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An analytic formula for the extrapolated range of electrons in condensed materials

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Abstract

A single analytic formula for the extrapolated range r_{ex} of electrons in condensed materials of atomic numbers from 4 to 92 is given. It has the form of the product of the continuous-slowing-down approximation (CSDA) range r_0 and a factor f_d related to multiple scattering detours. The factor f_d is expressed as a function of incident electron energy T_0 and atomic number Z of medium. Values of adjustable parameters in f_d have been optimized for data on the ratio r_{ex}/r_0, in which the Monte Carlo evaluated values of Tabata et al. [Nucl. Instr. Meth. B 95 (1995) 289] (from 0.1 to 100 MeV) and experimental data collected from literature (from 1 keV to 0.1 MeV) for r_{ex} have been used together with NIST-database values of r_0. For r_0 in the extrapolated-range formula, accurate database values or an approximate analytic expression developed as a function of T_0, Z, atomic weight A and mean excitation energy I of medium can be used. The maximum deviation of the resultant formula from the Monte Carlo data is about 2% for either option of r_0. The determination of the expression for f_d at energies below 0.1 MeV is tentative. By using an effective atomic number and atomic weight, the formula can also be applied to light compounds and mixtures.

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1. Introduction

The range of charged particles at a given energy is defined as the average of their path lengths in an unbounded uniform medium. In an approximation it can be evaluated by the integral of the reciprocal of the stopping power over energy from a final to an initial value. This integral is called the continuous-slowing-down approximation (CSDA) range. For experimental and application purposes, the projections of the paths of the particles in a given direction, e.g., the direction of the normal to the surface of an effectively semi-infinite medium, have more practical importance than the tortuous paths. For electrons the extrapolated range and the practical range are the quantities frequently used to represent the projected paths. The extrapolated range is commonly defined as the point where the tangent at the steepest point on the almost straight descending portion of the number-transmission curve meets the thickness axis. In another definition, the curve of the integral charge deposition as a function of depth is used instead of the transmission curve [1,2]. Values of the extrapolated ranges from the two definitions have been confirmed to show no significant difference [1]. The practical range is defined similarly by the depth–dose curve instead of the transmission curve, and is mostly used for the energy determination of medical electron beams (see Refs. [3,4] for the recent work on the practical range). The extrapolated and practical ranges have somewhat different values, and a detailed comparison of them will be published elsewhere. In the present paper the extrapolated range is considered.

The extrapolated range is used as a measure of the penetration depth in designing detectors for electrons and planning therapeutic treatment and industrial irradiation by electron beams. A universal semiempirical formula for the extrapolated range was proposed by Tabata et al. [5]. Such a formula can be incorporated into analytic expressions for

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transmission curves and depth–dose curves [6,7] to make it possible to evaluate these curves for arbitrary absorber material and incident electron energy. From the charge-deposition distributions calculated by the ITS-3.0 Monte Carlo code system [8,9], Tabata et al. [2] obtained a systematic set of values of the extrapolated range for electrons of energies from 0.1 to 100 MeV in elemental media of atomic numbers from 4 to 92. These values showed mostly good agreement with the experimental data reported earlier by Tabata et al. [10]. Comparison of the Monte Carlo results with the aforementioned semiempirical equation however showed some discrepancies originating from the lack of enough data at the time of formulating the equation. Therefore, it is considered appropriate to develop a better semiempirical formula on the basis of the Monte Carlo results.

In the present work we have developed a new formula for the extrapolated range of electrons in condensed materials. The energy region considered is from 0.1 to 100 MeV with a tentative extension of the formula down to lower energies. The formula is given in the form of the product of the CSDA range and a factor related to multiple scattering detours.

Values of the CSDA range can be obtained from ICRU Report 37 [11], the EPSTAR program available from the National Institute of Standards and Technology (NIST) in USA [12] or the ESTAR program provided by International Atomic Energy Agency [13]. Instead of such published or database values, an analytic expression can be used, provided that its precision is high enough not to deteriorate the resultant precision of the extrapolated-range formula. Many authors have proposed analytic expressions for the CSDA range [14–22] (a short review is given in Ref. [23]). With regard to universal formulas that cover a large number of materials and a wide region of energy, however, precision has been around 5% of the then available table at best, not satisfying the present demand. We have therefore made a new analytic expression for the CSDA range to incorporate it in the extrapolated-range formula.

It has been confirmed that the present extrapolated-range formula is applicable also to light compounds and mixtures by using an effective atomic number (and an effective atomic weight when the analytic expression for the CSDA range is used). A preliminary result of part of the present work was given in Ref. [23].

2. Formulation

2.1. Outline

We assume that the extrapolated range r_{ex} can be expressed as the product of a factor f_d and the CSDA range r_0:

$$r_{ex} = f_d r_0, \quad (1)$$

where f_d is a function of incident electron energy and the parameters that characterize the medium. We also assume that the parameters of the medium other than atomic number Z is unimportant here. The reciprocal of f_d is a quantity that gives a measure of multiple scattering detours of electrons [2]. In recent publications [4,24] the "detour factor" has been defined as the ratio of the projected range (the average depth of penetration) to the CSDA range rather than its reciprocal, and its behavior is similar to that of f_d.

If we use database values of r_0 in Eq. (1), the necessary task is only to find an expression for f_d. As was described in the previous section, however, we have also tried to make an analytic expression for r_0 well fitted to database values.

2.2. Data used

The main data used for r_{ex} in determining the present formula have been the Monte Carlo results [2] described in Section 1. These are available at energies from 0.1 to 100 MeV for the media of $_4$Be, $_6$C, $_{13}$Al, $_{29}$Cu, $_{47}$Ag, $_{79}$Au and $_{92}$U. Additionally experimental data collected from literature (Schonland [25,26], Lane and Zaffarano [27], Kanicheva and Burtsev [28], Kanter and Sternglass [29], Cosslett and Thomas [30]) have been used at energies below 0.1 MeV.

To study the applicability of the extrapolated-range formula to compounds and mixtures, r_{ex} values for such materials have been determined from the charge-deposition distributions computed by the ITS-3.0 Monte Carlo code [8,9] for 0.1 to 100 MeV electrons. Materials considered are tissue-equivalent plastic A-150, polymethyl methacrylate, "solid water" WT1, water, air-equivalent plastic C-552 and air. The charge-deposition distributions were obtained at the same time as the energy-deposition distributions reported in Ref. [31]. The method used to determine r_{ex} from the charge-deposition distributions has been the same as that described in Ref. [2].

The data used to formulate the expression for r_0 have been generated by the EPSTAR program [12] described in Section 1. The media considered are $_4$Be, $_6$C, $_{13}$Al, $_{14}$Si, $_{22}$Ti, $_{26}$Fe, $_{29}$Cu, $_{32}$Ge, $_{42}$Mo, $_{47}$Ag, $_{50}$Sn, $_{74}$W, $_{78}$Pt, $_{79}$Au, $_{82}$Pb and $_{92}$U. Initially the data in the energy region from 10 keV to 100 MeV were used. Finally the region considered was extended down to 1 keV for the media of Z from 4 to 42, because it was found that the extension affected little to the maximum deviation. It is to be noted however that the uncertainties of the r_0 values presently available

[2] For high energy electrons incident on low atomic number media $1/f_d$ becomes less than unity. This indicates that in these cases energy-loss straggling contributes more to this factor than the multiple scattering detours.

are rather large at energies below 10 keV because of the lack of a theory for shell corrections for the stopping power for the electron [32].

The applicability of the expression for r_0 to compound materials has been studied by the use of r_0 data for polyethylene, polyethylene terephthalate, polymethyl methacrylate, polystyrene, sodium iodide and water. These data have also been generated by EPSTAR. The expression for r_0 includes the mean excitation energy as one of the input parameters as described later, and the values for this quantity have been taken from ICRU Report 37 [11].

2.3. Expression for f_d

The form of f_d as a function of incident electron energy has been determined empirically as

$$f_d = 1/\left[a_1 + a_2/(1 + a_3/\tau_0^{a_4} + a_5\tau_0^{a_6})\right], \quad (2)$$

where the symbols a_i ($i = 1, 2, \ldots, 6$) denote constants for a given medium, and τ_0 is the kinetic energy of the incident electron in units of the rest energy of the electron. The term with a_3 governs the behavior of f_d at lower energies, and the term with a_5, the behavior at higher energies.

Eq. (2) has been fitted to the data on the ratio r_{ex}/r_0 for each elemental medium. From the analysis of the results the expressions for a_i as a function of Z have been determined as follows:

$$a_1 = b_1 Z^{b_2}, \quad (3)$$

$$a_2 = b_3 + b_4 Z, \quad (4)$$

$$a_3 = b_5 Z^{b_6 - b_7 \ln Z}, \quad (5)$$

$$a_4 = b_8/Z^{b_9}, \quad (6)$$

$$a_5 = b_{10} Z^{b_{11} - b_{12} \ln Z}, \quad (7)$$

$$a_6 = b_{13} Z^{b_{14}}, \quad (8)$$

where the symbols b_j ($j = 1, 2, \ldots, 14$) denote constants independent of medium. The forms of Eqs. (5) and (6) have been chosen to be analogous to Eqs. (7) and (8), and are rather tentative for the reason to be described in Section 3.1. Values of b_j have been sought by the method of least squares, in which the sum of squares of the relative deviations of the formula from the data has been minimized. In the final determination of b_j, heavier weights have been given to some of the data to reduce the maximum deviation.

2.4. Expression for r_0

Tabata et al. [5] derived the following functional form to approximate the CSDA range r_0:

$$r_0 = k_1\left[(1/\alpha)\ln(1 + \alpha\tau_0) - k_2\tau_0/(1 + k_3\tau_0)\right], \quad (9)$$

Table 1
Values of constants b_j ($j = 1, 2, \ldots, 14$) in the expression for f_d

j	b_j	j	b_j
1	0.3879	8	14.03
2	0.2178	9	0.7406
3	0.4541	10	4.294×10^{-3}
4	0.03068	11	1.684
5	3.326×10^{-16}	12	0.2264
6	13.24	13	0.6127
7	1.316	14	0.1207

where k_l ($l = 1, 2, 3$) and α are constants for a given medium. This form has been used as the starting point to obtain the present expression for r_0. Eq. (9) was obtained by assuming the following form for the stopping power S:

$$S = (1 + \alpha\tau_0)/\left\{k_1\left[1 - k_2(1 + k_3\tau_0)^{-2}\right]\right\}, \quad (10)$$

where the term multiplied by α represents the radiative stopping power, and the other, the collision stopping power. The expression for the collision stopping power was taken from Weber [14]. When k_2 and k_3 are equal to unity, Eq. (10) reduces to the constant stopping number approximation [33]. In Eq. (9) the terms of the order α/k_3 ($\approx 10^{-3} \times Z$) have been neglected compared with those of order unity.

To improve fits to data, we have modified Eq. (9) by including a modified stopping number B and two additional constants as follows:

$$r_0 = \frac{c_1}{B}\left[\frac{\ln(1 + c_2\tau_0^{c_3})}{c_2} - \frac{c_4\tau_0^{c_5}}{1 + c_6\tau_0}\right], \quad (11)$$

where the symbols c_m ($m = 1, 2, \ldots, 6$) denote constants for a given medium, and B consists of the main terms of the stopping number:

$$B = \ln\left(\frac{\tau_0}{I + c_7\tau_0}\right)^2 + \ln\left(1 + \frac{\tau_0}{2}\right). \quad (12)$$

In Eq. (12) I is the mean excitation energy of the medium expressed in units of the rest energy of the electron, and c_7 is a constant for a given medium. The term $c_7\tau_0$ has been incorporated in Eq. (12) to take into account the fact that the relative importance of the collision stopping power, compared with that of the radiative stopping power, and accordingly the effect of I on r_0 become smaller with increasing energy.

By analyzing the results of the fit to the r_0 data for each elemental medium, expressions for c_m ($m = 1, 2, \ldots, 7$) as a function of atomic number Z (and atomic weight A for c_1) of the medium have been determined as follows:

$$c_1 = d_1 A/Z^{d_2}, \quad (13)$$

$$c_2 = d_3 Z^{d_4}, \quad (14)$$

$$c_3 = d_5 - d_6 Z, \quad (15)$$

Table 2
Values of constants d_n ($n = 1, 2, \ldots, 14$) in the expression for r_0

n	d_n	n	d_n
1	3.600	8	1.303×10^{-4}
2	0.9882	9	1.02441
3	1.191×10^{-3}	10	1.2986×10^{-4}
4	0.8622	11	1.030
5	1.02501	12	1.110×10^{-2}
6	1.0803×10^{-4}	13	1.10×10^{-6}
7	0.99628	14	0.959

$$c_4 = d_7 - d_8 Z, \tag{16}$$

$$c_5 = d_9 - d_{10} Z, \tag{17}$$

$$c_6 = d_{11}/Z^{d_{12}}, \tag{18}$$

$$c_7 = d_{13} Z^{d_{14}}, \tag{19}$$

where the symbols d_n ($n = 1, 2, \ldots, 14$) denote constants independent of medium. Values of d_n have been determined by a method similar to that used in determining b_j in the expression for f_d.

3. Results and discussion

3.1. Values of constants and deviations of expressions from data

Values of b_j in the expression for f_d and those of d_n in the expression for r_0 have been determined as given in Tables 1 and 2. The analytic expression for $1/f_d$ is plotted in Fig. 1, and is compared with the ratio r_0/r_{ex}, in which the EPSTAR values are used for r_0. The values of the same ratio in which the previous formula of Tabata et al. [5] is used for r_{ex} are also shown (for Be, the correction factor for the formula given in Ref. [34] has been applied). In this figure $1/f_d$ is used rather than f_d, because a similar plot for energies up to 10 MeV was given earlier by Harder [35]. He pointed out that the behavior of r_0/r_{ex} is analogous to the behavior of the "scattering function" and that of the backscattering coefficient of electrons. At higher

Table 3
The maximum and root-mean-square (rms) relative deviations of the expressions for r_0 and f_d and of the extrapolated-range formula with the analytic expression for r_0 (r_{ex} with analytic r_0). Data considered are those for elemental media. Details of data are given in Section 2.2

Expression	Data	Deviation (%)	
		Maximum	Rms
r_0	EPSTAR	1.5	0.7
f_d or r_{ex} with EPSTAR	Monte Carlo	1.9	0.9
	Experiment [a]	28	14
r_{ex} with analytic r_0	Monte Carlo	2.0	0.9
	Experiment [a]	28	14

[a] Among the data used in the fitting of the expression for f_d, data for Ag and Au in the energy region from 1 to 10 keV were excluded to compare deviations within the regions in which the precision of the analytic expression for r_0 is guaranteed.

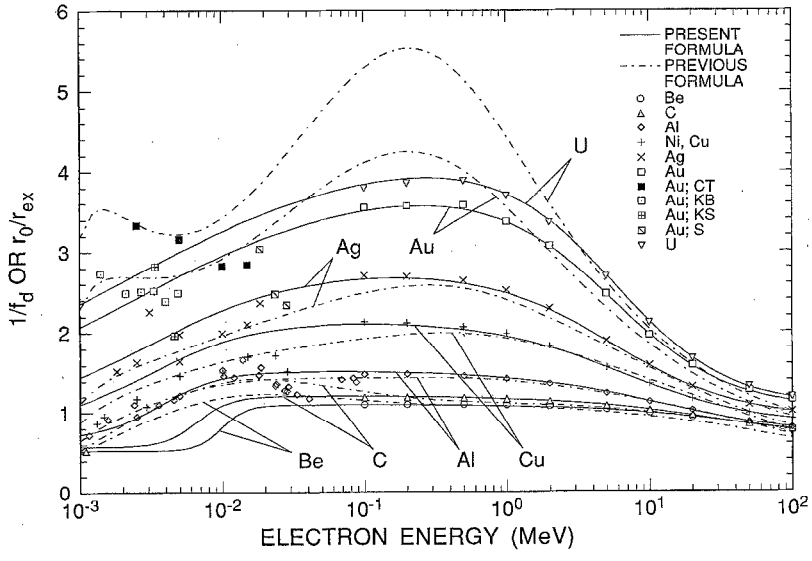

Fig. 1. The present expression for $1/f_d$ (solid lines) is compared with the values of the ratio r_0/r_{ex} (points and dash-dot lines). Points at energies below 0.1 MeV are from experiment [25–30], and the abbreviations given after "Au" in the legend denote authors; CT, Cosslett and Thomas [30]; KB, Kanicheva and Burtsev [28]; KS, Kanter and Sternglass [29]; S, Schonland [25]. Points at and above 0.1 MeV, from Monte Carlo data [2]; dash-dot lines, from the previous semiempirical formula for r_{ex} [5].

energies the present expression for $1/f_d$ approaches the form:

$$1/f_d = a_1 + a_2/(1 + a_5\tau_0^{a_6}). \quad (20)$$

Apart from a_1, the form of the right-hand side of Eq. (20) is the same as that of the empirical formula of Tabata et al. [36] for the backscattering coefficient of electrons for energies above about 50 keV.

For energies above 0.1 MeV, Fig. 1 indicates that the present expression for f_d, together with accurate r_0 data or a good analytic expression for r_0, gives a much better extrapolated-range formula compared with the previous one.

The uncertainties of EPSTAR values of r_0 are considered to be rather large at energies below 10 keV as described in Section 2.2. In Fig. 1 the r_0/r_{ex} data from each reference for Au as well as the curve of r_0/r_{ex} obtained from the previous extrapolated-range formula for Au and U show an unexpected trend of decrease with increasing energy in this region [3]. It is possible that this is a spurious trend caused by the use of the unreliable r_0 values. Further the values of r_0/r_{ex} given by the experimental r_{ex} values at energies below 0.1 MeV show large fluctuations. Therefore, the determination of the expression for f_d in the energy region below 0.1 MeV has been made only tentatively.

In Table 3 we show the maximum and root-mean-square (rms) relative deviations of the analytic expressions for r_0 and f_d as well as the deviations of the extrapolated-range formula with the analytic expression for r_0 (called r_{ex} with analytic r_0 in the following). The deviations of the expressions for f_d are the same thing as those of the extrapolated-range formula with EPSTAR r_0 values. Although the *precision* known from Table 3 of the expression for r_0 against the EPSTAR values is guaranteed down to the energy of 1 keV for Z from 4 to 42, it should be noted again that the *accuracy* of the EPSTAR values is uncertain at energies below 10 keV.

From the deviations of the expressions for r_0 and f_d one might expect that the deviations of r_{ex} with analytic r_0 would be significantly larger than those of f_d. Table 3 shows however that the deviations of these, i.e., the deviations of the extrapolated-range formula with the different options of r_0, are almost the same, being about 2% for the Monte Carlo data. The reason for this can be understood from Fig. 2, in which the deviations of the expressions from the data are plotted as a function of energy for r_0, f_d and r_{ex} with analytic r_0 (only the energy region in which the Monte Carlo data for r_{ex} are available is shown). Fig. 2a shows the deviations for the Be medium, for which the deviation of r_{ex} with analytic r_0 takes on the maximum value (among all the Monte Carlo data used for elemental media) of 2.0% at 0.5 MeV, and Fig. 2b shows the deviations for Al, for which the deviation of f_d and that of r_{ex} with analytic r_0 take on the maximum values of 1.9% and 2.0%, respectively, at 0.1 MeV. The figure indicates the following: (1) The numbers of extremal points of the deviation curves of r_0 and f_d for a given medium are rather small, so that the extremal points occur rarely at or nearly the same energy for r_0 and f_d. (2) The deviations of the expressions for r_0 and f_d have often opposite signs and partially cancel out each other in r_{ex} with analytic r_0. Owing to these characteristics of the deviations, we can use the analytic expression for r_0 instead of EPSTAR or other database values in the extrapolated-range formula without appreciable deterioration in precision. This is especially convenient when a database for r_0 is not at hand.

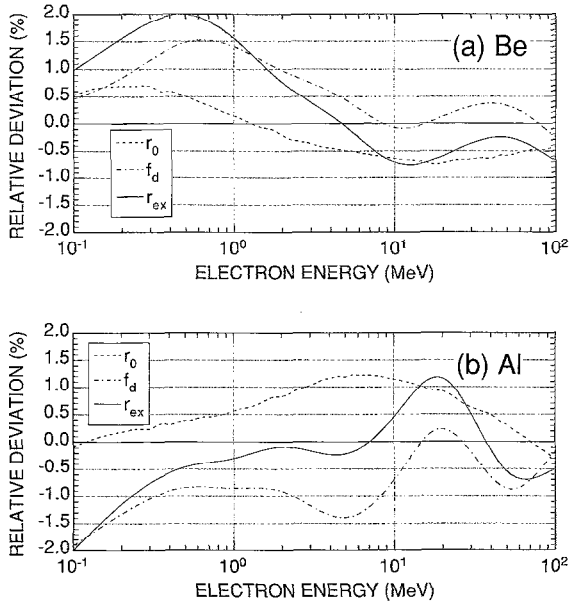

Fig. 2. Deviations of the expressions from the data are plotted as a function of electron energy for r_0, f_d and r_{ex} with the analytic expression for r_0: (a) the medium of Be, (b) the medium of Al. Curves have been drawn by spline fit to data points. Small irregularities on the curves of r_0 are not seen on the curves of r_{ex} because of the smaller number of data points for the latter than for the former.

A FORTRAN sample code to evaluate the present extrapolated-range formula is given in Fig. 3 together with an optional code for the analytic expression for r_0. In evaluating Eq. (11) at the lowest energies considered here, it is necessary to use double-precision arithmetic to avoid truncation errors. The input parameter FI in the latter code is the mean excitation energy in eV, as distinct from the definition of I in Eq. (12).

[3] The reason for the similarity of the behavior of the previous formula and that of the experimental data in this energy region is that the previous formula was fitted directly to the experimental data without the intervention of r_0.

```
      FUNCTION REX(T0,Z,R0)
C
C     PURPOSE
C       CALCULATE THE EXTRAPOLATED RANGE OF ELECTRONS
C
C     DESCRIPTION OF PARAMETERS
C       T0      - INCIDENT KINETIC ENERGY OF ELECTRONS IN MEV
C       Z       - ATOMIC NUMBER OF MEDIUM
C       R0      - CSDA RANGE IN G/CM2
C       ATW     - ATOMIC WEIGHT OF MEDIUM
C       FI      - MEAN EXCITATION ENERGY OF MEDIUM IN EV
C       REX     - EXTRAPOLATED RANGE OF ELECTRONS IN G/CM2
C
      TAU0=T0/0.511
      ALZ=LOG(Z)
      A1=0.3879*Z**0.2178
      A2=0.4541+0.03068*Z
      A3=3.326E-16*Z**(13.24-1.316*ALZ)
      A4=14.03/Z**0.7406
      A5=4.294E-03*Z**(1.684-0.2264*ALZ)
      A6=0.6127*Z**0.1207
      REX=R0/(A1+A2/(1.+A3/TAU0**A4+A5*TAU0**A6))
      RETURN
      END
C
      FUNCTION R0(T0,Z,ATW,FI)
C
C     PURPOSE
C       CALCULATE THE CONTINUOUS SLOWING-DOWN APPROXIMATION (CSDA)
C       RANGE OF ELECTRONS TO GIVE AN INPUT VALUE FOR FUNCTION REX
C
C     DESCRIPTION OF PARAMETERS
C       T0      - INCIDENT KINETIC ENERGY OF ELECTRONS IN MEV
C       Z       - ATOMIC NUMBER OF MEDIUM
C       ATW     - ATOMIC WEIGHT OF MEDIUM
C       FI      - MEAN EXCITATION ENERGY OF MEDIUM IN EV
C       R0      - CSDA RANGE IN G/CM2
C
      DOUBLE PRECISION C2,W
      TAU0=T0/0.511
      FI1=FI*1.E-6/0.511
      C1=LOG((TAU0/(FI1+1.1E-6*Z**0.959*TAU0))**2*(TAU0+2.)/2.)
      C1=3.6*ATW/Z**0.9882/C1
      C2=1.191E-3*Z**0.8622
      C3=1.02501-1.0803E-4*Z
      C4=0.99628-1.303E-4*Z
      C5=1.02441-1.2986E-4*Z
      C6=1.03/Z**1.11E-2
      W=LOG(1.D0+C2*DBLE(TAU0**C3))/C2
      W=W-DBLE(C4*TAU0**C5)/(1.D0+DBLE(C6*TAU0))
      R0=C1*SNGL(W)
      RETURN
      END
```

Fig. 3. FORTRAN codes to evaluate the extrapolated-range formula and the analytic expression for the CSDA range.

3.2. Applicability to compounds and mixtures

The expression for r_0 has been compared with the EPSTAR values for the seven kinds of compounds mentioned in Section 2.2. For these media we have used the effective atomic number Z_{eff} and the effective atomic weight A_{eff} given by the following formulas:

$$Z_{\text{eff}} = \frac{\sum_i f_i Z_i^2 / A_i}{\sum_i f_i Z_i / A_i}, \tag{21}$$

$$A_{\text{eff}} = (Z/A)_{\text{eff}}^{-1} Z_{\text{eff}}, \tag{22}$$

Table 4
The maximum and root-mean-square (rms) relative deviations of the expressions for r_0 and f_d and of the extrapolated-range formula with the analytic expression for r_0 (r_{ex} with analytic r_0). Data considered are those for compounds and mixtures. Details of data are given in Section 2.2

Expression	Data	Deviation (%)	
		maximum	rms
r_0	EPSTAR	2.1	1.0
f_d or r_{ex} with EPSTAR	Monte Carlo	2.7	1.4
r_{ex} with analytic r_0	Monte Carlo	2.8	1.3

Table 5
Extrapolated ranges of electrons in tissue-equivalent plastic A-150, polymethyl methacrylate (PMMA), "solid water" WT1, water, air-equivalent plastic C-552 and air. T_0 is the incident electron energy. Values in the second row are effective atomic number and atomic weight defined by Eqs. (21) and (22)

T_0 (MeV)	A-150	PMMA	WT1	Water	C-552	Air
	5.49, 10.00	5.85, 10.85	5.95, 11.04	6.60, 11.89	7.12, 14.41	7.36, 14.74
0.1	1.222E − 02	1.242E − 02	1.238E − 02	1.193E − 02	1.327E − 02	1.310E − 02
0.2	3.837E − 02	3.903E − 02	3.889E − 02	3.735E − 02	4.153E − 02	4.103E − 02
0.5	1.528E − 01	1.542E − 01	1.546E − 01	1.476E − 01	1.652E − 01	1.625E − 01
1	3.852E − 01	3.878E − 01	3.891E − 01	3.698E − 01	4.157E − 01	4.067E − 01
2	8.876E − 01	8.931E − 01	8.951E − 01	8.521E − 01	9.562E − 01	9.254E − 01
5	2.438E + 00	2.457E + 00	2.461E + 00	2.347E + 00	2.639E + 00	2.482E + 00
10	5.024E + 00	5.084E + 00	5.063E + 00	4.853E + 00	5.443E + 00	4.987E + 00
20	1.005E + 01	1.016E + 01	1.014E + 01	9.720E + 00	1.093E + 01	9.692E + 00
50	2.423E + 01	2.445E + 01	2.431E + 01	2.332E + 01	2.600E + 01	2.243E + 01
100	4.417E + 01	4.430E + 01	4.416E + 01	4.182E + 01	4.623E + 01	4.015E + 01

where $(Z/A)_{\text{eff}}$ is given by

$$(Z/A)_{\text{eff}} = \sum_i f_i Z_i / A_i, \tag{23}$$

f_i is the fraction by weight of the ith constituent element with the atomic number Z_i and the atomic weight A_i. The maximum and rms relative deviations of the expression for r_0 from the data are given in Table 4.

The r_{ex} values obtained for the six kinds of compounds and mixtures mentioned in Section 2.2 are given in Table 5. The effective atomic numbers of these media (see the second row of Table 5) are rather close to the atomic number of C. However, the extrapolated ranges in PMMA, WT1 and A-150 are smaller by 8–9% and the extrapolated range in water is smaller by 12–14% than in C at all the energies considered, mainly because of the differences of $Z_{\text{eff}}/A_{\text{eff}}$ from Z/A of C. This makes it meaningful to check the applicability of the extrapolated-range formula to these media.

The extrapolated range in air shows less increase with increasing energy than in the other materials given in Table 5. This is due to the fact that the density effect on the stopping power is negligible for the gaseous material. In relation to this, it should be noted that for gaseous materials the present expressions for r_0, f_{d} and r_{ex} with analytic r_0 can be used only at energies below about 2 MeV, because above this energy the formulas have been tailored to express the ranges in condensed materials having appreciable density effects. Therefore, deviations of f_{d} and r_{ex} with analytic r_0 from the Monte Carlo data for the compounds and mixtures have been evaluated by excluding the data for air at energies above 2 MeV, and are given in the last two rows of Table 4. Again the deviations of f_{d} and r_{ex} with analytic r_0 are almost the same. All the deviations in Table 4 are a little larger than the corresponding values of Table 3 for elemental media, but are considered to indicate the usefulness of the present extrapolated-range formula for different light materials so far as accurate and systematic data are lacking. Concerning the applicability to the compounds and mixtures that include the elements of high atomic numbers, a study has to be made in the near future.

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