread function from the point spread function, the **edge spread function** can be derived as the integration of a large number of differential elements which behave as line sources:

$$e(x) = \int_{x'}^{+\infty} l(x')dx'$$

where x' is the position denoting the beginning of an opaque edge with the border of the edge oriented parallel to the y axis. The integral is then taken to $+\infty$.

Notably, the line spread function is seen to be simply the derivative of the edge spread function:

$$l(x) = \frac{d[e(x)]}{dx}$$

This makes it easy to evaluate the line spread function from the edge spread function and subsequently evaluate the modulation transfer function from the line spread function.

Units of the Noise Power Spectrum

$N_p(E,f)$ and $NPS(E,f)$

An ideal detector recording $Q_o(E)$ monoenergetic photons/mm^2 has a signal to noise ratio of $\sqrt{Q_o(E)}$ (i.e. $Q_o(E)/\sqrt{Q_o(E)}$). The image recorded by a detector in an actual imaging system will exhibit larger noise and lower signal to noise ratio than the ideal detector. We can define $Q_{eq}(E)$ as the signal to noise ratio observed in a recorded image for regions large enough that the observed noise is uncorrelated. $Q_{eq}(E)$ is often referred to as the noise equivalent number of quanta or $NEQ(E)$ and has units of photons/mm^2.

The frequency dependent noise characteristics of an image recorded from a uniform field of monoenergetic radiation are described by the autocorrelation function or the associated noise power spectrum, $N_p(E,f)$. (Note: we write here the noise power spectrum in terms of one dimension of spatial frequency assumed to be in a particular direction.) When $N_p(E,f)$ is properly normalized, it has units of mm^2 and the value at zero frequency is equal to $1/Q_{eq}(E)$. That is, the $N_p(E,f)$ at $f=0$ reflects uncorrelated noise associated with very large correlation distances. $N_p(E,0)$ thus reflects noise which would be observed if the signal was measured for very large image elements.

The noise power spectrum can thus be separated into a magnitude term, $1/Q_{eq}(E)$, and a relative frequency dependence term, $NPS(E,f)$:

$$N_p(E,f) = \frac{1}{Q_{eq}(E)} NPS(E,f)$$

This provides a convenient form to consider the noise transfer characteristics of a radiation imaging system.

$D_{QE}(E)$ and D_{QE}

For radiation detected by an ideal detector, the (signal/noise)^2 for monoenergetic detectors is simple $Q_o(E)$ #/mm^2 and for a spectrum of radiation can be computed by the first and second energy moments of the spectrum and defined as Q_o. We previously emphasized that Q_o can not be obtained by integrating $Q(E)$.

Similarly we have seen how the (signal/noise)^2 in the image recorded by a detector can be interpreted as a noise equivalent flux, $Q_{eq}(E)$. For a detector recording a spectrum of radiation determination of Q_{eq} may not be obtained by analyticaly integrating $Q_{eq}(E)$. Independent analysis of the signal and noise transfer properties of the detector is required.

A single measure of detector performance can be defined as the ratio of the (signal/noise)^2 input to a detector to the (signal/noise)^2 observed in the recorded image. For monoenergetic radiation this detective quantum efficiency is defined as

$$D_{QE}(E) = \frac{Q_{eq}(E)}{Q_o(E)}$$

The response of a system to a spectrum of radiation can be similarly defined as

$$D_{QE} = \frac{Q_{eq}}{Q_o}$$

and D_{QE} also can not be obtained by integration of $D_{QE}(E)$

Signal/Noise Transfer
as a function of spatial frequency

$D_{QE}(E,f)$ and $DQE(E,f)$

Since $MTF(E,f)$ describes the relative signal transfer characteristics of a system as a function of frequency and $NPS(E,f)$ describes the relative noise transfer characteristics of the system, we can define the relative frequency dependance of the detective quantum efficiency as

$$DQE(E,f) = \frac{MTF^2(E,f)}{NPS(E,f)}$$

Since the MTF represents signal transfer it appears as a squared term wheras the NPS reflects noise variance and thus has units of (noise)^2.

The overall frequency dependent detective quantum efficiency reflecting the (signal/noise)^2 transfer is then expressed as

$$D_{qe}(E,f) = \frac{Q_{eq}(E)}{Q_o(E)} DQE(E,f)$$

or

$$D_{qe}(E,f) = D_{qe}(E) DQE(E,f)$$

where the first term is the large area, zero frequency detective quantum efficiency in absolute efficiency units and the frequency dependence is described by $DQE(E,f)$ as a relative function with $DQE(E,0) = 1.0$ as is the case with $MTF(E,f)$ and $NPS(E,f)$.

We will consider later how to determine the detective quantum efficiency for a detector exposed to a spectrum of radiation.