Point Spread-Function, Line Spread-Function, and Modulation Transfer Function

Tools for the Study of Imaging Systems[1]

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IN RECENT YEARS the transfer theory of linear, invariant communication systems has been of increasing usefulness in the study of radiographic imaging systems (1–4). Indeed, its application is not confined to the imaging systems themselves but can be extended to the analysis of the entire radiological process involving exposing, imaging, and visual detection operations (5). This analysis will eventually result in quantitative descriptions of the inherent limitations of present radiological processes and, hopefully, in the development of improved processes yielding increased diagnostic certainty.

At present the number of investigators working in this field is very small—too small, in fact, to insure a satisfactory rate of progress toward the important goal. This is partly due to the fact that the physics and mathematics involved are highly specialized and cannot readily be assimilated from the existing literature. Any author in this field is faced with the dilemma of writing lucidly for readers having diverse backgrounds of scientific training, since meaningful investigations in this field cannot be carried out by physicists alone but must be made in cooperation with radiologists who are the only ones, after all, who fully appreciate the operational aspects of the radiological process. In this cooperation each investigator will tend to contribute most in the field for which he was trained. On the other hand, it is helpful if all investigators develop a common language and an understanding of basic concepts. With this in mind, the present discussion of some important concepts of optical communication theory is presented in nonmathematical form. A brief mathematical derivation is given in the APPENDIX.

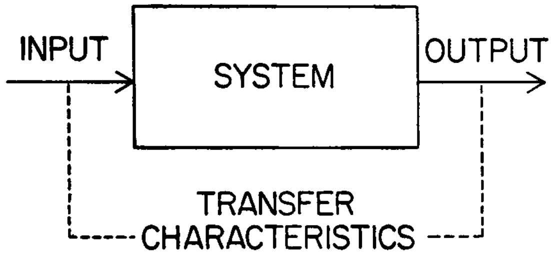

Fig. 1. Schematic representation of a transducer from the viewpoint of communication theory.

GENERAL DESCRIPTION OF BASIC CONCEPTS

The Point Spread-Function

In the analysis of a physical system, methods of communication theory are used to determine the performance of the system as a transducer in converting a system input to an output. It is not the aim of communication theory to investigate in detail the interior of a system but rather to characterize a system terminally by establishing a general dependence of the output on the input. As indicated in Figure 1, the problem can be stated as follows: Given a black box (the system), determine its transfer characteristics so that the output resulting from any conceivable input can be uniquely predicted.

The practical importance of knowing the system transfer characteristics is obvious. For example, if the system is a sound transmitter the "fidelity" of the output can be predicted; in the case of imaging systems the image deterioration introduced for any given object can be predetermined. The present discussion will be confined to imaging systems.

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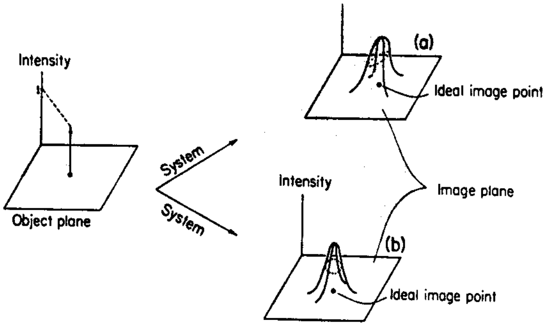

Fig. 2. The point spread-function is the image of a unit intensity point source (arrow). *a*, nonisotropic system; *b*, isotropic system.

In the general case, this system analysis is extremely complicated. It can be simplified, however, if the investigation is restricted to a particular type of system having the following properties:

1. Linearity, which is distinguished by the two basic characteristics: (*a*) the output corresponding to a sum of inputs is equal to the sum of the outputs corresponding to the inputs acting separately; and (*b*) multiplication of the input by a constant multiplies the output by the same constant.

2. Invariance, which requires that the image of a point retains its shape as the object point is moved in the object plane. This property is frequently called *isoplanatism*. Often this requirement is not fulfilled in the entire image plane. In that case the image plane can be decomposed into isoplanatic patches over each of which the shape of the point image does not vary appreciably. The shape of the point image, however, does vary from patch to patch. For the sake of clarity the following discussion will be confined to systems which are isoplanatic as a whole. Conclusions reached, however, will be applicable within each isoplanatic patch in systems which are not isoplanatic as a whole.

The isoplanatic property enables us to define a unique characteristic of the imaging system which is called the *point spread-function* of the system and is defined as the radiation intensity distribution in the image of an infinitely small aperture

In a perfect imaging sytem the radiant energy emanating from a point source in the object plane would be concentrated at a point in the image plane, the ideal image point. In practical systems, however, optical imperfections result in a "smearing-out" of the energy around the ideal image point and, therefore, in unsharp imaging of the point source. The point spread-function provides a measure of this unsharpness. In Figure 2 the unit point source is shown as an arrow of unit length standing on the object plane and the point spread-function as a "hump" on the image plane. As indicated schematically in Figure 2, A, the point spread-function is unsymmetric in general. For certain systems, however, the point spread-function possesses rotational symmetry as shown in Figure 2, B. Systems of this type are called *isotropic.* The isotropic property results in a simplified description of the transfer characteristics of the system which will be discussed later. From the above, it is apparent that the point spread-function is, in fact, a transfer characteristic of the system for a specific input in that it provides a unique relation between a unit point source input and the corresponding output. It will now be shown that the point spread-function is of much broader significance for linear systems.

From the first property associated with system linearity described previously it follows that, if an arbitrary number of unit point sources is located in the object plane, each of them will be imaged independently of the others as a point spread-function in the image plane. The total image of all unit point sources is then simply the sum of all corresponding point spread-functions over the image plane. This is known as the *superposition principle* of linear imaging.

From the second property of linear systems it follows that, if the intensity of a unit point source is multiplied by some constant, then the corresponding point spread-function will be multiplied by the same constant to yield the image of the point source of nonunit intensity. This property in conjunction with the superposition principle leads to the conclusion that, if the input consists of a field of point sources of arbitrary intensity, the output or the total image of the field of point sources is simply the sum of the corresponding point spread-functions, each multiplied by an appropriate constant to take account of the intensity of the corresponding point source. Figure 3 illustrates this phenomenon for two point sources.

It is conceptually not difficult to extend the case of a finite number of point sources to the practical case of a continuous, two-dimensional object which can be considered as an aggregate of an infinite number of point sources of different intensities. The image of each point in the object is the point spread-function multiplied by an appropriate intensity factor, and the total image is the sum of all the point images. Thus, from a knowledge of the input intensity distribution in the object and of the system point spread-function the output intensity distribution in the image can be determined. Therefore, the point spread-function is a general transfer characteristic of linear, isoplanatic imaging systems. The mathematical operation of multiplying each point in the object intensity distribution by the system point spread-function and summing over the entire object distribution is known as *convolution* of the input with the point spread-function.

Before this convolution can be carried out, the point spread-function of a given system needs to be measured by using a small aperture as a source. Direct measurement of the point spread-function, however, is difficult for two experimental reasons. First, in order to approximate a point source of radiation the aperture must be made quite small relative to the size of the point spread-function. Under practical conditions this yields a very low input radiation intensity into the system. Second, measurement of the resulting intensity distribution in the image plane requires scanning with a small aperture exactly through the center of the distribution which causes alignment difficulties. These experimental problems which cause inaccurate measurements can be overcome by measuring another transfer characteristic of the system from which the point spread-function can be calculated.

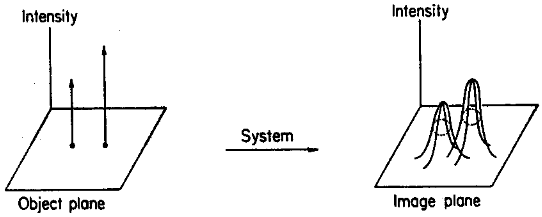

Fig. 3. The image of two point sources formed by a linear, isotropic system (superposition principle).

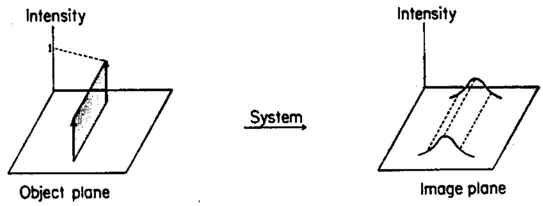

Fig. 4. The line spread-function is the image of a unit intensity line source.

The Line Spread-Function

For linear, isoplanatic imaging systems, a second transfer characteristic can be defined. This is the *line spread-function* which represents the radiation intensity distribution in the image of an infinitely narrow and infinitely long slit (line source) of unit intensity. In a perfect imaging system the radiant energy emanating from a line source in the object plane would be concentrated in a line in the image plane. In practical systems, however, optical imperfections result in a "smearing-out" of the energy around the ideal line image and, therefore, in unsharp imaging of the line source. The line spread-function provides a measure of this unsharpness. In Figure 4 the unit line source is shown as a "knife edge" of unit height standing on the object plane, and the line spread-function as a "welt" on the image plane. It will be shown later that the line spread-function is a system transfer characteristic

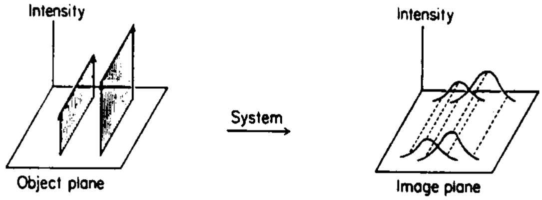

Fig. 5. The image of two line sources formed by a linear system (superposition principle).

in that it provides a unique relation between a certain class of arbitrary inputs and the corresponding outputs.

In practice the line spread-function is measured by approximating a line source with a slit which is narrow and long relative to the size of the point spread-function, and by scanning the resulting output (the slit image) with a narrow slit. This experimental technic eliminates both difficulties associated with the direct measurement of the point spread-function. Determining the point spread-function from the measured line spread-function, however, is no simple matter in general. This is due to the fact that the line spread-function is a one-dimensional function obtained from a rectilinear scan of a one-dimensional intensity distribution, whereas the point spread-function is two-dimensional. This can be explained by noting the relationship between the two functions.

It can be shown mathematically (see APPENDIX) that the direct measurement of the line spread-function described above is equivalent to scanning the point spread-function with a slit which is narrow and long relative to the size of the point spread-function. Since the point spread-function is often unsymmetric, as indicated in Figure 2, A, the shape of the line spread-function depends on the direction in which the point spread-function is scanned. For the calculation of the point spread-function the line spread-functions corresponding to all possible orientations of the scanning slit must be known (6). In terms of the directly measured line spread-function this means that the line source must be placed in all possible orientations in the object plane. Matters are simplified considerably when the imaging system is isotropic (7). In that case the point spread-function is rotationally symmetric, as indicated in Figure 2, B, and the shape of the line spread-function is independent of the orientation of the line source in the object plane and is also symmetric. Thus, if the system is isotropic, one measurement of the line spread-function suffices for the calculation of the point spread-function.

To summarize, the line spread-function serves as an experimentally accurate tool for determining the point spread-function which is a system transfer characteristic for the most general case of two-dimensional radiation intensity distributions in the object plane. In addition, it can be shown that the line spread-function is a system transfer characteristic for the special case of one-dimensional inputs. The reasoning is analogous to the case of the point spread-function in the foregoing section and will, therefore, be presented only in its essentials.

As discussed previously, the point spread-function is a unique characteristic of isoplanatic imaging systems. Therefore, the line spread-function is also a unique system characteristic for any one orientation of the line source relative to nonisotropic systems or for any orientation relative to isotropic systems. If the system is also linear, an input consisting of a field of line sources of arbitrary intensity will result in an output which is the sum of the corresponding line spread-functions, each multiplied by an appropriate intensity factor. Figure 5 illustrates this for two line sources. If the input is a continuous object over which the radiation intensity varies in one dimension only, such as a straight-edge or a bar pattern, the object can be considered as an aggregate of an infinite number of line sources of different intensities. The corresponding output is calculated by multiplying each line source in the object intensity distribution by the system line spread-function and summing over the entire object distribution. This one-dimensional convolution operation is illustrated in Figure 6. Therefore, the line

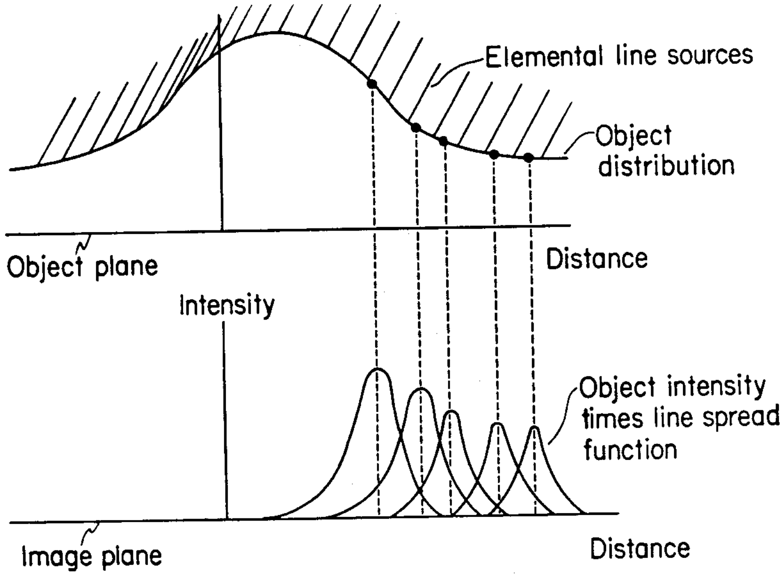

Fig. 6. Schematic illustration of one-dimensional convolution. Only a few of an infinite number of elemental line images are shown.

spread-function is a transfer characteristic of linear, isoplanatic imaging systems for the special case of one-dimensional inputs.

It must be emphasized that the line spread-function does not serve as a magical shortcut to reduce a two-dimensional transfer problem to a one-dimensional one. To describe the transfer of two-dimensional inputs, the point spread-function is needed, and the line spread-function is merely an accurate experimental tool to determine the point spread-function. The line spread-function leads to a simplification of the overall problem only in the case of one-dimensional inputs. Even in this case, the two-dimensional character of the imaging system cannot be ignored entirely when the system is nonisotropic. The slit which is used to measure the line spread-function must be oriented in the same direction relative to the imaging system in which the direction of constant intensity in the one-dimensional object is oriented. Only in the study of isotropic systems can the orientation of the object relative to the system be ignored.

The Modulation Transfer Function (MTF)

In the foregoing sections two methods of describing the optical transfer characteristics of imaging systems have been discussed. It was seen that by means of the system point spread-function, the output resulting from an arbitrary, two-dimensional input can be predicted. Similarly, the system line spread-function can be used directly to predict the output corresponding to an arbitrary, one-dimensional input. The great utility of these concepts derives from the fact that the study of the transmission of complex object intensity distributions is reduced to the study of the transmission of very simple intensity distributions—namely, a point source or a line source. Once the relatively uncomplicated experiments to measure the transmission of these simple object distributions have been carried out on a given system, the transmission of any conceivable object distribution can be calculated.

In this context the study of the transmission of a third simple object intensity distribution in which the intensity varies sinusoidally with distance in the object plane is of particular usefulness. The solid curves in Figure 7 depict two such distributions having different *spatial frequencies* measured in cycles/mm. The term spatial frequency does not imply a vibration or change in time of the intensity distribution. The distribution is considered stationary in time and space just as the line

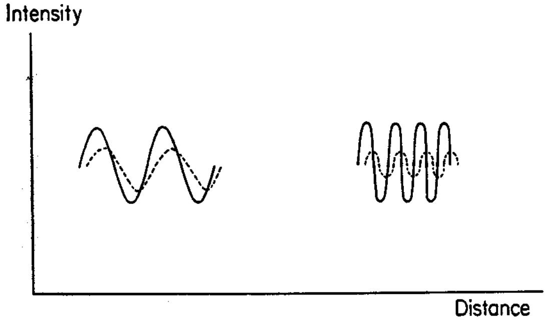

Fig. 7. Sinusoidal intensity distributions in space. Solid lines, input distributions of different spatial frequency; dotted lines, output from a linear, nonisotropic system.

pattern in a resolution chart which is described as having a certain number of lines/mm.[2] When such a sinusoidal intensity distribution in the object plane is imaged by means of a linear, isoplanatic, nonisotropic system of unit magnification, the intensity distribution in the image plane will also be sinusoidal with the same spatial frequency as the object distribution. The optical imperfections of the system, however, will cause the amplitude of the image distribution to be reduced, and the distribution as a whole will be shifted laterally relative to the object distribution as shown by the broken curves in Figure 7. The reduced amplitude indicates a loss of resolution in the system, and the lateral shift is described as a phase shift. It is customary to characterize the sinusoidal distribution in terms of its modulation rather than its amplitude. *Modulation* is defined as the ratio of the amplitude to the average value of the distribution. Note that the average value of a sinusoidal distribution of radiation intensity cannot be smaller than the amplitude of the distribution since negative values of intensity are physically meaningless. Therefore, the modulation cannot be greater than unity.

In practice, several sinusoidal intensity distributions having different spatial frequencies but identical amplitudes and modulations are introduced in the object plane, and the modulation and phase shift of each is measured in the image plane. The modulations and phase shifts in the image will vary with spatial frequency. The ratio of the output modulation to the input modulation together with the phase shift expressed as a function of spatial frequency is called the *optical transfer function* of the system (8). The ratio of the output modulation to the input modulation alone, expressed as a function of spatial frequency, is called the *modulation transfer function* of the system. In mathematical terms the modulation transfer function is the absolute value of the optical transfer function. For a complete description of the transfer of sinusoidal inputs through nonisotropic systems the optical transfer function is required since a phase shift occurs in these systems. In isotropic systems the phase shift is zero, so that the modulation transfer function completely describes the transfer of sinusoidal inputs. For the sake of simplicity the following discussion will be confined primarily to the modulation transfer function.

The mere fact that the modulation transfer function provides a description of the imaging of sinusoidal intensity distributions is not sufficient reason for introducing it, since the same can be done by convolving the sinusoidal distribution with the system line spread-function. The great significance of the modulation transfer function lies in the fundamentally different manner in which it describes the transfer of sinusoidal inputs. It will be recalled that the calculation of the output from the input by means of the point or line spread-functions proceeds from a point-by-point knowledge of the intensity distribution in the object plane. Convolution of the object distribution with the point or line spread-function, which is expressed in terms of distance in the image plane, results in a point-by-point description of the intensity distribution in the output plane. In other words, point and line spread-function are transfer characteristics of the system in the *spatial domain*. Calculation of the output from the input sinusoidal distribution by

[2] In the analysis of systems for the imaging of moving phenomena, *e.g.*, fluoroscopic systems, an additional temporal variation of the sinusoidal input needs to be introduced. This case will not be discussed here.

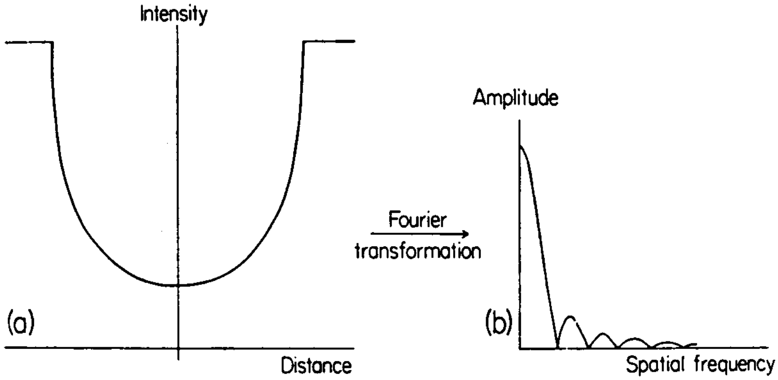

Fig. 8. Two equivalent methods of describing a signal: *a*, x-ray pattern of a blood vessel; *b*, corresponding amplitude spectrum.

means of the modulation transfer function, on the other hand, proceeds from a knowledge of the modulation and the spatial frequency of the input. Multiplication of the input modulation by the modulation transfer function, which is expressed in terms of spatial frequency, results in the modulation of the output having the same spatial frequency. Thus, the modulation transfer function describes the transfer of sinusoidal inputs in the *spatial frequency domain.* Note that the mathematically complicated convolution operation in the spatial domain is replaced by simple multiplication in the spatial frequency domain. (Compare equations (2) and (5) in the APPENDIX.) It will now be shown that the modulation transfer function also provides a unique relation between arbitrary, not necessarily sinusoidal inputs and the corresponding outputs and is therefore a general system transfer characteristic.

It is well known (9) that most nonperiodic variations of a quantity in time or in space (signals, inputs, etc.) can be represented as a sum of an infinite number of sinusoidal component signals of different amplitudes and frequencies. The mathematical process for accomplishing this harmonic analysis is called *Fourier transformation.* A plot of the amplitudes of the component signals as a function of frequency is known as the *amplitude spectrum*[3] of the original signal. In the temcycles/mm as described above. Figure 8 shows an example of a signal and its amplitude spectrum. This particular signal is the input to a radiographic imaging system (5). In principle, describing the input as an intensity distribution in the spatial domain and as an amplitude distribution in the frequency domain is analogous to describing it in two different languages. The two descriptions are equally valid and comprehensive,[4] and the Fourier transform is the means for translating from one language to the other. For example, in Figure 8 the width of the input in the spatial domain translates into the magnitude of the amplitudes at high frequencies or, simply, the high-frequency content in the frequency domain. Similarly, the study of the transfer of intensity distributions in the spatial domain becomes a study of the transfer of amplitude spectra in the frequency domain. Since the modulation transfer function describes the transfer of sinusoidal inputs in the frequency domain, it also describes the transfer of amplitude spectra. Therefore, the modulation transfer function is a general transfer character-

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[3] In this discussion the amplitude spectrum is defined as the absolute value of the complex Fourier spectrum, which is in keeping with mathematical and engineering terminology. It is not to be confused with the Fourier spectrum of an input amplitude distribution which is often called an amplitude spectrum in optics.

[4] In general the description in the frequency domain must also include the phases of the sinusoidal components. Since the present discussion is concerned mainly with the modulation transfer function and not with the

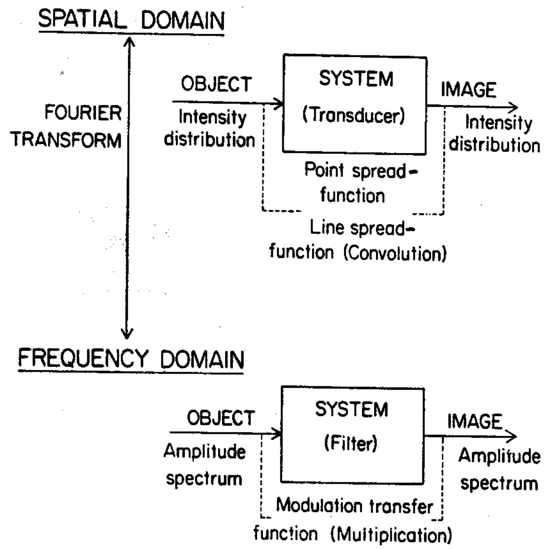

Fig. 9. Summary of concepts used in optical communication theory.

istic of linear, isoplanatic, isotropic systems in the frequency domain, linking arbitrary inputs with the corresponding outputs. By the same reasoning for nonisotropic systems the optical transfer function is a general transfer characteristic. Thus, just as in the case of the point and line spread-function, the introduction of the modulation transfer function has reduced the study of the transmission of complicated inputs to the study of the transmission of very simple inputs—namely, sinusoidal intensity distributions. In keeping with the language of the frequency domain, imaging systems are often called *filters* of spatial frequencies (see APPENDIX).

To complete the equivalence of the spatial and frequency domains, the counterpart of the modulation transfer function in the spatial domain is the point spread-function, since one can be calculated from the other by two-dimensional Fourier transformation. Thus, the modulation transfer function is two-dimensional in the frequency domain and is rotationally unsymmetric for nonisotropic systems. This means that, if one-dimensional sinusoidal inputs are used to measure the modulation transfer characteristics of nonisotropic systems, they must be placed in all possible orientations in the object plane to obtain the complete, two-dimensional modulation transfer function. When the system is isotropic, the modulation transfer function is rotationally symmetric and the orientation of the sinusoidal input in the object plane is immaterial. In this case the modulation transfer function is usually plotted in one dimension only.

It is of great experimental significance for situations in which it is difficult to generate sinusoidal inputs, *e.g.*, x-ray inputs, that the modulation transfer function along a certain direction can be calculated by one-dimensional Fourier transformation from the line spread-function measured by means of a line source oriented perpendicular to that direction (3). In the case of isotropic systems, the orientation of the line source is immaterial, and the general modulation transfer characteristics of the system can be calculated simply from the line spread-function by one-dimensional Fourier transformation. As in the spatial domain, this apparent reduction of a two-dimensional problem to a one-dimensional problem should not be misinterpreted. To analyze the transfer of two-dimensional inputs which are described by two-dimensional amplitude spectra in the frequency domain, the two-dimensional modulation transfer function is needed. Only in the case of one-dimensional inputs is the one-dimensional modulation transfer function sufficient to relate the output to the input.

Figure 9 broadly summarizes the transfer characteristics of imaging systems in the spatial domain and in the spatial frequency domain which have been discussed. It remains to be shown in what manner the description in the frequency domain aids in the solution of the transfer problem. As noted previously, calculation of the output corresponding to a given input in the spatial domain requires convolution of the input with the point spread-function or the line spread-function of the system. In the frequency domain the output is calculated simply by multiplying the input amplitude spectrum by the modulation transfer function of the system. This reduction in mathematical complexity is especially useful for the analysis of *cascaded systems* consisting of several linear, isoplanatic components in series so that the output from one component is the input to the next component. In the spatial domain, calculation of the overall transfer characteristics of such systems from the component transfer characteristics would require a complicated multiple convolution. On the other hand, the total modulation transfer function of the complex system is simply the product of all component modulation transfer functions. This makes it easy, for example, to determine the weakest link in the chain of imaging components comprising a composite system.

Furthermore, working in the frequency domain makes it possible to study the combined effects on the image of the optical system characteristics and of noise, such as electronic system noise in image intensifier-television chains, grain noise in films, and input quantum noise in all radiographic imaging systems. Noise is best described quantitatively by means of the *Wiener spectrum* which is derived from Fourier analysis of the random noise pattern. The Wiener spectrum indicates the spatial frequency content of noise just as the amplitude spectrum is used to describe the spatial frequency content of nonrandom intensity distributions. For example, the modulation transfer function in conjunction with the noise spectrum is useful in predicting the transmission of input noise through the system.

Besides the two specific advantages of the frequency domain representation cited above, treating optical imaging systems as filters of spatial frequencies brings to bear on the optical transfer problem a great body of knowledge derived from electrical communications theory (10). For example, a close analogy exists between the design of an optical system and the construction of a device to amplify an electrical pulse. Depending on the shape of the pulse which determines its frequency content, the amplifier must have a frequency response sufficiently wide (*band width*) to accommodate the whole frequency band into which the pulse can be resolved. Otherwise, distortion will result. Similarly, the optical degradation of an image can be related to the modulation transfer function of an imaging system.

RADIOGRAPHIC APPLICATION

In order to relate the concepts discussed above to practice, their application to radiographic screen-film systems will be described. The details of the experimental procedure have been discussed in previous publications (3, 11) and will only be summarized here.

The most commonly used screen-film system consists of a pair of fluorescent screens in intimate contact with both sides of a radiographic film which has photographic emulsion coated on both sides of a support. The thickness of the phosphor layer on each screen is about 100 μ, and each emulsion is about 10 μ thick. The thickness of the support is about 180 μ. Before the above concepts of communication theory are applied to this system, it needs to be established if the system satisfies the conditions of linearity and isoplanatism.

The exposure response of the screen-film system is nonlinear when it is expressed in terms of photographic density by means of the characteristic curve of the film. It has been determined experimentally, however, that the system is linear if the output is expressed in terms of effective exposure or illuminance in the film. From two slit images whose exposures differed by a factor of four, it was found that the ratio of the two effective illuminance distributions in the film was equal to the measured x-ray exposure ratio and independent of the distance from the center of the line image. Thus, the screen-film system can be "linearized" by using as the output the effective illuminance in the emulsions. This procedure is commonly applied in the analysis of photographic films.

The isoplanatic condition requires that the point spread-function is independent of the angle at which x rays are incident on

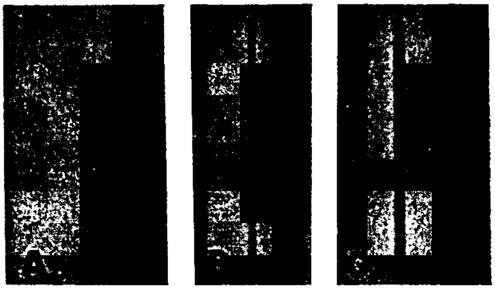

Fig. 10. Radiographs of a 10-μ slit on Royal Blue film. A. Without intensifying screens. B. With medium-speed screens. C. With fast screens.

the screen-film system. This is not the case, in general, since oblique incidence will result in an increase of the effective thickness of the screens and in lateral displacement of images. The influence of oblique incidence on the point spread-function, however, is estimated to be negligible for practical exposure setups involving focal-spot-screen distances which are very large relative to the thickness of the system for films of normal size. In any case, the isoplanatic condition will certainly be satisfied sufficiently well if only objects near the perpendicular to the screen-film system are considered.

In additon to being linear and isoplanatic, screen-film systems can also be expected to be isotropic. This is known to be the case for photographic emulsions and should also be true for intensifying screens if the screens are uniform, since it is difficult to see how a uniform screen could introduce directional effects. Experimental results have confirmed this expectation.

In applying linear communication theory to radiographic screen-film systems involving double-coated films, it should be noted that the process of image formation is quite different from that usually encountered in optics. The radiographic image is essentially a shadowgram of a three-dimensional structure formed by penetrating radiation. The source of radiation is of finite size and the x-ray beam is diverging. As a result, geometrical unsharpness and enlargement exist in the input to the imaging system. Under these conditions it is convenient to define the object plane as a plane in space immediately adjacent to the intensifying screen nearest the x-ray tube, and the input as the x-ray intensity pattern in this plane. Geometrical unsharpness and enlargement effects are thereby separated from the characteristics of the screen-film system and are described by separate transfer characteristics of the exposing process (12).

The line spread-function of screen-film systems is then measured by means of a 10 μm slit formed by platinum jaws, 2000 μm thick, which is mounted in the aluminum front of a vacuum-exposure holder. The screen-film system to be investigated is placed inside the holder in intimate contact with the slit and exposed to x rays on an optical-bench arrangement. The finite width of the slit is negligible relative to the unsharpness of present screen-film systems, so that the slit is effectively a line source of radiation. Exposed and processed slit images similar to those shown in Figure 10 are traced on a microdensitometer (13), and the line spread-function is calculated by means of the microdensitometer calibration curve and the characteristic curve of the film. It should be noted that this method of measurement results in a line spread-function whose physical meaning is somewhat different from that used in optics. Since the double-coated film sandwich is quite thick and since the total slit image is composed of partial images in the two emulsions which are separated by the film base, a well-defined image plane does not exist within the system. In fact, the only image plane which can be defined rigorously is the final image plane in the microdensitometer during the scanning operation. Therefore, scanning of the slit image and subsequent application of the conventionally measured characteristic curve of the film do not yield the corresponding exposure or illuminance distribution in an image plane, but rather what might be looked upon as an "effective" exposure or illuminance distribution in the film. It has been shown experimentally (14) that it is valid to call this effective distribution a line spread-function of the screen-film system.

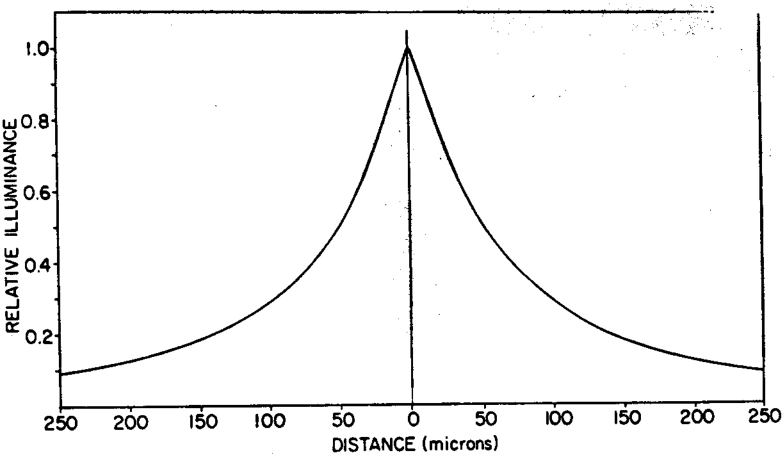

Fig. 11. Normalized line spread-function of a medium-speed screen-film system.

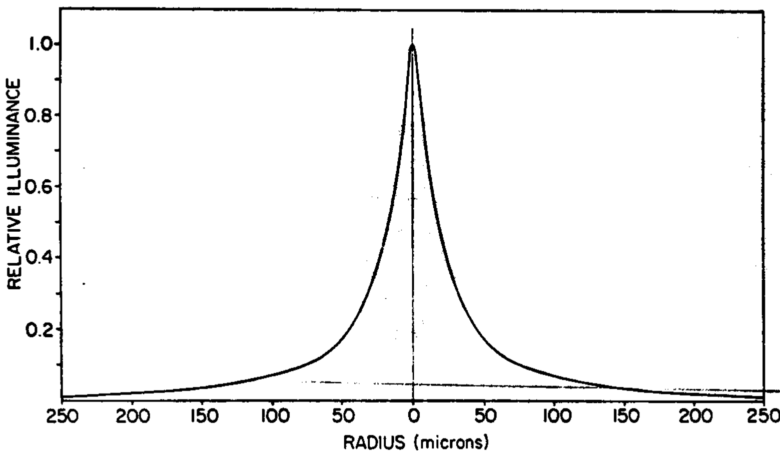

Fig. 12. Normalized point spread-function calculated from Figure 11.

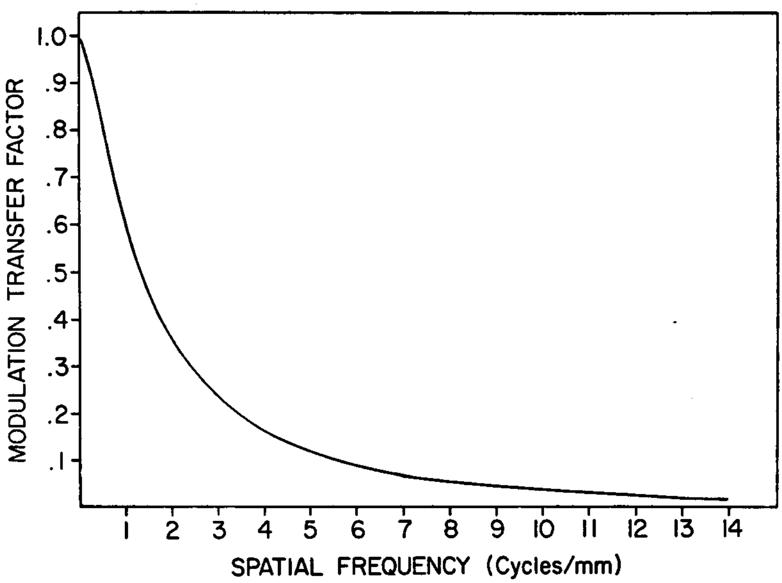

Fig. 13. Modulation transfer function calculated from Figure 11.

From the measured line spread-function, one can calculate the point spread-function using a method described by Marchand (7), and the modulation transfer function by Fourier transformation. Results for a medium-speed screen-film system are shown in Figures 11–13. Application of these transfer characteristics to the calculation of outputs from certain inputs has been discussed elsewhere (5).

The analysis of radiographic imaging systems by means of such communication theory methods can provide the following valuable results: (*a*) objective measurement of the optical system performance; (*b*) determination of the optical degradation of diagnostically important detail in the image; and (*c*) prediction of optical system parameters for the design and use of systems providing optimum visibility of diagnostically important detail.

The same concepts can be applied to the analysis of other phenomena affecting radiographic-image quality, such as geometrical and motion unsharpness and the recording of quantum noise.

SUMMARY

Concepts of communication theory are discussed which can be used to determine the optical performance of imaging systems in converting a system input to an output. Three transfer characteristics of linear, invariant imaging systems are described in a nonmathematical manner: the point spread-function and line spread-function in the spatial domain, and the modulation transfer function in the spatial frequency domain. The physical meaning of these concepts and the relationships between them are explained in detail. A brief mathematical derivation is given in the APPENDIX.

The application of linear communication theory to radiographic screen-film systems is discussed. Measurement of the line spread-function of these systems is explained, and transfer characteristics are given for a medium-speed screen-film system.

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APPENDIX

Discussion of Spatial Frequency Concept

The description of an x-ray pattern in space as a sum of many sinusoidal components of different amplitudes and spatial frequencies can be illustrated best by means of a specific example.

Let us assume that the x-ray pattern of interest is the square wave, $f(x)$, shown in Figure 14. By means of a mathematical technic called Fourier analysis, this function of distance can be written as a sum of infinitely many sinusoidal functions,

$$f(x) = \frac{\pi}{2} + 2 \sin x + \frac{2}{3} \sin 3x + \frac{2}{5} \sin 5x + \cdots$$

The first term in this Fourier series representation of the function is a constant and is often called the DC-term. Subsequent terms are sinusoidal and each is characterized by an amplitude A and a frequency ν in keeping with the conventional nomenclature for sinusoidal distributions (Fig. 15).

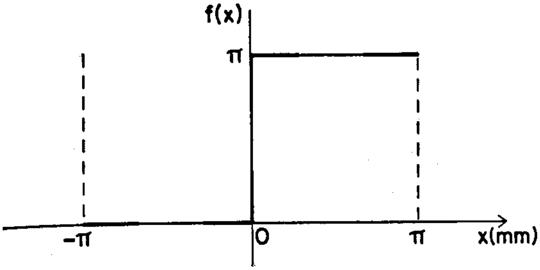

Fig. 14. Hypothetical x-ray intensity distribution in space: $f(x) = 0$ for $-\pi < x < 0$; $f(x) = \pi$ for $0 < x < \pi$.

$$y = A \sin 2\pi\nu_x.$$

Since the distributions are sinusoidal in space, their frequencies are expressed in cycles/mm and are called spatial frequencies. The plot of amplitude *versus* spatial frequency shown in Figure 16 is called the spatial frequency spectrum or amplitude spectrum of the x-ray pattern. It expresses the x-ray pattern in a form which is advantageous for studying the manner in which it is imaged.

The great significance of the amplitude spectrum is that it tells us exactly and uniquely what sinusoidal distributions need to be added together to synthesize the original x-ray pattern. In other words, the amplitude spectrum describes the spatial frequency content of the x-ray pattern. If any of the component sinusoidal distributions are not added, the original x-ray pattern will not be obtained. This is illustrated in Figure 17, which shows the original x-ray distribution and the results of plotting the sums of an increasing number of terms in the Fourier series up to the fourth term. It is seen that the original distribution is approximated more closely the more terms are added. Note that one effect of not adding all sinusoidal distributions which are contained in the original pattern is a rounding of the corners.

To translate these facts into the language of image formation, we look upon the amplitude spectrum of the x-ray pattern as the object for an imaging system. If the system is able to transmit the entire frequency content of the object, all component sinusoidal distributions necessary to synthesize the object will also be present in the image. This means that the image will look like the object or that the system is able to image the object perfectly. If, on the other hand, the system is not able to transmit the entire frequency content of the object, *i.e.*, if it acts like a filter of spatial frequencies having insufficient band width, then the absence of these frequencies from the image will cause the image to appear different from the object. Specifically, if the system transmits only sinusoidal distributions at the low frequency end of the object spectrum, *i.e.*, if it is a low-pass filter, the effect on the image will be similar to that shown in Figure 17. The

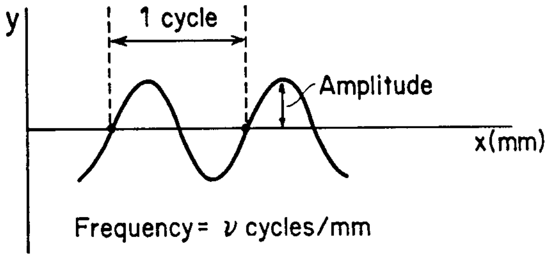

Fig. 15. Conventional nomenclature for sinusoidal distributions.

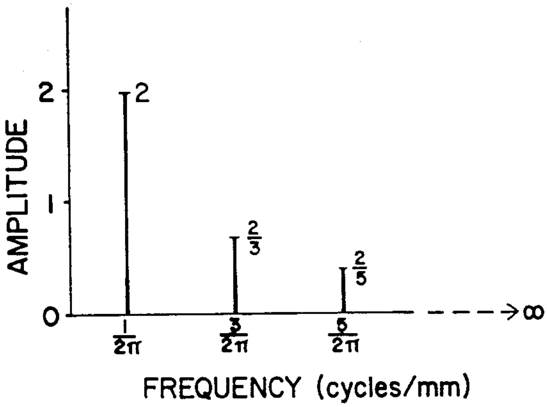

Fig. 16. Amplitude spectrum of intensity distribution in Figure 14.

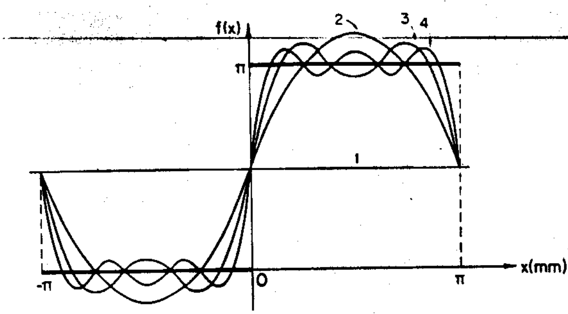

Fig. 17. Synthesis of intensity distribution of Figure 14 by adding sinusoidal distributions given by spectrum of Figure 16. Curve 1: $y = \pi/2$; Curve 2: $y = \pi/2+2 \sin x$; Curve 3: $y = \pi/2+2 \sin x+2/3 \sin 3x$; Curve 4: $y = \pi/2+2 \sin x+2/3 \sin 3x+2/5 \sin 5x$.

rounded corners resulting from the limited frequency content of the image will give the impression of unsharp imaging of the object.

It follows, therefore, that a knowledge of the filtering characteristics of an imaging system would enable us to predict how the object is transmitted by the system and what the appearance of the resulting image will be. In other words, the filter function of the system is a measure of the optical quality of the system. This filter function is called the system modulation transfer function (MTF).

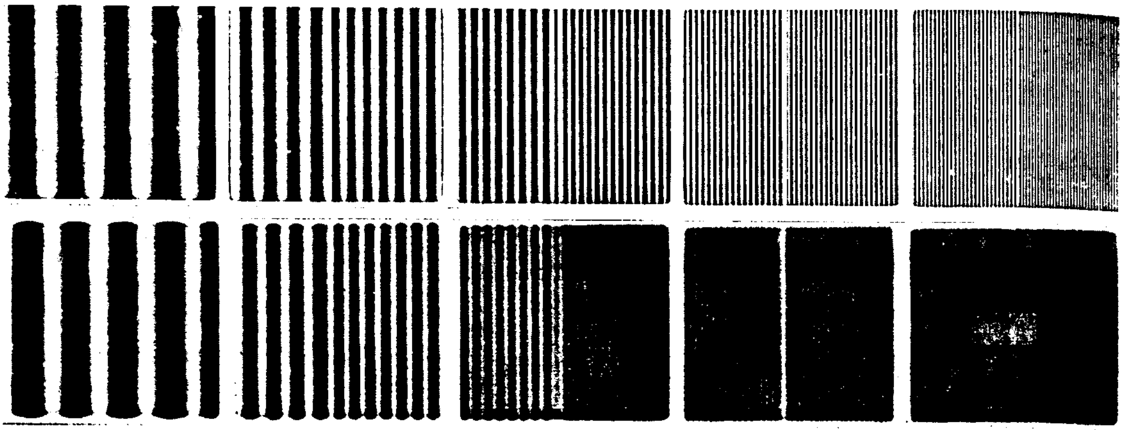

Fig. 18. Top. Part of a sinusoidal transmittance test object for the direct measurement of modulation transfer functions in optics. Bottom. Unsharp image of the test object.

The direct measurement of the *MTF* can best be illustrated by using an example from optics. Figure 18, A shows a reproduction of a part of a transparent piece of film which can be used as a test object for the measurement of the *MTF* of lens-film combinations, for example. The photographic density within each small rectangular test patch on the film is such that the light intensity transmitted by the film varies sinusoidally with distance along one dimension of the patch. The spatial frequency of the sinusoidal light intensity distribution is different from patch to patch. On this particular test object, it ranges from 3/8 to 6 cycles/mm. When these sinusoidal light intensity distributions are used as objects for an unsharp lens, *i.e.*, if they are imaged unsharply by a lens on a piece of film, the resulting image will appear as in Figure 18, B. Note that the lens has not transmitted the higher spatial frequencies contained in the object. The light intensity distributions in the object and in the image can be drawn schematically as in Figure 19, A. The object has equal amplitude at all spatial frequencies. In the image, the amplitude decreases with increasing frequency. A plot of the ratio of image amplitude to object amplitude *versus* the spatial frequency of the corresponding patch is the modulation transfer function of the lens-film combination, as indicated schematically in Figure 19, B.

The *MTF* shows that the lens-film combination is a low-pass spatial frequency filter. When the amplitude spectrum of any arbitrary object is multiplied by the *MTF*, the amplitude spectrum of the image is obtained.

In the case of x-ray imaging systems, the system *MTF* is not usually measured directly in this manner, since it is difficult to generate a sinusoidal spatial distribution of x-ray intensity which can serve as a test object. The *MTF* is calculated from the measured line spread-function instead.

Mathematical Derivation

Definitions: The one-dimensional Fourier transform of a function $f(x)$ is defined to be

$$F(\nu) = \int_{-\infty}^{\infty} f(x) e^{-2\pi i \nu x} dx.$$

It exists if

$$\int_{-\infty}^{\infty} |f(x)| dx < \infty \text{ and } f(x) \text{ has a finite number of discontinuities.}$$

The two-dimensional Fourier transform of a function $f(x,y)$ is defined to be

$$F(\nu_x,\nu_y) = \iint_{-\infty}^{\infty} f(x,y) e^{-2\pi i(\nu_x x + \nu_y y)} dxdy.$$

It exists if

$$\iint_{-\infty}^{\infty} |f(x,y)| dxdy < \infty \text{ and } f(x,y) \text{ has a finite number of discontinuities.}$$

The one-dimensional delta function $\delta(x)$ is defined by

$$\int_{-\infty}^{\infty} \delta(x) dx = 1 \text{ and } \delta(x) = 0 \text{ for } x \neq 0.$$

The two-dimensional delta function $\delta(x,y)$ is defined by

$$\iint_{-\infty}^{\infty} \delta(x,y) dxdy = 1 \text{ and } \delta(x,y) = 0 \text{ for } x^2 + y^2 \neq 0.$$

The Point Spread-Function

Consider an imaging system which operates on an input $i(\xi,\eta)$ to produce an output $o(x,y)$

$$L\{i(\xi,\eta)\} = o(x,y),$$

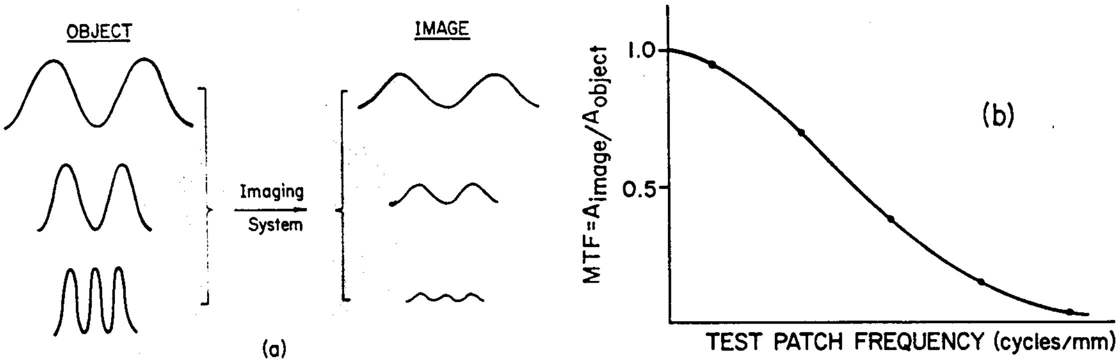

Fig. 19. A. Light intensity distributions transmitted by three of the test patches in Figure 18 (schematic). B. Modulation transfer function calculated from intensity distributions in Figure 18 (schematic).

where (ξ,η) and (x,y) are rectilinear coordinates in the object plane and in the image plane, respectively. Assume that the system is linear, such that

$$L\{a_1i_1(\xi,\eta) + a_2i_2(\xi,\eta)\} = a_1o_1(x,y) + a_2o_2(x,y) = a_1L\{i_1(\xi,\eta)\} + a_2L\{i_2(\xi,\eta)\}.$$

Any two-dimensional input can be expressed as a superposition of two-dimensional delta functions or point source inputs,

$$i(\xi,\eta) = \iint_{-\infty}^{\infty} \delta(\xi - \xi',\eta - \eta')\cdot i(\xi',\eta')d\xi'd\eta'.$$

Then,

$$L\{i(\xi,\eta)\} = \iint_{-\infty}^{\infty} i(\xi',\eta') \cdot L\{\delta(\xi - \xi',\eta - \eta')\}d\xi'd\eta' \quad (1)$$

because of the linearity condition.

If the system is isoplanatic, a point spread-function (PSF) or an output resulting from a point source input can be defined, whose shape is independent of the position of the point source in the object plane,

$$L\{\delta(\xi,\eta)\} = PSF(x - \xi,y - \eta) = L\{\delta(\xi - \xi',\eta - \eta')\}.$$

Then, equation (1) becomes

$$o(x,y) = \iint_{-\infty}^{\infty} i(\xi,\eta)PSF(x - \xi,y - \eta)d\xi d\eta. \quad (2)$$

This is a two-dimensional convolution integral, which states that the point spread-function is a transfer characteristic of the system.

The Line Spread-Function

Experimentally it is more convenient to work with line source inputs $\delta(\xi)$. For nonisotropic systems, the orientation of the line source in the object plane will affect the resulting line image. To emphasize this fact, the line source is written $\delta(\xi',\theta)$ which indicates that the coordinate system in the object plane has been rotated through an angle θ relative to the original coordinate system (ξ,η). This rotation results in new coordinate systems (ξ',η') and (x',y'). According to equation (2), the resulting output becomes

$$o(x',\theta) = \iint_{-\infty}^{\infty} \delta(\xi',\theta)PSF(x' - \xi', y' - \eta',\theta)d\xi'd\eta' = \int_{-\infty}^{\infty} PSF(x',y' - \eta',\theta)d(y' - \eta') = LSF(x',\theta)$$

which is the line spread-function of the system along a certain direction. For isotropic systems the directional effect does not exist. This definition indicates that the line spread-function can be obtained either by using a line source input or by scanning the point spread-function with a slit.

To calculate outputs resulting from two-dimensional inputs by means of equation (2), the point spread-function is needed. In can be calculated from the measured line spread-function (6, 7). In the case of one-dimensional inputs $i(\xi',\theta)$, however, we obtain from equation (2)

$$o(x',\theta) = \iint_{-\infty}^{\infty} i(\xi',\theta)PSF(x' - \xi', y' - \eta',\theta)d\xi'd\eta' = \int_{-\infty}^{\infty} i(\xi',\theta)\left[\int_{-\infty}^{\infty} PSF(x' - \xi',y' - \eta',\theta)d\eta'\right]d\xi' = \int_{-\infty}^{\infty} i(\xi',\theta)LSF(x' - \xi',\theta)d\xi'. \quad (3)$$

This is a one-dimensional convolution integral, which states that the line spread-function is a system transfer characteristic for one-dimensional inputs.

The Modulation Transfer Function

To express the system transfer characteristics in the spatial frequency domain, we take the two-dimensional Fourier transform of both sides of equation (2),

$$\iint_{-\infty}^{\infty} o(x,y)e^{-2\pi i(\nu_x x + \nu_y y)}dxdy =$$

$$\iint_{-\infty}^{\infty}\left[\iint_{-\infty}^{\infty} i(\xi,\eta)PSF\,(x-\xi,y-\eta)d\xi d\eta\right] \times$$

$$e^{-2\pi i(\nu_x x + \nu_y y)}dxdy = \iint_{-\infty}^{\infty} i(\xi,\eta) \times$$

$$e^{-2\pi i(\nu_x \xi + \nu_y \eta)}\left[\iint_{-\infty}^{\infty} PSF(x-\xi,y-\eta) \times\right.$$

$$\left. e^{-2\pi i[\nu_x(x-\xi) + \nu_y(y-\eta)]}\,d(x-\xi)d(y-\eta)\right]d\xi d\eta.$$

This can be written

$$O(\nu_x,\nu_y) = I(\nu_x,\nu_y) \times H(\nu_x,\nu_y) \qquad (4)$$

where O and I are the complex spatial frequency spectra of the output and the input respectively and H is the optical transfer function of the system. This equation indicates that $H(\nu_x,\nu_y)$ is a system transfer characteristic in the frequency domain. Note that the optical transfer function is the two-dimensional Fourier transform of the point spread-function.

Taking absolute values of the functions in equation (4), we obtain

$$|O(\nu_x,\nu_y)| = |I(\nu_x,\nu_y)| \cdot MTF(\nu_x,\nu_y) \qquad (5)$$

which is a relation between the output and input amplitude spectra and the modulation transfer function of the system. The modulation transfer function is a general system transfer characteristic for isotropic systems, since in that case $H(\nu_x,\nu_y)$ is real.

To express the transfer of one-dimensional inputs in the frequency domain, we form the one-dimensional Fourier transform of both sides of equation (3),

$$\int_{-\infty}^{\infty} o(x',\theta)e^{-2\pi i\nu_{x'}x'}dx' = \iint_{-\infty}^{\infty} i(\xi',\theta)LSF\,(x' -$$

$$\xi',\theta)e^{-2\pi i\nu_{x'}x'}dx'd\xi' = \int_{-\infty}^{\infty} i(\xi',\theta)e^{-2\pi i\nu_{x'}\xi'} \times$$

$$\left[\int_{-\infty}^{\infty} LSF(x'-\xi',\theta)e^{-2\pi i\nu_{x'}(x'-\xi')} \times\right.$$

$$\left. d(x'-\xi')\right]d\xi'.$$

This can be written

$$O(\nu_{x'},\theta) = I(\nu_{x'},\theta) \cdot H(\nu_{x'},\theta),$$

which shows that the Fourier transform of the line spread-function is the optical transfer function along a certain direction $\nu_{x'}$. For isotropic systems, the directional dependence vanishes.

The connection between the optical and modulation transfer functions and the transfer of one-dimensional sinusoidal inputs can be derived by substituting

$$i(\xi',\theta) = A + A_i \cos 2\pi\nu_{\xi'}\xi'$$

in equation (3). Letting $x' - \xi' = \sigma$, the output becomes

$$o(x',\theta) = A + A_i\int_{-\infty}^{\infty} LSF(\sigma,\theta)\cos 2\pi\nu_{\xi'}(x'-\sigma)d\sigma,$$

and expanding the integrand,

$$o(x',\theta) = A + A_i[H_c(\nu_{\xi'})\cos 2\pi\nu_{\xi'}x' + H_s(\nu_{\xi'})\sin 2\pi\nu_{\xi'}x'], \qquad (6)$$

where H_c and H_s are the cosine and sine transforms of the line spread-function respectively. This can be written

$$o(x',\theta) = A\{1 + M_0\cos[2\pi\nu_{\xi'}x' - \Phi(\nu_{\xi'})]\}, \qquad (7)$$

where $M_0 = A_0/A$ is the output modulation.

Letting $\cos\Phi(\nu_{\xi'}) = H_c(\nu_{\xi'})/(H_c^2 + H_s^2)^{1/2}$ and $\sin\Phi(\nu_{\xi'}) = H_s(\nu_{\xi'})/(H_c^2 + H_s^2)^{1/2}$ in equation (6), the output becomes

$$o(x',\theta) = A\{1 + M_i \cdot (H_c^2 + H_s^2)^{1/2} \cdot \cos[2\pi\nu_{\xi'}x' - \Phi(\nu_{\xi'})]\}, \qquad (8)$$

where $M_i = A_i/A$ is the input modulation. Comparing equations (7) and (8), we obtain

$$M_0 = M_i \times (H_c^2 + H_s^2)^{1/2} = M_i \times MTF(\nu_{\xi'}),$$

since $(H_c^2 + H_s^2)^{1/2}$ is the absolute value of the optical transfer function. This result states that the system acts as a filter of spatial frequencies, the MTF being the filter function.

The phase shift $\Phi(\nu_{\xi'})$ in the output is

$$\Phi(\nu_{\xi'}) = \arctan H_s/H_c.$$

This equals zero when $H_s = 0$, which is the case for isotropic systems.