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Path following of underactuated marine surface vessels using line-of-sight based model predictive control ☆

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ABSTRACT

This paper presents a model predictive control (MPC) for a way-point tracking of underactuated surface vessels with input constraints. A three-degree-of-freedom dynamic model of surface vessels has been used for the controller design. In order for the control action to render good helmsman behavior, a MPC scheme with line-of-sight (LOS) path generation capability is formulated. Quadratic programming (QP) is used to solve a linear MPC by successive linearization along the LOS model of the surface vessel. Furthermore, we show that an LOS decision variable can be incorporated into the MPC design to improve the path following performance. The effectiveness of the developed control law is demonstrated via computer simulations.

1. Introduction

Trajectory tracking and path following of surface ships have been a long standing control problem that have attracted attention from the control community for many years. The under-actuated nature of these problems, namely with more variables to be controlled than the number of control actuators, coupled with the nonlinear characteristics of the hydrodynamics associated with the ship motion, makes the control problem both interesting and challenging.

The underactuated ship motion control problem has been tackled in a number of research studies in the last few years. For example, the recursive technique proposed in Jiang and Nijmeijer (1999) for the standard chain form systems is used in Pettersen and Nijmeijer (2001) to provide a local tracking result. By applying the cascaded approach, a global tracking result was obtained in Breivik and Fossen (2004) and Lefeber (2000), where the stability analysis is performed relying on the theory of linear time varying systems. Using backstepping and Lyapunov's direct method, a controller is developed that guarantees global exponential stability of straight line and circle trajectories (Do et al., 2002). Path following approach based on a line-of-sight (LOS) algorithm, which is often used in a way-point ship control practice, is presented in Pettersen and Lefeber (2001). In Fantoni et al. (2002), a simplified ship model has been examined based on passivity considerations and Lyapunov theory, discontinuous feedback control laws have been derived to ensure the global convergence to the origin.

However, in realistic implementations, performance of control systems is often limited due to constraints on inputs or states that naturally arise. None of the previously cited works has taken those constraints explicitly into account. The rudder magnitude and rate constraints can be addressed explicitly by employing a model predictive control (MPC) scheme. By taking into account these physical constraints, control actions that respect actuators limits can be generated.

As the pioneer work of MPC application in tracking control of marine surface vessels, Wahl and Gilles (1998) considers rudder saturation in their MPC controller and adopts a 1 DoF yaw dynamical model. Perez (2005) presented a model predictive rudder roll stabilization (RRS) control system. The control objective was to regulate the heading to a desired value and to reduce the roll angle under various sea conditions without considering the path following requirement. Recently, Li et al. (2009a) has explored the path following problem with roll constraints using the rudder as the only control input. However, traditional guidance and control schemes used to steer the vehicle along the prescribed trajectory are designed separately, using well established design methods for control and simple strategies such as line-of-sight (LOS) for guidance. This paper proposes a new MPC based methodology for the design of guidance and control systems for surface vessels whereby the two systems are designed simultaneously.

In particular, we explore the use of linear MPC for the nonlinear ship maneuvering control. The linear MPC is chosen mainly to ease the computational burden in calculating the MPC solution. The developed ship model is linearized to obtain the time varying linear predictive model along the prescribed

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line-of-sight guidance path and a linear MPC is formulated for the linearized system, thereby simplifying the optimization task. The guidance algorithm by LOS is then implemented to follow the desired geometrical paths and minimize the cross-track error defined as the shortest distance between the vessel and the straight line. A unique feature of this work is that the LOS guidance parameter is embedded in the MPC controller design as an additional decision variable and its utility is explored through the simulations. The underlying idea of changing the LOS vector norm to improve the convergence of the LOS algorithm has been also employed in a previous work (Moreira et al., 2007). To our knowledge, while MPC studies have been reported for marine vessel applications, studies on the MPC control design integrated with the LOS guidance scheme for a surface vessel path following under limited rudder deflection and rudder rate have not been seen in open literature to date.

The remainder of this paper is organized as follows: in the next section the kinematic and dynamic model of the surface vessel is presented. The MPC algorithm is developed in Section 3. Simulation results are presented and analyzed in Section 4, followed by conclusions.

2. A surface vessel model

Consider the following generic mathematical model that captures the ship motion characteristics in the horizontal plane (Perez and Fossen, 2004):

$$m(\dot{u} - vr) = X_h + \tau_x,$$

$$m(\dot{v} + ur) = Y_h + Y_\delta \delta,$$

$$I_z \dot{r} = N_h + N_\delta \delta, \tag{1}$$

where $(\cdot)_h$ are the hydrodynamic forces and moment. τ_x and δ are the propeller thrust and the rudder deflection, respectively. m and I_z are the vehicle mass and mass moment of inertia. u, v are the body-fixed linear velocities (surge and sway), and r is the yaw rate. Y_δ and N_δ are rudder coefficients associated with sway force and yaw moment, respectively. The hydrodynamic forces are often modeled as a nonlinear function of surge velocity, sway velocity, and yaw rate, for example, as shown in Bertram (2000).

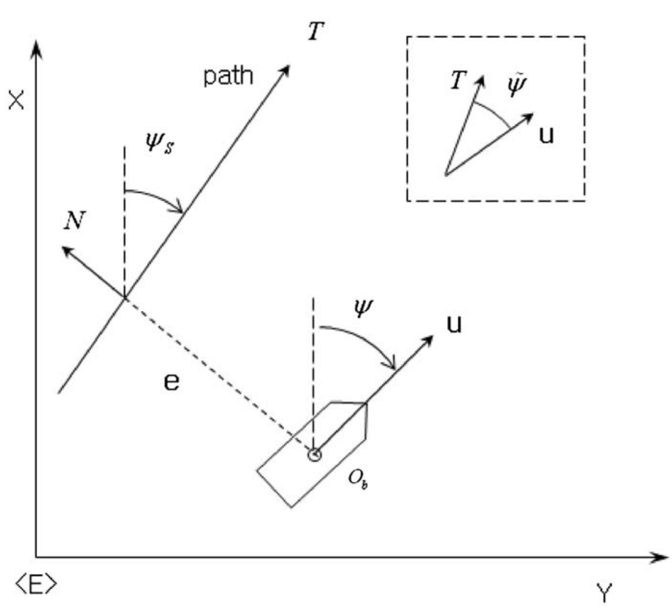

Fig. 1. A schematic representation of ship kinematic variables for a straight-line path following.

For surface vessels operating in the open sea, the path is often a straight line or way-point path, which consists of piecewise straight lines. In these cases, the path curvature is zero, therefore the path following error dynamics could be simplified into

$$\dot{e} = u\sin(\overline{\psi}) + v\cos(\overline{\psi}),$$

$$\dot{\overline{\psi}} = r, \tag{2}$$

where e (defined as the distance between the center of gravity of the vessel O_b and the path, see Fig. 1 for the geometric interpretation) and $\overline{\psi} = \psi - \psi_S$ are referred to as the cross-track error and relative heading error, respectively. ψ is the heading angle of the vessel and ψ_S is the path direction as shown in Fig. 1. N, T in Fig. 1 are the unit normal and unit tangent vectors of the curve.

The control objective of the path following problem is to drive e and $\overline{\psi}$ to zero. Furthermore, in this study, we maintain a constant propeller speed rather than a constant surge speed, which is more realistic for most ship maneuvering operating conditions. Hence, the surge velocity is assumed to be bounded and non-zero, but time varying throughout the paper.

3. Line-of-sight based model predictive control

3.1. Line-of-sight guidance system

A conventional surface vehicle tracking control system is usually implemented using a standard PID autopilot in series with an LOS guidance as shown in Fig. 2. The LOS guidance principle embodies an intuitive understanding of the behavior of a ship and the action of a helmsman, namely, the desired heading angle is computed on the basis of cross tracking error e and a lookahead distance $\Delta > 0$ which is referred to as the lookahead distance, a guidance parameter used to shape the convergence of the ship to the path tangential. LOS schemes have been applied to surface ships by Fossen (2002). Based on e and Δ, the LOS angle denoted by $\overline{\psi}_{LOS}$ can be constructed as follows (see Fig. 3):

$$\overline{\psi}_{LOS} = -\frac{e}{\sqrt{e^2 + \Delta^2}}. \tag{3}$$

In this paper, the LOS given by Eq. (3) is adopted as a reference trajectory for a ship to follow in the MPC control design. Note that other shaping functions with arctan-like properties are also possible candidates. The advantage of using the linear-like LOS

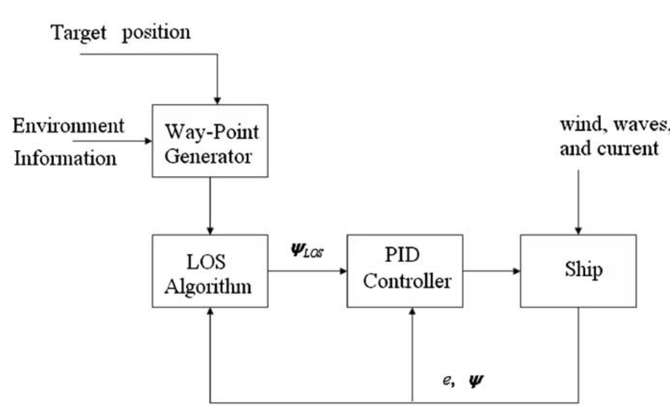

Fig. 2. Schematic of an autopilot system combining a PID control system with a LOS guidance system.

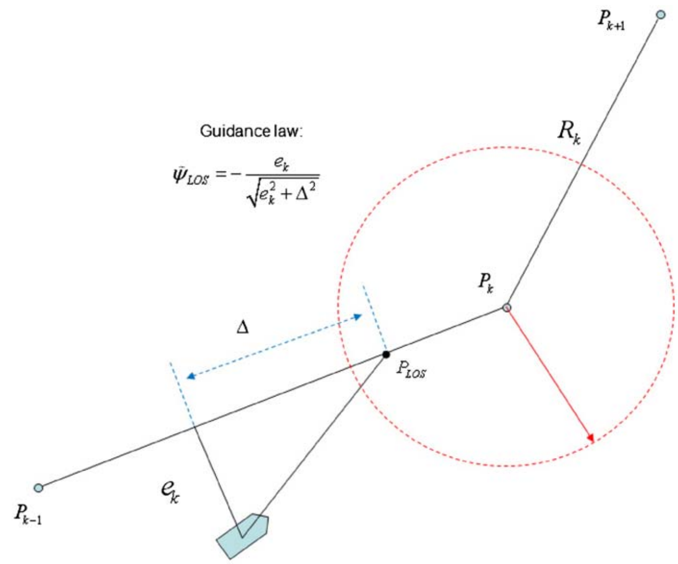

Fig. 3. Line-of-sight guidance for a straight line.

angle expression ($\bar{\psi}_{LOS} = -ke,\ k = 1/\sqrt{e^2+\Delta^2} > 0$) over arctan-like functions has been elaborated in Li et al. (2009b).

In the ship path following literature, Δ is typically expressed as a distance of n ship length (nL_{ship}) (Breivik, 2003).

We consider a generic desired geometric path composed of a collection of way-points $(\ldots, P_{k-1}, P_k, P_{k+1}, \ldots)$. The LOS position P_{LOS} is a point located somewhere along the straight line segment connecting the previous way-point (P_{k-1}) and current way-point (P_k). The position P_{LOS} moves along the line as the ship is approaching the desired path. A criteria for switching to the next way-point, located at P_{k+1}, is for the ship to be within a circle of a radius R_k of the current way-point P_k. The guideline to choose R_k is examined in Breivik (2003). In our simulation, R_k is assumed to be equal to two ship lengths, that is $R_k = 2L_{ship}$ as shown in Moreira et al. (2007).

If a ship heading follows exactly what is specified by the LOS algorithm Eq. (3), one can establish the desired path following performance. Note, from Eq. (2), that

$$\dot{e} = u\sin(\bar{\psi}) + v\cos(\bar{\psi}) = U\sin\left(\bar{\psi} + \arctan\frac{v}{u}\right), \tag{4}$$

where $U = \sqrt{u^2+v^2} > 0$. If the heading of the ship ($\bar{\psi}$) is made to satisfy

$$\bar{\psi} + \arctan\frac{v}{u} = \bar{\psi}_{LOS}, \tag{5}$$

we have

$$\begin{aligned}\dot{e} &= u\sin\bar{\psi}_{LOS} \\ &= U\bar{\psi}_{LOS}\frac{\sin\bar{\psi}_{LOS}}{\bar{\psi}_{LOS}} \\ &= -\frac{U}{\sqrt{e^2+\Delta^2}}\frac{\sin\bar{\psi}_{LOS}}{\bar{\psi}_{LOS}}e \le 0, \quad \bar{\psi}_{LOS} \in (-\pi, \pi),\end{aligned} \tag{6}$$

from which it can be derived that the origin of e is asymptotically stable if the ship's absolute speed is positive, $U = \sqrt{u^2+v^2} > 0$. Here, e, the cross track error, is positive.

3.2. The linear MPC approach with fixed LOS

Model predictive control is a control strategy that determines control action at each sampling time by solving an optimization problem over a horizon based on a model prediction. Although prediction and optimization are performed over a future horizon, only the values of the inputs for the current sampling interval are used and the same procedure is repeated at the next sampling time when new measurements become available. For complex, constrained, multivariable control problems, MPC has become an accepted standard in the process industry where plant dynamics are slow and computational resources are abundant. However, for systems with fast and/or nonlinear dynamics, the implementation of MPC technique remains fundamentally limited due to high demand in computational resource associated with optimization. Though most physical systems are inherently nonlinear in nature, the majority of MPC applications are based on the linear dynamic model, mainly to take the computational advantages of linear MPC. This is also the approach taken in this work.

For the sake of simplicity, we assume in this work that the states of the plant are available for measurement. The path following kinematic and dynamic model, represented by Eqs. (1) and (2), can be written in a more compact form as

$$\dot{\mathbf{x}} = f(\mathbf{x}, \mathbf{u}), \tag{7}$$

where $\mathbf{x} = [e\ \ \bar{\psi}\ \ v\ \ r]^T$ and $\mathbf{u} = \delta$.

A linear model of the system dynamics can be obtained by computing an error model with respect to reference signals $\mathbf{x}_r = (0, \bar{\psi}_{LOS}, 0, 0)^T$ and $\mathbf{u}_r = 0$. Note that we assumed $\mathbf{x}_r(3,1) \approx 0$ to simplify the derivation of the linear MPC where $\mathbf{x}_r(3,1)$ stands for the third component of the reference signals. This property approximately holds for $\Delta \gg |e|$ and $|v/U| \ll 1$. $\dot{\mathbf{x}}_r = \mathbf{f}(\mathbf{x}_r, \mathbf{u}_r)$ also holds under this assumption. The linear model of Eq. (7) will take the form:

$$\dot{\tilde{\mathbf{x}}} = \mathbf{f}_{\mathbf{x},r}\tilde{\mathbf{x}} + \mathbf{f}_{\mathbf{u},r}\tilde{\mathbf{u}}, \tag{8}$$

where $\mathbf{f}_{\mathbf{x},r}$ and $\mathbf{f}_{\mathbf{u},r}$ are the partial derivatives of $\mathbf{f}$ with respect to $\mathbf{x}$ and $\mathbf{u}$, respectively, evaluated around the reference point $(\mathbf{x}_r, \mathbf{u}_r)$. $\tilde{\mathbf{x}} = \mathbf{x} - \mathbf{x}_r$ represents the error with respect to the reference points and $\tilde{\mathbf{u}} = \mathbf{u} - \mathbf{u}_r$ is its associated control input.

The approximation of $\dot{\tilde{\mathbf{x}}}$ using forward differences gives the following discrete-time system model:

$$\tilde{\mathbf{x}}_{k+1} = A_k\tilde{\mathbf{x}}_k + B_k\tilde{\mathbf{u}}_k, \tag{9}$$

with

$$A_k = \begin{bmatrix} 1 & U\cos\psi_r(k)T & -\sin\psi_r(k)T & 0 \\ 0 & 1 & 0 & T \\ 0 & 0 & 1+a_vT & a_rT \\ 0 & 0 & b_vT & 1+b_rT \end{bmatrix},$$

$$B_k = \begin{bmatrix} 0 \\ 0 \\ Y_\delta T \\ N_\delta T \end{bmatrix}. \tag{10}$$

At time k, consider the quadratic cost function,

$$\Phi_k = \sum_{j=1}^{N} \tilde{\mathbf{x}}_{k+j}^T Q\tilde{\mathbf{x}}_{k+j} + \tilde{\mathbf{u}}_{k+j-1}^T R\tilde{\mathbf{u}}_{k+j-1}, \tag{11}$$

where N is the prediction horizon and Q, R are weighting matrices, with $Q \ge 0$ and $R > 0$. The notation $\tilde{\mathbf{x}}_{k+j}$ indicates the value of $\tilde{\mathbf{x}}$ at the instant $k+j$ predicted at instant k.

Minimizing the cost function equation (11) subject to

$$\begin{aligned}&\tilde{\mathbf{x}}_{k+j} = A_k\tilde{\mathbf{x}}_{k+j-1} + B_k\tilde{\mathbf{u}}_{k+j-1}, \quad -\delta_{max} \le \tilde{\mathbf{u}}_{k+j-1} \le \delta_{max}, \\ &-\Delta\delta_{max} \le \Delta\tilde{\mathbf{u}}_{k+j-1} \le \Delta\delta_{max},\end{aligned} \tag{12}$$

for $j = 1, 2, \ldots, N$, one can determine a control sequence $\tilde{\mathbf{u}}_k^*, \ldots, \tilde{\mathbf{u}}_{k+j-1}^*$. Eq. (12) incorporates maximum rudder deflection and maximum rudder rate limitations. According to MPC formulation, the first control action of the sequence of optimal control, $\tilde{\mathbf{u}}_k^*$, is used as the control at time k.

After some algebraic manipulation using the system dynamic equation (9), we can rewrite the cost function equation (11) in a standard quadratic form:

$$\Phi(k) = \tfrac{1}{2}\overline{\mathbf{u}}_k^T H_k \overline{\mathbf{u}}_k + f_k^T \overline{\mathbf{u}}_k + d_k, \tag{13}$$

with

$$H_k = 2(\overline{B}_k^T \overline{Q}\,\overline{B}_k + \overline{R}),$$

$$f_k = 2\overline{B}_k^T \overline{Q}\,\overline{A}_k \tilde{\mathbf{x}}_k, \tag{14}$$

$$d_k = \tilde{\mathbf{x}}_k^T \overline{A}_k^T \overline{Q}\,\overline{A}_k \tilde{\mathbf{x}}_k,$$

where $\overline{Q} = \mathrm{diag}(Q;\cdots;Q)$, $\overline{R} = \mathrm{diag}(R;\cdots;R)$, $\overline{\mathbf{u}}_k = [\tilde{\mathbf{u}}_k^T,\ldots,\tilde{\mathbf{u}}_{k+N-1}^T]^T$, and $\overline{A}_k$ and $\overline{B}_k$ are given by

$$\overline{A}_k = \begin{bmatrix} A_k \\ A_{k+1}A_k \\ \vdots \\ A_{k+N-1}\cdots A_k \end{bmatrix},$$

$$\overline{B}_k = \begin{bmatrix} B_k & 0 & \cdots & 0 \\ A_{k+1}B_k & B_{k+1} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ A_{k+N-1}\cdots B_k & A_{k+N-1}\cdots B_{k+1} & \cdots & B_{k+N-1} \end{bmatrix}. \tag{15}$$

The matrix H_k is a Hessian matrix, and it is always positive definite for positive R. It describes the quadratic part of the cost function, and the vector f_k describes the linear part. d_k is independent of $\overline{\mathbf{u}}_k$ and has no influence in the determination of $\overline{\mathbf{u}}^*$. Note that now only the control variables are used as decision variables. Furthermore, constraints for the initial condition and model dynamics are not necessary anymore, since now these informations are implicit in the cost function equation (13). The optimization problem can be solved using standard QP algorithm.

3.3. MPC formulation augmented with an LOS parameter

The path following capability of a vessel motion control system is usually achieved using rudder only. Because the maximum deflection and turning speed often limit the control authority of the rudder, the control signal can easily position the rudder to operate at its limit during transients. The large rudder motion often causes adverse effects on the maneuvering performance and ride quality as they induce large roll motion. The proper planning of the vessel trajectory can be an alternative solution to alleviate the excessive rudder motion and thereby decreasing the period of rudder saturation time. In this section, the LOS lookahead distance is considered as an additional decision variable in MPC design, and the simulation studies will be presented to compare the path following performance of the control system with fixed and variable lookahead distance. In order to adjust the LOS lookahead distance within the MPC framework, we introduce a new design parameter $\bar{l}_k$ in the LOS guidance law equation (3) as follows:

$$\tilde{\psi}_{LOS} = -(1+\bar{l}_k)\frac{e_k}{\sqrt{e_k^2+\Delta^2}} \tag{16}$$

and modify the cost function equation (11) to include $\bar{l}_k$:

$$\Phi(k) = \sum_{j=1}^{N}\left(\tilde{\mathbf{x}}_{k+j}^T Q \tilde{\mathbf{x}}_{k+j} + \sum_{j=1}^{N} \tilde{\mathbf{u}}_{k+j-1}^T R \tilde{\mathbf{u}}_{k+j-1}\right) + L\bar{l}_k^2, \tag{17}$$

where L is a constant weighting factor. Note that for the fixed LOS lookahead distance, we have $\bar{l}_k = 0$.

With the predictive model equation (9) and

$$\mathbf{x}_{r,k} = \cdots = \mathbf{x}_{r,N} = -(1+\bar{l}_k)\frac{e_k}{\sqrt{e_k^2+\Delta^2}}\underbrace{[0,1,0,0]^T}_{c},$$

the cost function can be formulated into the standard QP format in terms of new variable $\overline{\mathbf{u}}_k, \bar{l}_k$:

$$\Phi(k) = \begin{bmatrix} \overline{\mathbf{u}}_k \\ \bar{l}_k \end{bmatrix}^T \begin{bmatrix} \overline{B}_k^T\overline{Q}\,\overline{B}_k + \overline{R} & \dfrac{e_k}{\sqrt{e_k^2+\Delta^2}}\overline{B}_k^T\overline{Q}\,\overline{L}_k c \\ \left(\dfrac{e_k}{\sqrt{e_k^2+\Delta^2}}\overline{B}_k^T\overline{Q}\,\overline{L}_k c\right)^T & L + \dfrac{e_k^2}{e_k^2+\Delta^2}M_k(\overline{L}_k c) \end{bmatrix} \begin{bmatrix} \overline{\mathbf{u}}_k \\ \bar{l}_k \end{bmatrix}$$
$$+ \begin{bmatrix} \overline{\mathbf{u}}_k \\ \bar{l}_k \end{bmatrix}^T \begin{bmatrix} 2\overline{B}_k^T\overline{Q}\overline{A}_k\left(x_k - \dfrac{e_k}{\sqrt{e_k^2+\Delta^2}}\overline{L}_k c\right) \\ \dfrac{2e_k}{\sqrt{e_k^2+\Delta^2}}M_k x_k + 2\dfrac{e_k^2}{e_k^2+\Delta^2}M_k\overline{L}_k c \end{bmatrix}$$
$$+ \frac{e_k^2}{e_k^2+\Delta^2}M_k\overline{L}_k c + 2\frac{e_k}{\sqrt{e_k^2+\Delta^2}}M_k\mathbf{x}_k + \mathbf{x}_k^T\overline{A}_k^T\overline{Q}\,\overline{A}_k\mathbf{x}_k, \tag{18}$$

where M_k, $\overline{L}_k$ and c is given by

$$M_k = (\overline{L}_k c)^T \overline{A}_k^T \overline{Q}\overline{A}_k,$$

$$\overline{L}_k = \begin{bmatrix} -A_k + I_{4\times4} \\ -A_{k+1}A_k + I_{4\times4} \\ \vdots \\ A_{k+N-1}\cdots A_k + I_{4\times4} \end{bmatrix}, \quad c = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}. \tag{19}$$

Since the last three terms in Eq. (18) are independent of $\overline{\mathbf{u}}_k, \bar{l}_k$, the optimal solution remains the same if we replace the cost function by

$$\Phi(k)' = \begin{bmatrix} \overline{\mathbf{u}}_k \\ \bar{l}_k \end{bmatrix}^T$$
$$\times \overbrace{\left[\overline{B}_k^T\overline{Q}\,\overline{B}_k + \overline{R}\,\frac{e_k}{\sqrt{e_k^2+\Delta^2}}\overline{B}_k^T\overline{Q}\,\overline{L}_k c\left(\frac{e_k}{\sqrt{e_k^2+\Delta^2}}\overline{B}_k^T\overline{Q}\,\overline{L}_k c\right)^T L + \frac{e_k^2}{e_k^2+\Delta^2}M_k\overline{L}_k c\right]}^{H}$$
$$\times \begin{bmatrix} \overline{\mathbf{u}}_k \\ \bar{l}_k \end{bmatrix} + \underbrace{\begin{bmatrix} \overline{\mathbf{u}}_k \\ \bar{l}_k \end{bmatrix}^T}_{\overline{\mathbf{u}}^T}$$
$$\times \overbrace{\left[2\overline{B}_k^T\overline{Q}\left(\overline{A}_k\left(x_k + \frac{e_k}{\sqrt{e_k^2+\Delta^2}}\overline{L}_k c\right)\right) + \frac{2e_k}{\sqrt{e_k^2+\Delta^2}}M_k x_k + 2\frac{e_k^2}{e_k^2+\Delta^2}M_k\overline{L}_k c\right]}^{f} \tag{20}$$

which can be readily solved using standard optimization software. When the constraints are not active, the optimal solution is computed by $\overline{\mathbf{u}} = -\frac{1}{2}H^{-1}f$ where the condition $H > 0$ is equivalent to (refer to Boyd et al., 1994)

$$\overline{R} + \overline{B}_k^T\overline{Q}\,\overline{B}_k - V_k T_k^{-1} V_k^T > 0, \tag{21}$$

where

$$T_k = L + \frac{e_k^2}{e_k^2+\Delta^2}(\overline{L}_k c)^T \overline{A}_k^T \overline{Q}\,\overline{A}_k \overline{L}_k c$$

Table 1
Model ship principal particulars.

Item	Symbol	Value
Length	L	1.6 m
Breadth	B	0.38 m
Height	H	0.17 m
Mass	m	30
Inertia	I_z	0.3 kg m^2
Nominal speed	u_n	0.4 m/s

Table 2
Experimentally identified parameters.

$X_{\dot{u}}$	−6.8558	X_u	3.0211
$X_{u\|u}$	−12.9059	X_{uuu}	2.7759
$Y_{\dot{v}}$	−17.5	Y_v	−20.5
$Y_{\|v\|v}$	−24.2	$N_{\dot{v}}$	0
N_v	1.1965	$N_{\|v\|v}$	0.0016
$Y_{\dot{r}}$	0.0	Y_r	−0.835
$Y_{\|r\|r}$	−0.63	$Y_{\|v\|r}$	0.0
$Y_{\|r\|v}$	−0.14	N_r	−1.2522
$N_{\dot{r}}$	−1.2	$N_{\|v\|r}$	0.1
$N_{\|r\|r}$	−0.3302	$N_{\|r\|v}$	0.04
Y_δ	$1.8902*10^5$	Y_δ	0.022
N_δ	−0.01		

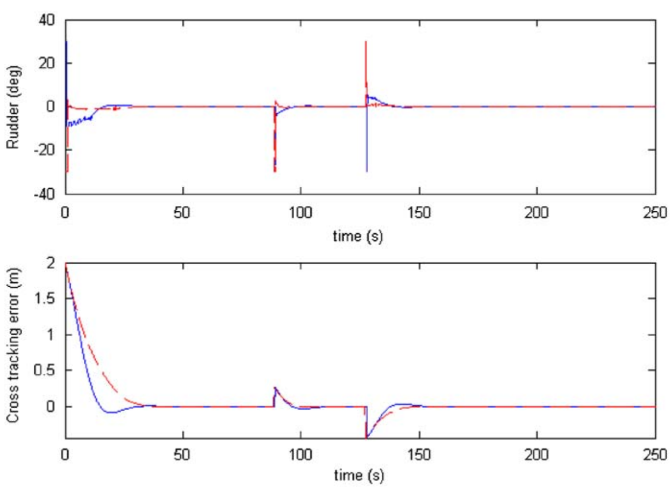

Fig. 4. Rudder and cross tracking error for LOS-based MPC (with fixed lookahead distance, dashed line) and non-LOS MPC (solid line) schemes.

and

$$V_k = \frac{e_k}{\sqrt{e_k^2+\Delta^2}} \overline{B}_k^T \overline{Q}\,\overline{A}_k \overline{L}_k c.$$

Eq. (21) is equivalent to

$$\overline{R}+\overline{B}_k^T\overline{Q}\,\overline{B}_k - \frac{(\overline{B}_k^T\overline{Q}\,\overline{A}_k\overline{L}c)(\overline{B}_k^T\overline{Q}\,\overline{A}_k\overline{L}c)^T}{L_1+(\overline{L}c)^T\overline{A}_k^T\overline{Q}\,\overline{A}_k\overline{L}c} > 0. \tag{22}$$

where $L_1 = (e_k^2+\Delta^2)/e_k^2 L$.

Using the matrix inversion lemma:

$$(F+PGT)^{-1} = F^{-1} - F^{-1}P(G^{-1}+TF^{-1}P)^{-1}TF^{-1}, \tag{23}$$

with

$$F=\overline{Q}^{-1}, \quad P=\overline{A}_k\overline{L}c, \quad T=P^T, \quad G=L_1^{-1} \tag{24}$$

we can show that Eq. (22) is equivalent to

$$\overline{R}+\overline{B}_k^T(\overline{Q}^{-1}+\overline{A}_k\overline{L}cL_1^{-1}c^T\overline{L}^T\overline{A}_k^T)^{-1}B_k > 0. \tag{25}$$

Since the second term is positive semi-definite, the inequality condition equation (21) or (22) holds as long as $\overline{R}>0$. Therefore, the positive definiteness of H is automatically guaranteed by selecting $R>0$. The practical way of guaranteeing stability of MPC is to use a prediction horizon that is large compared with the settling time of the plant. Our recent study explored the effect of MPC parameters such as a prediction horizon, a sampling time, and penalty matrices on a ship maneuvering performance and stability (Li et al., 2009a). The reader is also referred to Marjanovi and Lennox (2004) for a further survey on the stability of MPC.

4. Simulation evaluation of the control system

For the simulation and verification of the guidance and control design, a mathematical model of the ship is required. The dynamics of a 1:50 scaled model ship of an offshore supply vessel is described in Oh et al. (2009) for horizontal motion with surge, sway and yaw as variables. The main geometric dimension parameters of the model are listed in Table 1 and the hydrodynamic coefficients are presented in Table 2. The values of the coefficients are used to simulate the MPC path following capability. The form of the hydrodynamic models used in Oh et al. (2009) is taken from Abkowitz (1964) and is shown in Eqs. (26)–(28):

$$X_h = X_{\dot{u}}\dot{u}+X_u u+X_{|u|u}|u|u+X_{uuu}u^3, \tag{26}$$

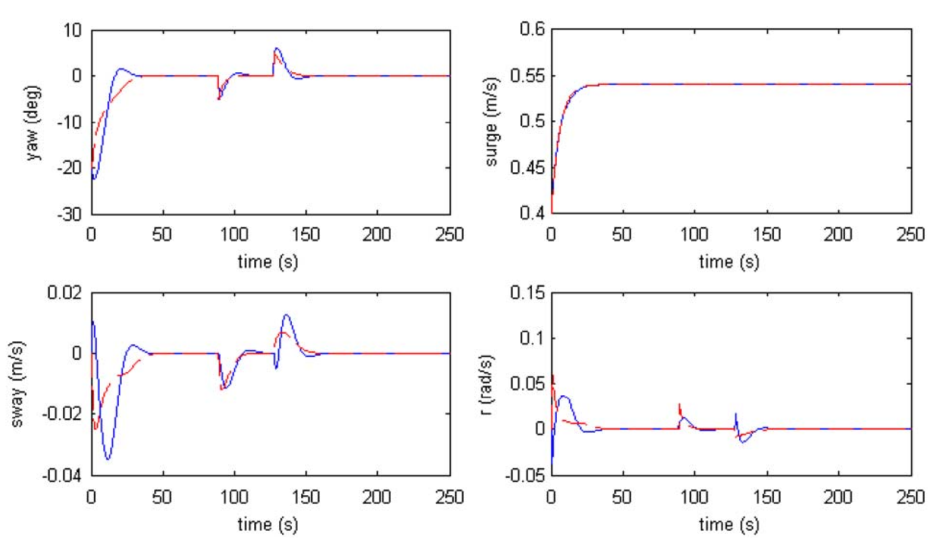

Fig. 5. Ship motion variables for LOS-based MPC (with fixed lookahead distance, dashed line) and non-LOS MPC (solid line) schemes.

$$Y_h = Y_{\dot{r}}\dot{r} + Y_{\dot{v}}\dot{v} + Y_v v + Y_r r + Y_{|v|v}|v|v + Y_{|v|r}|v|r + Y_{|r|v}|r|v + Y_{|r|r}|r|r, \tag{27}$$

$$N_h = N_{\dot{r}}\dot{r} + N_{\dot{v}}\dot{v} + N_v v + N_r r + N_{|v|v}|v|v + N_{|v|r}|v|r + N_{|r|v}|r|v + N_{|r|r}|r|r. \tag{28}$$

The mathematical model is discretized with a sampling time $T_s = 0.5$ s. Two different simulations are carried out to illustrate the effectiveness of the LOS based MPC design: First, the LOS based MPC is compared with its counterpart non-LOS MPC. Second, the LOS parameter is further used as a decision variable along with the primary control variable (i.e., the rudder deflection angle). The optimization problem has been solved with the MATLAB routine *quadprog* using the linearized dynamical model of a scale model ship described in Section 3. For simulations, four way points are set as $P_1 = (0,0)$, $P_2 = (50,0)$, $P_3 = (70,2)$, $P_4 = (130,2)$. The desired straight paths are computed by connecting two adjacent way-points. The ship is initially positioned at (0, 2). Hence, the ship starts with a cross tracking error of 2 m. The lookahead variable for computing the LOS heading angle is chosen to be $\Delta = 3L_{ship}$ (taken from Breivik, 2003) during the entire run. The weighting matrices used are $Q = \mathrm{diag}(1, 1, 0.5, 0.1)$ and $R = 0.1\mathbf{I}_{4\times4}$. The prediction horizon is $N = 10$ and the sampling time is 0.5 s. The rudder has a saturation constraint as $\delta_{max} = 30$ deg.

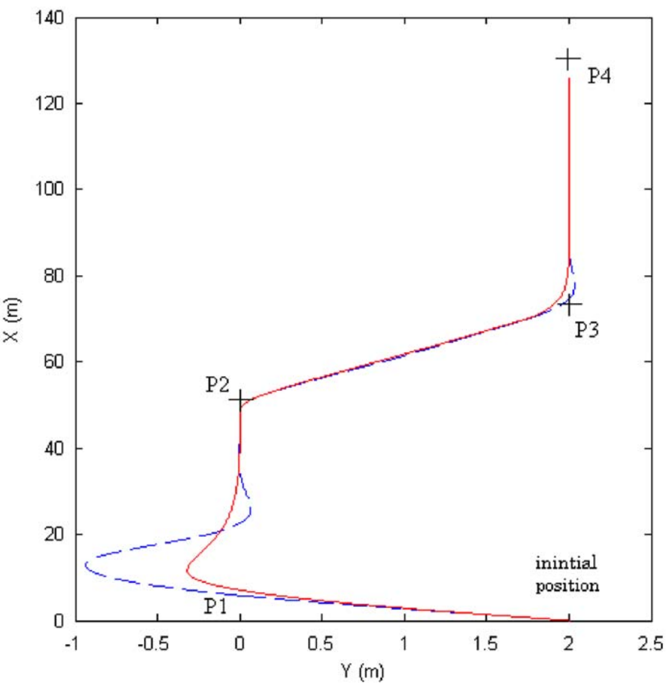

Fig. 6. Trajectory of the model ship in the x–y plane with fixed LOS (dashed line) and variable LOS (solid line) MPC path following control.

Fig. 4 shows the path following results of the LOS-based MPC and non-LOS MPC schemes. The dashed line stands for the LOS based MPC. With a constant speed of 0.4 m/s and a lookahead variable Δ of three ship lengths, the cross tracking error gradually converges to the desired path without transient overshoot. Give the same weighting matrices Q, R, the non-LOS MPC result (the solid line) shows the overshoot in the path following. The difference in these two path following performance is expected since tracking LOS points rather than the final target point can prevent excessive control effort and therefore result in smooth maneuvering. In Fig. 5, it can be seen that the control input is within the limits imposed by the saturation constraint. Fig. 6 displays all other variables of interest in the ship motion as functions of time in the path following process.

To illustrate the working of MPC path following with a varying LOS, both the LOS parameter $\bar{l}$ and the rudder command δ are treated as decision variables in solving the augmented QP problem to minimize the cost function given in Eq. (18). To evaluate the performance of the control scheme with a fixed LOS parameter versus that with an adjustable LOS, the simulations are carried out under more stringent limitations on the rudder amplitude and rate constraints, namely 10 deg and 10 deg/s, respectively and $-0.1 < \bar{l} < 0.1$ $\bar{l}$ is imposed to avoid excessive deviation from the desired LOS angle. The trajectories of the ship are plotted in Fig. 6 for fixed and variable lookahead distance cases. They are referred as fixed LOS and variable LOS, respec-

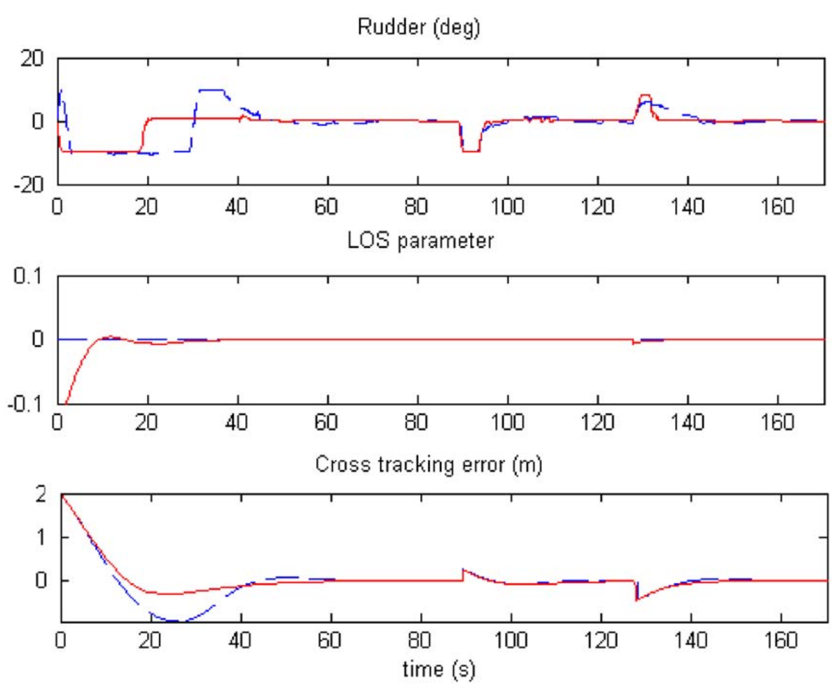

Fig. 7. LOS parameter, cross tracking error, and the rudder deflection angle for fixed LOS (dashed line) and variable LOS (solid line) MPC schemes.

tively. The benefits of incorporating the LOS parameter in the optimization procedure can be clearly identified from Fig. 6. Better tracking performance is achieved with a reduced overshoot. Fig. 7 shows the rudder deflection angle and the LOS parameter of the two simulation runs. Adjusting the LOS parameter has shortened the time period in which the rudder saturates, and as a result, the ship path following overshoot has reduced significantly. We also noticed that there were no significant differences in the computational time for fixed LOS and variable LOS MPC schemes. The CPU time required to compute the QP solution for both methods on a 2.2 GHz Pentium 4 PC with 2G RAM are around 0.0234 s, which indicates that the proposed MPC algorithms are applicable and the real-time implementation for the ship path following is possible.

5. Conclusions

This paper proposed an MPC design method, aiming at integrating an LOS guidance algorithm with a path following control for surface vessels. The choice of MPC for this application is primarily motivated by the need to explicitly handle constraints in the form of rudder saturation and rudder rate limit. The LOS guidance to mimic a good helmsman behavior was incorporated into the MPC design in order to generate reference trajectories for the ship to follow. The solution of the optimization problem through a standard QP solver was shown to be effective in achieving path following for vessels in the presence of input constraints. The simulation results showed that the MPC controllers combined with the LOS guidance scheme outperformed those obtained using MPC method alone. Moreover, incorporating the LOS lookahead variable as a design variable in MPC optimization provided an additional degree of freedom in shaping the path following control performance. Numerical simulations also confirmed the benefits of the variable LOS scheme.

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