Election Forensics: Strategies versus Election Frauds in Germany*

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June 18, 2015

*Prepared for presentation at the 2015 Annual Conference of the European Political Science Association, Vienna, Austria, June 25–27, 2015. An earlier version was presented at the 2015 Annual Meeting of the Midwest Political Science Association, Chicago, April 16–19, 2015.

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Abstract

Many statistical methods that have been proposed to use low-level election vote counts to detect election frauds have a hard time distinguishing distortions in vote counts that stem from voters' strategic behavior from distortions that originate in election frauds. Even a finite mixture model implementation of the recently proposed positive empirical approach of Klimek, Yegorov, Hanel and Thurner (2012) has this limitation. We use latent variable models for polling station voting data and postelection complaint data from the 2005 and 2009 German Bundestag elections to show that the types of multimodality emphasized in the Klimek et al. (2012) model are valid symptoms of the localized incidents that trigger the complaints. The postelection complaints in Germany certainly do not represent the full variety of election frauds that occur in various election systems. They reflect a mix of simple administrative failures, concerns about voter privacy, worries about ballot access and disagreements with the way Germany's mixed system operates—a banal "fraud" latent variable at best. But the fact that some methods that can be shown to discriminate strategies from frauds—even if imperfectly—is an important advance for election forensics.

1 Introduction

A key challenge for election forensics—the field devoted to using statistical methods to try to determine whether the results of an election are accurate—is to be able to tell whether patterns in election results that may appear anomalous in statistical estimates and tests are the results of election frauds or of strategic behavior. Many methods for trying to detect election frauds have been proposed (e.g. Myagkov, Ordeshook and Shaikin 2009; Levin, Cohn, Ordeshook and Alvarez 2009; Shikano and Mack 2009; Mebane 2010; Breunig and Goerres 2011; Pericchi and Torres 2011; Cantu and Saiegh 2011; Deckert, Myagkov and Ordeshook 2011; Beber and Scacco 2012). Both strategic behavior and frauds can cause the patterns such methods look for (Mebane 2013, 2014). Here we examine whether a finite mixture model implementation of the key ideas in Klimek, Yegorov, Hanel and Thurner (2012) can help avoid possible confusion and clearly identify circumstances in which frauds occur. The finite mixture model we use improves the one introduced by Mebane, Egami, Klaver and Wall (2014) (improvements to be described elsewhere).

We use data from the 2005 and 2009 federal elections in Germany to help assess the efficacy of the modified Klimek et al. (2012) model. Mebane et al. (2014) show that the parameters for the probability of each of the two kinds of frauds that the Klimek et al. (2012) model uses—Incremental Fraud and Extreme Fraud—are correlated with measures of strategic voting in the 2009 German *Bundestagswahl* (federal election) if either the original Klimek et al. (2012) algorithm or a chi-squared modification of the algorithm is used to estimate the parameters separately for the *Erststimmen* (single-member district plurality rule votes) of each *Wahlkreis* (district). Ample evidence exists to demonstrate that strategic voting occurs in the mixed system used in German federal elections (Bawn 1999; Pappi and Thurner 2002; Gschwend 2007). The *Erststimmen*, being plurality votes for a single winner, are affected by "wasted vote" reasoning such as Cox (1994) analyzes. The *Zweitstimmen* (proportional representation tier votes) exhibit "threshold insurance" strategic behavior intended to insure that key smaller parties gain seats in the *Bundestag*

(Herrmann and Pappi 2008; Shikano, Herrmann and Thurner 2009).

But Mebane et al. (2014) also show that there is some kind of relationship between the "fraud" probability estimates and the occurrence of complaints about the election filed with the *Bundestag* after the election. While the complaints do not necessarily concern what might be considered genuine frauds, they have face validity as imperfect measures of potentially serious irregularities. Ziblatt (2009) uses such complaints to measure the occurrence of election frauds in Germany during the years 1871–1912, and Breunig and Goerres (2011) make a similar usage with regard to more recent elections, although Breunig and Goerres (2011, 3–4) mention that the *Bundestag* has never responded to a complaint in a way that "questioned the validity of an election."

We use latent variable models to assess how the distribution of complaints across districts relates to the distribution of the fraud probability parameter estimates. We code complaints after the 2005 and 2009 elections by type, using a scheme adapted from the Election Incident Reporting System (EIRS) developed for elections in the United States (Verified Voting Foundation 2005; Hall 2005; Johnson 2005). We use an improved version of the finite mixture variant of the Klimek et al. (2012) model developed by Mebane et al. (2014) to estimate the fraud probabilities used in the latent variable exercise.

2 Data

The manifest variables we use in our latent variable models are based on two kinds of measures. These are fraud probabilities estimated using an improved version of Mebane et al. (2014)'s finite mixture variant of the Klimek et al. (2012) model and the extended EIRS codes representing postelection complaints filed with a committee of the *Bundestag*. We show how the "fraud" probability estimates are strongly related to measures of strategic behavior, relationships that pose sharply the measurement challenge for election forensics.

2.1 "Fraud" Probabilities

In the Klimek et al. (2012) model the baseline assumption is that votes in an election with no fraud are produced through the interaction of processes whose effects can be summarized by two Normal distributions: there is one distribution for turnout proportions and another, independent distribution for the proportion of votes going to the winner. Klimek et al. (2012) assume that election fraud means that votes are added to the votes for the winner. Some votes are transferred to the winner from the opposition, and some are transferred from nonvoters. The two kinds of election fraud refer to how many of the opposition and nonvoters votes are shifted: with "incremental fraud" moderate proportions of the votes are shifted; with "extreme fraud" almost all of the votes are shifted. Klimek et al. (2012) have parameters that specify the probability that each unit experiences each type of election fraud: f_{i} is the probability of incremental fraud and f_{e} is the probability of extreme fraud. Other parameters fully describe the bimodal and trimodal distributions that the model characterizes as being consequences of election frauds.

Mebane et al. (2014) implement a finite mixture model that uses the ideas about election frauds expressed in the Klimek et al. (2012) algorithm but produces results that better fit the model to data than the original algorithm does. We use an improved version of the Mebane et al. (2014) specifications with polling station vote count data from the 2005 and 2009 elections (Bundeswahlleiter 2010*a*,*b*) to estimate parameters f_{i} and f_{e} for the *Erststimmen* in each district. For each district i, $i = 1, \ldots, 299$, we obtain estimates $\hat{f}_{\mathrm{i}i}$ and $\hat{f}_{\mathrm{e}i}$.

Figures 1 and 2 show the distributions of the fraud probability estimates across the German districts.[1] Colors correspond to rescaled values $\hat{f}_{\mathrm{i}i}/\max(\hat{f}_{\mathrm{i}i})$ and $\hat{f}_{\mathrm{e}i}/\max(\hat{f}_{\mathrm{e}i})$. The color is red for values equal to 1.0 and blue for 0.0, and intermediate colors correspond to intermediate values. The maximum values of $\hat{f}_{\mathrm{i}i}$ are large: $\max(\hat{f}_{\mathrm{i}i}) = .74$ in 2005 and

[1] District (*Wahlkreis*) shapefiles are `Geometrie_Wahlkreise_16DBT_VG1000.shp` for 2005 and `Geometrie_Wahlkreise_17DBT_VG1000.shp` for 2009, found via `http://www.bundeswahlleiter.de/de/bundestagswahlen/`.

$\max(\hat{f}_{\mathrm{i}i}) = .9$ in 2009. The maximum values of $\hat{f}_{\mathrm{e}i}$ are very small: $\max(\hat{f}_{\mathrm{e}i}) = ..0047$ in 2005 and $\max(\hat{f}_{\mathrm{i}i}) = .0037$ in 2009. A few districts in each year have $\hat{f}_{\mathrm{i}i}$ or $\hat{f}_{\mathrm{e}i}$ values near the maximum values, but most values are zero or very near zero.

*** Figures 1 and 2 about here ***

Strategic Entanglements: The estimates of fraud probabilities from the Klimek et al. (2012) model are related to measures of strategic voting. Mebane et al. (2014) show that estimates of f_{i} and f_{e} for each district using the original and a slightly modified version of the Klimek et al. (2012) algorithm are significantly related to measures of strategic voting in Germany in 2009, and in 2005 such relationships are if anything more prevalent. Estimates of $\hat{f}_{\mathrm{i}i}$ and $\hat{f}_{\mathrm{e}i}$ using our current finite mixture method relate to measures of strategic voting in both years. Relations in 2005 are much more extensive than in 2009.

We use two-dimensional nonparametric regressions[2] in which the fraud probability estimates $\hat{f}_{\mathrm{i}i}$ and $\hat{f}_{\mathrm{e}i}$ are the outcome variable to estimate the relationships between fraud probability estimates and strategic voting measures. For covariates we use variables that have been argued to measure effects of strategic voting. Germany's mixed system gives opportunity to observe different distributions of votes being cast in the same district at the same time under both plurality (*Erststimme*) and proportional representation (*Zweitstimme*) rules. The difference between those votes is often used as a measure of strategic behavior (e.g. Cox 1997, 83; Bawn 1999). With the plurality election results alone, measures such as the difference between the winner's and the third-place candidate's votes connect to strategic behavior (Cox 1994). Mebane (2013, 2014) finds that the conditional mean of the second significant digits of votes both for the winning and second-place candidates in a plurality election varies in relation to strategic behavior.

Figure 3 illustrates the strong relationships between fraud probability estimates and strategic voting measures in 2005. In all four regressions displayed in the parts of Figure 3,

[2] We use the `sm` (Bowman and Azzalini 1997) package of **R** to compute the nonparametric regressions.

the rug plot along the x-axis shows the distribution of the proportional difference between the *Zweitstimmen* and *Erststimmen* cast for CDU-CSU in each district. That difference is one of the covariates in all of the regressions. The other covariates, with district distributions shown along the y-axes, are (a) the district *Erststimme* winner's proportion of the votes, (b) the margin between the winner and the third-place candidate, (c) the mean of the second significant digits in the winner's vote in each polling station, and (d) the mean of the second significant digits in the second-place candidate's vote in each polling station.[3] The titles above the graphics in each part of the figure report the p-value for a test of the nonparametric regression model compared to the model of no effects.

*** Figure 3 about here ***

In three of the four regressions in Figure 3, the nonparametric regression relationships are significant. In Figure 3(a), the relationship concerns the tendency for $\hat{f}_{\mathrm{i}i}$ to increase as the winner's proportion of the vote increases.[4] In Figure 3(b) the relationship concerns the tendency for $\hat{f}_{\mathrm{i}i}$ to increase as negative values of the *Zweitstimmen−Erststimmen* difference become even more negative and the winner's margin over the third-place finisher increases. In Figure 3(d) $\hat{f}_{\mathrm{i}i}$ tends to increase as the mean of the second digits of the second-place finisher's polling station vote counts decreases and as the *Zweitstimmen−Erststimmen* difference becomes more negative.

Figure 4 shows the same kinds of regressions except using the *Zweitstimmen−Erststimmen* difference for SPD instead of CDU-CSU. The relationships are significant in all four regressions. Similar patterns occur in regressions estimated for the FDP, Green and Left parties.

*** Figure 4 about here ***

[3]Polling stations include both in-person (*Urnenwahlbezirke*) and mail (*Briefwahlbezirke*) vote districts.

[4]In 2005 there are two districts in which the number of *Zweitstimmen* for CDU-CSU is greater than the number of *Erststimmen*: district 176 (Main-Kinzig-Kreis (part), Hesse) and district 236 (Stadt Weiden in der OberPfalz, Bavaria). These districts appear prominently on the right in Figure 3(a), but neither of them is responsible for the significant association between $\hat{f}_{\mathrm{i}i}$ and the various covariates.

Figures 5 and 6 show the same kinds of regressions as in Figures 3 and 4 except for 2009 instead of 2005. In this case, the only significant relationships—in Figures 5(c) and 6(c)—show that $\hat{f}_{\text{i}i}$ tends to increase as the mean of the second digits of the district winner's polling station vote counts decreases. Similar patterns occur in regressions estimated for the FDP, Green and Left parties.

*** Figures 5 and 6 about here ***

In 2005 $\hat{f}_{\text{e}i}$ has significant relationships with several measures of strategic voting, in patterns that are difficult to summarize. In 2009 $\hat{f}_{\text{e}i}$ shows no association with measures of strategic voting.

2.2 Election Verification in Germany

A mandate for *Wahlprüfung* (Election Verification) was first instituted in Germany in 1871 as part of the constitution of the newly minted German *Reich* (Ziblatt 2009). This institution has evolved over time and its current constitutional form was developed as part of the 1949 *Grundgesetz* (Basic Law). The *Wahlprüfung* concept, in its post World War II configuration includes the *Bundestag* as initial arbiter for complaints and also allows for complaints to be further forwarded to the *Bundesverfassungsgericht* (Federal Constitutional Court). The *Bundestag* committee that primarily receives and adjudicates postelection complaints is the *Ausschuss für Wahlprüfung, Immunität und Geschäftsordnung* (Committee for Election Verification, Immunity and Rules of Procedure, or AWIG).

In contrast to other election complaint systems—such as that used in Mexico—political parties in Germany do not play a central role in the complaint process. Complainants tend to be individuals who either directly experienced a failure of election administration or who are otherwise unsatisfied with the prevailing electoral system or political order more generally in Germany.

It is usually believed that election fraud is rare in Germany, but rare does not mean nonexistent. The AWIG and hence the *Bundestag* did not overturn any election results in 2005 or 2009 as a result of the complaints, although they did issue several recommendations on ways Germany's elections could be improved. One such situation involved the use of electronic voting machines in 2005. Several complaints in 2005 question the use of certain electronic voting machines due to their lack of a paper trail. The lack of a paper trail prevents voters from being able to positively ascertain that their vote has been recorded correctly. While the committee accepted the complaint as valid, it did not take any action to annul the results as the complainants had not actually demonstrated that the results had been manipulated using these machines. However, the committee did issue a recommendation that called on the government to investigate the allegations made by the complainants about the vulnerabilities of electronic voting (Bundestag 2000). The complainants would go on to lodge a successful complaint with the Federal Constitutional Court, which found the use of such electronic voting technology unconstitutional.

The AWIG has a standard that unless a complaint is shown to be "mandatsrelevant" the complaint will be rejected. To be "mandatsrelevant," the circumstance a complaint addresses must demonstrably change the composition of the *Bundestag* or make this a possibility that cannot be ruled out (Ausschuss für Wahlprüfung, Immunität und Geschäftsordnung 2013). The *Bundestag*'s failure to act on a complaint does not necessarily imply that the complaint is unfounded.

Facts motivating a complaint need not relate to election frauds in the sense of maleficent acts that distort votes. Because complaints generally come from citizens and not from political parties, they may be unlikely to result from partisan motivations. But some complaints are effectively partisan, such as those that relate to candidates' inability to get on the ballot, or would-be parties' frustration at not being treated as parties. Citizens may also act strategically with their complaints just as they do with their votes.

Special Oddities in 2005: Two unusual events relating to election administration dominated public perceptions of the *Bundestagswahl* in 2005: mismatched *Briefwahl* (mail ballots) in Dortmund and the special election in Dresden. Press coverage of the election made ample mention of these events and they subsequently show up frequently in the complaints from 2005.

Regarding the *Briefwahl in Dortmund*, the city of Dortmund contracted a private company to mail absentee ballots to voters in both of Dortmund's districts, but about 10,000 ballots were mistakenly sent to absentee voters in the incorrect district (Bundestag March 3, 2006). This provoked substantial confusion and outrage in Dortmund. While the AWIG acknowledged the administrative failure, they failed to annul the results from Dortmund because the complainants could not prove that these 10,000 ballots would have changed the outcome of either district race in Dortmund (Bundestag March 3, 2006).

In the state of Saxony in the eastern part of the country there was also a widely publicized controversy relating to the 2005 election. Eleven days before the 2005 election, a district candidate in Dresden's first district died, causing the election to be postponed, the so-called *Nachwahl in Dresden* (Behnke 2008). The situation in Dresden was controversial due to the strategic advantage held by those voters who would cast their ballots already knowing the outcome in the rest of the country. Germany's mixed system encourages strategic ticket-splitting generally, but in the case of a *Nachwahl* the additional knowledge is helpful because of a phenomenon known as "*negatives Stimmgewicht*" (negative vote weight). Due to the way votes were translated into *Bundestag* seats at the time, it was possible for a voter to cast a *Zweitstimme* (proportional representation tier vote) for their favored party and have that vote cost their party a seat (Behnke 2010). This is also related to the phenomenon of *Überhangmandate* (overhang mandates).[5] In the case of the voters in Dresden in 2005, it was possible to calculate the precise number of *Zweitstimmen* that would cause the CDU to lose an overhang mandate and thus reduce their parliamentary

[5]Negative vote-weight as enabled by overhang mandates was found unconstitutional by the *Bundesverfassungsgericht* in 2008 (Behnke 2010).

delegation by one. This knowledge encouraged conservative voters to behave strategically and cast a “coalition vote,” i.e., to cast their *Erststimmen* for the CDU and their *Zweitstimmen* for the FDP (Behnke 2008).

2.2.1 Complaints Types

Different types of postelection complaints occur with different frequencies in connection with the two elections. We code the complaint documents using a scheme that as much as possible follows the EIRS coding scheme (Verified Voting Foundation 2005; Hall 2005; Johnson 2005). We refer to our set of codes as EIRS+. Various modifications are necessary because of the particular features of the German election system. As Table 1 shows, overall we distinguish nineteen EIRS+ types of complaints that occur in 2005 and sixteen types in 2009.[6] Two additional “complaint” types that refer to either *Briefwahl in Dortmund* or *Nachwahl in Dresden* occur in 2005 that are measured partly using the subject field in the original files[7] and partly by reading the detailed descriptions in the body of each document.

*** Table 1 about here ***

The manifest variable we use in the model for the complaints of type k in district i, y_{ki}, is a binary indicator for whether at least one complaint of type k occurs for district i. As described in subsection 2.2.2, sometimes ambiguity about the district to which a complaint refers produces an ambiguous count of the number of complaint instances. When the ambiguity is between a count of zero and a positive value, ambiguity is induced in y_{ki}. Such ambiguity motivates the mixture likelihood of equation (3).

The ambiguity also produces variety in the totals of the binary indicators across districts. Table 1 shows two total counts for each type of complaint in each year, the least that can occur and the most. For most types the two counts are the same, and in a few instances the counts differ by one. Despite the variations, the types of EIRS+ complaints

[6]We describe the postelection complaint data collection and coding in more detail in the Data Appendix.
[7]See page 20.

that are the most frequent in 2005 are the same: Electoral System; Absentee-ballot Related Problem; and Polling Place Problem. In 2009 Party List Not on Ballot is the second most frequent complaint type, behind Electoral System, and Absentee-ballot Related Problem is third.

2.2.2 Complaints Locations

To identify whether at least one complaint of a specific type occurs in a particular district, we associate each instance of each type of complaint with a district. To do this we use district description files, postal codes that appear on most documents,[8] and shapefiles for postal code and district boundaries (details are in the Data Appendix). In some instances the association between a complaint instance and districts is ambiguous: the complaint instance may be associated with two or more districts. Usually this ambiguity is eliminated by associating zip code information with subcity level administrative divisions that we can match with documents (*Wahlkreisbeschreibung*) describing the electoral districts in terms of these same subcity administrative divisions. In some cases the ambiguity cannot be eliminated. Location ambiguity arises for several reasons. In some cases the zip code covers parts of multiple districts. In a few cases a complaint refers to only a city name—e.g., "Berlin," "Munich," "Dortmund" or "Dresden"—and the city comprises multiple districts.[9] In some instances of the "Absentee-ballot Related Problem" type of complaint the document refers to relevant claimed occurrences in two distinct districts, for example the district where the vote was supposed to be counted and the district in which the complainant is resident.

[8]Some complaints submitted by voters located outside of Germany lack any German postal code.

[9]Three documents in 2005 from voters outside of Germany mention only "Munich" (in two documents) or "Berlin," leading to ambiguities spanning, respectively, four and twelve districts. In 2009 one overseas complaint can be located only in "Berlin" (12 districts). We omit these complaints, where the ambiguity spans more than two districts, from the latent variable analysis. In 2009 one document from Berlin can be resolved no more precisely than down to three districts, but (see the District Association paragraph of the Data Appendix) one district has a much smaller number of voters in the referent zipcode area—the voters in only one *Wahlbezirke*—than do the other two districts, so for this observation we ignore the third potential district.

3 Model

We use latent variable models to study how variation in the district-specific fraud probability parameter estimates $\hat{f}_{ii}$ and $\hat{f}_{ei}$, based on the *Erststimmen*, relates to variation across districts in the occurrence of postelection complaints submitted to the AWIG. The specifications for the models are as follows.

Each of the K binary manifest complaint variables y_k, $k = 1, \ldots, K$, relates to an unobserved continuous variable x_k that is itself related to one or more common latent variables ξ_ℓ through equations for each district $i = 1, \ldots, 299$ of the following form,

$$x_{ki} = c_k + \sum_{\ell=1}^{J} \lambda_{k\ell}\xi_{\ell i}\,, \quad k = 1, \ldots, K+2\,, \tag{1}$$

where the values of c_k and of $\lambda_{k\ell}$ that are not constant[10] have Normal distributions and the ξ_ℓ are multivariate Normal with mean $\boldsymbol{\gamma} = (\gamma_i, \ldots, \gamma_J)'$ and precision matrix $\boldsymbol{\Upsilon}$.[11] When the district associated with a complaint is known with certainty, the manifest variables y_k take the (probit) index variable form

$$\text{Prob}(y_{ki} = 0) = \int_{-\infty}^{0} \phi(x_{ki}, \psi_k)\, df \tag{2a}$$

$$\text{Prob}(y_{ki} = 1) = \int_{0}^{\infty} \phi(x_{ki}, \psi_k)\, df \tag{2b}$$

where $\phi(x, \psi)$ is the Normal density with mean x and precision ψ.[12] But when uncertainty about the assignment of a complaint to a district leads to uncertainty about whether the observation y_{ki} should be $y_{ki} = 0$ or $y_{ki} = 1$, we mix over the two possible values. Let v_1 and v_2 be the number of registered voters in the portion of each of the two districts for

[10] See the discussion on page 13.

[11] Adapting specifications given by Lee (2007), the prior for each mean γ_ℓ is Normal, and the prior for $\boldsymbol{\Upsilon}$ is Wishart. Further details about the prior specifications for the three chains used for each Markov Chain Monte Carlo (MCMC) run are in the Model Appendix.

[12] The precisions ψ_k have gamma priors.

which the district assignment is uncertain.[13] Generate $r_i \in \{0,1\}$ using probabilities that have Dirichlet priors $\mathcal{D}([v_1, v_2])$ by[14]

$$\pi_i \sim \mathcal{D}([v_1, v_2])$$
$$L_i \sim \mathrm{dcat}(\pi_i)$$
$$r_i = L_i - 1$$

With this prior π_i has mean $\left(\frac{v_1}{v_1+v_2}, \frac{v_2}{v_1+v_2}\right)$. Let

$$\mathrm{Prob}(y_{ki}) = r_i \int_{-\infty}^{0} \phi(x_{ki}, \psi_k)\, df + (1-r_i) \int_{0}^{\infty} \phi(x_{ki}, \psi_k)\, df\,. \tag{3}$$

We generate π_i—separately for each pair of districts—because we are uncertain about the chances that any ambiguously locatable complaint should be associated with one of the two districts to which it could relate. The components of each π_i range over the unit interval and each r_i switches between the values zero and one as the MCMC algorithm proceeds.

For the manifest variables $\hat{f}_{\mathrm{i}i}$ and $\hat{f}_{\mathrm{e}i}$ we use M_i, the number of polling stations for district i, to compute the expected number of polling stations that $\hat{f}_{\mathrm{i}i}$ and $\hat{f}_{\mathrm{e}i}$ imply are affected by either incremental frauds or extreme frauds. The variables $z_{1i} = \mathrm{rnd}(\hat{f}_{\mathrm{i}i} M_i)$ and $z_{2i} = \mathrm{rnd}(\hat{f}_{\mathrm{e}i} M_i)$ are the expected numbers of fraudulent polling stations, rounded to the nearest integer. These variables connect $\hat{f}_{\mathrm{i}i}$ and $\hat{f}_{\mathrm{e}i}$ to the latent variables ξ_ℓ through a binomial likelihood and a logistic functional form: for $k' = 1, 2$,

$$z_{k'i} \sim \mathrm{Binomial}(M_i, p_{k'i})\,, \quad p_{k'i} = \frac{1}{1+\exp(-x_{K+k',i})}\,. \tag{4}$$

The scales and means of the latent variables are set by matching them to particular manifest variables, as follows. The coefficients ("factor loadings") $\lambda_{k\ell}$ are fixed equal to

[13] Details are in District Association paragraph of the Data Appendix.

[14] The dcat() function in the following specification returns positive integer values for L_i in $\{1,2\}$ (Lunn, Jackson, Best, Thomas and Spiegelhalter 2013, 352).

zero when the variable y_k or $z_{k'}$ is not a measure of latent variable ξ_ℓ. For each latent variable there is one $\lambda_{k\ell}$ that is fixed equal to 1.0 (using a different k for each ℓ), thus establishing a unit of measurement (scale) for the latent variables. Each latent variable has $\lambda_{k\ell}$ fixed equal to 1.0 for one distinct value k, and for that value k $\lambda_{k\ell'} = 0$ for all $\ell' > \ell$, while the $\lambda_{k\ell}$ are free to take on any value for all other combinations of manifest and latent variables. To set the mean of each latent variable we fix $c_k = 0$ for the values k that have $\lambda_{k\ell}$ fixed equal to 1.0. These restrictions on $\lambda_{k\ell}$ and c_k are sufficient to identify the parameters of an "exploratory" factor analysis model (Anderson and Amemiya 1988, 760).

4 Results

The model specifications we use feature three latent dimensions in both years. Using more or fewer latent dimensions in either year produces posterior distributions for the mean (c_k and $\boldsymbol{\gamma}$) and loading ($\lambda_{k\ell}$) parameters that are severely multimodal, featuring parameters with both positive and negative modes.[15] Posteriors in the specifications we use are all unimodal and for the most part symmetric. The latent variable structures estimated for the two years share important features.

First, in both cases the common latent variables are most likely uncorrelated. Although the posterior means of the covariances between latent variables are not exactly zero (see $\boldsymbol{\Phi} = \boldsymbol{\Upsilon}^{-1}$ in Table 6 in the Model Appendix), the 95% credible intervals for the covariance parameters all include zero.

Second, in both models one common latent variable underlies complaints about the electoral system as a whole while the other common latent variables relate to complaints with a more localized focus. This pattern is clearest in 2009. Table 4 shows 95% credible intervals and posterior medians for the loading parameters $\lambda_{k\ell}$ in 2009. The scale for the second common latent variable ξ_2 is set by the "Electoral System" manifest variable, and

[15]For 2009 some factor loading parameters have multimodel posterior distributions even with the three-dimensional model.

the "Electoral System" manifest variable does not have a loading that differs significantly from zero on the first latent variable (its loading on the third latent variable is zero by construction).

*** Tables 2 3, 4 and 5 about here ***

In 2005 the Dortmund and Dresden events complicate matters, but a common latent variable relatively focused on electoral system concerns does appear. In Table 2, which shows 95% credible intervals and posterior medians for $\lambda_{k\ell}$ for the first two common latent variables in 2005, ξ_2 again has its scale set by the "Electoral System" manifest variable and the loadings for the "Counting of the Votes," "Improper Campaigning Influence" and "Police Harassment" are clearly signed and positive only for this latent variable, while the loading for "Problems with the Creation of Party Lists" is clearly signed and negative. Both the Dortmund and Dresden manifest variables have positive loadings on ξ_1, Dresden has a positive loading on ξ_2 (Dortmund comes close), and Dortmund has a positive loading on ξ_3. Because the concerns about the *Nachwahl in Dresden* largely focused on "negative vote weight," which is an electoral system phenomenon, the large loading for the Dresden variable on ξ_2 supports interpreting that latent variable as primarily about electoral system concerns.

The estimates $\hat{f}_{\mathrm{i}i}$ and $\hat{f}_{\mathrm{e}i}$ relate to the common latent variables in diverse ways. Both $\hat{f}_{\mathrm{i}i}$ and $\hat{f}_{\mathrm{e}i}$ are positvely related to ξ_3 in 2005 and to ξ_1 in 2009. $\hat{f}_{\mathrm{e}i}$ is postively related to ξ_2 in 2005 while $\hat{f}_{\mathrm{i}i}$ is negatively related to ξ_2 in 2009. The other loadings involving $\hat{f}_{\mathrm{i}i}$ or $\hat{f}_{\mathrm{e}i}$ have credible intervals that include zero.

If the complaints variables and the latent variables they have in common reflect real irregularities in the administration of the two elections, do the relationships between those latent variables and $\hat{f}_{\mathrm{i}i}$ and $\hat{f}_{\mathrm{e}i}$ suggest that $\hat{f}_{\mathrm{i}i}$ and $\hat{f}_{\mathrm{e}i}$ merit being described as "fraud" probabilities? In each year, the loadings for $\hat{f}_{\mathrm{i}i}$, $\hat{f}_{\mathrm{e}i}$ or both suggest that the probabilities do relate in a meaningful way to localized irregularities. In 2005, both $\hat{f}_{\mathrm{i}i}$ and $\hat{f}_{\mathrm{e}i}$ relate positively to a common latent variable (ξ_3) that also relates strongly to the Dortmund variable and to the "Polling Place Problem" type of complaint.[16] In 2009, both $\hat{f}_{\mathrm{i}i}$ and $\hat{f}_{\mathrm{e}i}$ relates positively to the common latent variable (ξ_1) that also relates positively to all of the complaint variables except "Electoral System," "Disability Access Problem" and "Problems with the Creation of Party Lists." In 2009 $\hat{f}_{\mathrm{i}i}$ is negatively associated with the common latent variable (ξ_2) that seems to refer not to localized complaints as much as to complaints generally concerning the electoral system. In 2005 $\hat{f}_{\mathrm{e}i}$ relates positively to the second common latent variable, which refers to electoral system concerns but has localized entanglements via the late election in Dresden.

Our methods are not designed to assess whether the connections between $\hat{f}_{\mathrm{i}i}$ and $\hat{f}_{\mathrm{e}i}$ and localized incidents are enough to outweigh the way $\hat{f}_{\mathrm{i}i}$ in 2005 relates to measures of strategic behavior, but the connections between the probabilities and localized irregularities seems strong. Whether these irregularities should be called *frauds* is an interpretive matter we will not try to resolve.

5 Discussion

To be able to distinguish strategic behavior from election frauds is a key challenge for election forensics. If some voters change how they vote based on strategic considerations, then the distribution of votes differs from what it would have been had the voters not done that. Votes can also change due to fraudulent manipulations. Statistical methods for detecting frauds that focus on identifying "unusual" patterns in votes need to be insensitive to patterns induced by strategic behavior, else users of the methods need to be aware of the potential for confused inferences. Mebane (2013, 2014) argues especially that methods based on vote counts' second significant digits are highly sensitive to strategic behavior. The concern is that all election forensic methods may be sensitive to strategic behavior.

Parameters of the Klimek et al. (2012) model that purport to measure the probability

[16] It's worth noting that included among the "Polling Place Problem" complaints are complaints in four districts that refer to the electronic voting system that was used in 2005.

of election frauds sometimes also respond to strategic voting. The Klimek et al. (2012) parameters describe particular bimodal and trimodal distributions that are viewed as "unusual." When using the original Klimek et al. (2012) algorithm, estimates of both the Incremental and Extreme parameters—f_{i} and f_{e}—relate to measures of strategic behavior in the German federal elections of 2005 and 2009. An improved version of the finite mixture model variant of Mebane et al. (2014) produces estimates of the parameters that are less related to measures of strategic behavior, nonetheless strong relationships still exist.

Our latent variable analysis suggests that estimates of the "fraud" probability parameters $\hat{f}_{\mathrm{i}i}$ and $\hat{f}_{\mathrm{e}i}$ do relate meaningfully to the localized irregularities that provoke postelection complaints to the *Bundestag* in both of the German elections. The estimates tend to be higher in districts that have complaints than in districts that lack complaints, and the complaints that relate positively to $\hat{f}_{\mathrm{i}i}$ and $\hat{f}_{\mathrm{e}i}$ tend to be complaints that refer to localized problems and not to general electoral system concerns. Whether the incidents that provoke the complaints should be described as "frauds" is a matter of interpretation, but $\hat{f}_{\mathrm{i}i}$ and $\hat{f}_{\mathrm{e}i}$ appear to be valid but not perfect measures of those incidents. That is, to be a bit more precise, the bimodal and trimodal distributions that the Klimek et al. (2012) model highlights appear to be valid measures of the "frauds" that occur in German federal elections.

6 Appendices

The Data Appendix describes the EIRS+ data in greater detail. The Model Appendix reports BUGS code used for MCMC estimation of the latent variable models using OpenBUGS (Lunn, Spiegelhalter, Thomas and Best 2009; OpenBUGS 2013; Lunn et al. 2013), along with tables that report credible intervals for means in the models.

6.1 Data Appendix

6.1.1 German complaints data

Sources: One of the standing committees of the *Bundestag* is the *Ausschuss für Wahlprüfung, Immunität und Geschäftsordnung* (Committee for Election Verification, Immunity and Rules of Procedure). This committee deals with the rules of the *Bundestag*, possible criminal proceedings against *Bundestag* members and complaints about the administration of national elections (*Bundestagswahlen*, *Europawahlen*, etc.).

All of the complaints data come from the archives of the *Bundestag*'s website. The "*Drucksache*" field represents the document number in the form "Election Period/Document Number." There is also a file number associated with every complaint in the form "WP XX/Election Year." The *Drucksache* field and the file number allow for the easy finding of the original complaint's text. The name, location, and reason fields are all taken directly from the original documents published by the relevant *Bundestag* committee. The "EIRS Coded Reason" borrows from the Election Incident Reporting System (Verified Voting Foundation 2005; Hall 2005; Johnson 2005), with a few additions necessitated by the vagaries of the German electoral system and the type of complaints that it precipitates. To determine the *Wahlkreis* (district) that corresponds with the zip code given for each case, we use a shapefile of German zip codes[17] in conjunction with a shapefile that shows the district boundaries for the relevant election (occasionally, these borders were unclear, most likely due to projection differences between the two shapefiles). It should be noted that the locations given in the files are only the location of the complainant, i.e., it is entirely possible for someone to complain about an issue that they themselves did not experience—standing is not an issue.

EIRS+ Codes: Here is a list of all of the reason codes we applied to the complaints data. They translate the nature of the complaint as presented in the long-form text into a

[17] See `http://arnulf.us/PLZ`.

more database-friendly form.

- Absentee-ballot related problem: cases where complainants did not receive their absentee ballot, their absentee ballot came late, or where there were any other problems related to the preparation or administration of absentee voting.
- Registration related problem: cases where complainants were not able to vote or request an absentee ballot due to problems with their registration (not registered at all or they were registered in a different location) or in cases where there were problems mailing the *Wahlbenachrichtungen* (letters that notify registered German voters when an upcoming election will take place and where they are supposed to vote).
- Improper Campaigning Influence: cases where the complainant encountered improper campaign advertising (for example, advertising too close to a polling place) or felt that any of the parties' campaigns were conducted in an otherwise inappropriate, if not necessarily illegal, manner.
- ID related problem: as Germany does not have strict voter ID laws, many complainants demanded a more robust process for checking the identity of voters at the polling place.
- Criminal status related problem: cases where problems with the administration of federal elections in prisons were alleged.
- Disability access problem: cases where polling places or other voting-related buildings were not accessible to the disabled.
- Ballot related problem: all complaints related to the physical characteristics of the ballot and its design (the size of the ballot, the color of the ballot, the folding of the ballot, etc.)
- Polling place problem: includes problems related to the built environment of the polling place, the set-up of the voting booths and other temporary election structures, as well as problems with polling place workers. A few examples of problems with the built environment of a polling place would be the presence of surveillance cameras or an elevated balustrade that could hypothetically allow people to observe voters in the voting booths.

- Electoral System: includes complaints relating to specific aspects of the German electoral system (overhang mandates, the 5% threshold, the method used for turning votes into seats, etc.) Also includes complaints that do not criticize a specific aspect of the German electoral system, rather a broader issue that is related to the electoral system.

- Party List Not on Ballot/Other Ballot Access Issues: many parties whose party lists were not recognized by the *Bundeswahlleiter* (and therefore did not appear as options under the second vote) complained about this impediment. This category also includes complaints from independent candidates about their placement on the ballot.

- Problems with the creation of Party Lists: cases where complainants claim that party lists were improperly prepared. In Germany, the candidates and their order on the individual Landlists are determined by the parties themselves at a mass gathering of each party

- Counting of the votes: any complaint that alleges inconsistencies in vote counts or improper procedures in the preparation of those counts.

- Improper Statistics/Representative Election Statistic: any complaint that alleges the violation of the secret ballot through the preparation of certain election statistics. In Germany, the most salient of these statistics is the *Repräsentative Wahlstatistik* (Representative Election Statistic). This determines the voting patterns of Germans differentiated by sex and age range, which is accomplished through sampling precincts throughout Germany by having them distribute marked ballots (these ballots indicate the voter's sex and age-range). This process is controversial, as numerous complainants objected to the perceived invasion of privacy.

- Improper District Boundaries: indicates a complaint that alleged improprieties in the drawing of district boundaries

- Allegations of Official Corruption: complaints that accuse various government officials of involvement in various corruption schemes (this does not include allegations of improprieties against poll workers)

- Police Harassment: complaints of this type allege that the police improperly interfered in some aspect of the electoral process. Specifically, the complaints in 2005 allege that the police impeded the legal activities of an aspiring political party

- Voter Intimidation: complaints of this type allege intimidation by polling place officials or other persons that occurred while the complainant was casting their ballot (whether in-person or via the *Briefwahl*)

- Unspecified Other: includes complaints where the nature of the complaint could not be ascertained or non-sequitur complaints.

Many of the codes that originate with EIRS are closely related (for example, many complaints coded under "Absentee-ballot related problem" involve problems with voter registration and as such could also be coded as "Registration related problems"). The same issue presents itself with many of the codes developed specifically for the German case, as they deal with numerous specific complaints about the electoral system.

To determine how to code the reason for the complaint, we consulted the "*Betreff*" (subject) field that is contained in the original *Bundestag* files, in the table of contents alongside the corresponding file number. This field gives an approximation of the nature of the complaint as parsed by the committee. The documents also contain the specifics of every complaint as well as the response of the committee. As such, both the complaints and the committee's responses can be quite lengthy. The four documents—17/2250, 17/3100, 17/4600 and 17/6300—that relate to the 2009 *Bundestagswahl* are 56, 212, 136, and 144 pages long respectively. The reason codes in the database take into account both the subject of the complaint as assigned in the table of contents and the broader enumeration of the complaint found in the body of the document.

Some complaints are assigned multiple "reason" codes (up to six). This is usually precipitated by a telltale "*u.a.*" ("*unter anderem*," meaning among others) in the original *Betreff* of the complaint. The precise nature of each multifaceted complaint is completely enumerated in the main text of each complaint, as opposed to the *Betreff*. In order to receive multiple codes, a complaint had to enumerate multiple complaints that involved multiple EIRS+ codes, not simply multiple aspects of the same code. For example, in 2009 many people complained about overhang mandates, a complaint that was coded under "Electoral System." Many people also complained about the distribution of seats, which would also fall under an "Electoral System" complaint. In some cases, these complaints involved both the division of seats and overhang mandates, in which case the complaint was still simply coded as a single "Electoral System" complaint, regardless of the fact that there are two complaints about the electoral system. The codes are designed to show the subjects of the complaints, as opposed to their multiplicity.

District Association: To determine the district from which the complaint emanated, we use the postal codes given in the documents from the *Bundestag* in conjunction with several sets of shapefiles (one that shows the distribution of postal codes across Germany and the other that shows the division of the districts in a given *Bundestag* election). It should be noted that the locations given in the files are only the location of the complainant, i.e., it is entirely possible for someone to complain about an issue that they themselves did not experience—standing is not an issue.

Some complaints involved a more complex procedure for determining their geographic assignment. This is caused by the discrepancy in scale between the shapefile that displays all of Germany's zip codes and that which shows the boundaries of each district. There are significantly many more zip codes in Germany than districts, and occasionally it is difficult to discern whether the zip code in question is actually divided between districts or whether an apparent division is just an artifact of laying two differently scaled maps over one another. This problem is especially acute in the larger cities where there are often dozens upon dozens of zip codes. In some cases, due to the shape of both the district and zip code in question, it is clear that the discrepancy is purely a cartographic issue. In other cases, more investigation is required to determine whether a zip code straddles multiple districts.

To overcome this issue, we developed a procedure that takes this ambiguity into account. For zip codes that span multiple *Wahlkreise*, the relative portion of a given zip code's area that is in each *Wahlkreis* is used to develop weights that determine the likelihood of a complaint being located in a given district. These weights are determined by the size of the population in the portion of one zip code that is in a given *Wahlkreis*. The population numbers are the number of registered voters in each portion of the zip code that is coincident with a specific *Wahlkreis*. To determine the number of registered voters contained in a zip code and then subdivide this number between the *Wahlkreise* contained therein, we used a combination of zip code shapefiles and precinct shapefiles, augmented to include precinct-level voting data from the election in question. Here is a general outline of the procedures used to facilitate this type of investigation:

1. Open ArcMap

2. Under the "File" drop-down menu, click "Add Data" and select "Add data..."

3. Select the shapefiles that contain the relevant zip code information as well as those that contain the appropriate precinct information

4. Open the attribute table of the zip code shapefile by either right-clicking its name in the "Table of Contents" pane of the ArcMap display or alternatively by double-clicking the name of the shapefile while holding down the control key on the keyboard. Do the same for the shapefile that contains the precinct information.

5. Select the zip code in question by clicking on it in the attribute table and this will highlight the corresponding polygon in the map display. The next steps will outline the procedure for highlighting all of the precints encompassed by the zip code in question.

6. Under the "Selection" drop-down menu, click on "Select by Location..."

7. From the resulting window: 1) choose ”select features from” as the ”Selection method”; 2) select the ”Target layer”, which in this case will be the shapefile that contains the precinct information; 3) select the ”Source layer”, which in this case will be the shapefile containing the zip code information; 4) check the ”Use selected features” box, which is directly below the drop down list of source layers; 5) From the ”Spatial selection method for target layer feature(s)” drop-down menu, select the ”have their centroid in the source layer feature” option and click ”Apply”

8. The penultimate step is to manually select any other polygons from the precinct shapefile that are at least partially encompassed by the zip code, even if the centroids of these polygons are not contained within the zip code.

9. Once the number and identifying information for each precinct in each district has been obtained, the number of eligible voters in each district in a given zip code can be obtained by consulting the precinct level vote data for the election in question.

In the latent variable analysis we treat the complaints and their location in a binary way. The types of disputes are aggregated to the district level: if there is a positive number of complaints of a certain type then the whole district receives a “1” for that type, and if there are no complaints of a certain type in a district then it receives a “0”. If the count of complaints is positive only because one of the possible locations of a geographically ambiguous complaint is a particular district, then we have an instance where we are uncertain about the assignment of a complaint to a district; in such instances we are uncertain about whether the observation y_{ki} should be $y_{ki} = 0$ or $y_{ki} = 1$ (recall page 11).

Despite the fact that there are districts with more than one complaint of the same type, we decided that considering the multiplicity of a particular type of complaint in a given district would be ill-advised for several reasons. The first is that the complaint data was coded not to represent the number of complaints of a certain type in a district, but to illuminate the type of complaints levied by citizens in a given district. In many cases, determining the precise number of disputes of the same type contained in a given complaint is a nebulous task, as often these disputes are inextricably related to one another. This is especially true of complaints that allege the unconstitutionality of various aspects of the electoral system. Second, considering the multiplicity of complaints could bias the analysis in favor of well-publicized problems with electoral administration. Just because more people filed complaints of a given nature does not mean that these complaints are more valid or serious than a complaint filed by just one individual. One example where the preceding logic would be less applicable would be in a situation where many individuals allege that their votes had not been counted or that they had otherwise been illegally barred from voting. This is not the case regarding the German disputes, however. Third, even in districts where relatively more complaints originated, the number of complainants still represents a tiny percentage of the population and reading too much into the sheer number of complaints risks distorting the resulting model.

6.2 Model Appendix

6.2.1 BUGS Code

The code we use with `OpenBUGS` (Lunn et al. 2009; OpenBUGS 2013; Lunn et al. 2013) to run the MCMC algorithms is as follows. We show the model specifications for the preferred models in both years.

2005:

```
model{
  for(i in 1:N){
    for (j in 1:20) {
      # geo location indicator
      L[i,j] ~ dcat(pi[i,j,1:2])
      # prior for mixture probability vector
      alpha[i,j,1] <- w1[i,j]
      alpha[i,j,2] <- w2[i,j]
      pi[i,j,1:2] ~ ddirch(alpha[i,j,1:2])
    }
```

```
  #measurement equation model
  for(j in 1:20){
    r[i,j] <- L[i,j]-1
    y1[i,j]~dnorm(mu[i,j],psi[j])I(thd[1,z1[i,j]],thd[1,z1[i,j]+1])
    y2[i,j]~dnorm(mu[i,j],psi[j])I(thd[1,z2[i,j]],thd[1,z2[i,j]+1])
    y[i,j] <- r[i,j]*y1[i,j] + (1-r[i,j])*y2[i,j]
    ephat[i,j]<-y[i,j]-mu[i,j]
  }
  for(j in 21:22){
    z1[i,j+3] ~ dbin(p[i,j], z1[i,23])
    p[i,j]<-ilogit(mu[i,j])
    ephat[i,j]<-z1[i,j+3]-p[i,j]*z1[i,23]
  }
  # three factors
  mu[i,1]<- xi[i,1]                                                  # Absenteeb
  mu[i,2]<- lam[1]*xi[i,1]  + lam[20]*xi[i,2] + lam[38]*xi[i,3] +c[1]  # Allegatio
  mu[i,3]<- lam[2]*xi[i,1]  + lam[21]*xi[i,2] + lam[39]*xi[i,3] +c[2]  # Ballotrel
  mu[i,4]<- lam[3]*xi[i,1]  + lam[22]*xi[i,2] + lam[40]*xi[i,3] +c[3]  # Countingo
  mu[i,5]<- lam[4]*xi[i,1]  + lam[23]*xi[i,2] + lam[41]*xi[i,3] +c[4]  # Criminals
  mu[i,6]<- lam[5]*xi[i,1]  + lam[24]*xi[i,2] + lam[42]*xi[i,3] +c[5]  # Disabilit
  mu[i,7]<- lam[6]*xi[i,1]  + xi[i,2]                                # Electoral
  mu[i,8]<- lam[7]*xi[i,1]  + lam[25]*xi[i,2] + lam[43]*xi[i,3] +c[6]  # IDrelated
  mu[i,9]<- lam[8]*xi[i,1]  + lam[26]*xi[i,2] + lam[44]*xi[i,3] +c[7]  # ImproperC
  mu[i,10]<-lam[9]*xi[i,1]  + lam[27]*xi[i,2] + lam[45]*xi[i,3] +c[8]  # ImproperD
  mu[i,11]<-lam[10]*xi[i,1] + lam[28]*xi[i,2] + lam[46]*xi[i,3] +c[9]  # ImproperS
  mu[i,12]<-lam[11]*xi[i,1] + lam[29]*xi[i,2] + lam[47]*xi[i,3] +c[10] # PartyList
  mu[i,13]<-lam[12]*xi[i,1] + lam[30]*xi[i,2] + lam[48]*xi[i,3] +c[11] # PoliceHar
  mu[i,14]<-lam[13]*xi[i,1] + lam[31]*xi[i,2] + xi[i,3]               # Pollingpl
  mu[i,15]<-lam[14]*xi[i,1] + lam[32]*xi[i,2] + lam[49]*xi[i,3] +c[12] # Problemwi
  mu[i,16]<-lam[15]*xi[i,1] + lam[33]*xi[i,2] + lam[50]*xi[i,3] +c[13] # Registrat
  mu[i,17]<-lam[16]*xi[i,1] + lam[34]*xi[i,2] + lam[51]*xi[i,3] +c[14] # Unspecifi
  mu[i,18]<-lam[17]*xi[i,1] + lam[35]*xi[i,2] + lam[52]*xi[i,3] +c[15] # Voterinti
  mu[i,19]<-lam[18]*xi[i,1] + lam[36]*xi[i,2] + lam[53]*xi[i,3] +c[16] # Dortmund
  mu[i,20]<-lam[19]*xi[i,1] + lam[37]*xi[i,2] + lam[54]*xi[i,3] +c[17] # Dresden

  mu[i,21]<-lam[55]*xi[i,1] + lam[56]*xi[i,2] + lam[57]*xi[i,3] +c[18] # fi
  mu[i,22]<-lam[58]*xi[i,1] + lam[59]*xi[i,2] + lam[60]*xi[i,3] +c[19] # fe

  #structural equation model
  xi[i,1:3]~dmnorm(u[1:3],phi[1:3,1:3])
}# end of i

#thresholds
for(j in 1:20){
  thd[j,1]<-alpbot
  thd[j,2]<-alpmid
  thd[j,3]<-alptop
```

```
  }

  for(i in 1:3){u[i]<-gam[i]}

  #priors on loadings and coefficients
  var.lam[1]<-4.0*psi[3]     var.lam[2]<-4.0*psi[3]     var.lam[3]<-4.0*psi[3]
  var.lam[4]<-4.0*psi[3]     var.lam[5]<-4.0*psi[3]     var.lam[6]<-4.0*psi[6]
  var.lam[7]<-4.0*psi[7]     var.lam[8]<-4.0*psi[8]     var.lam[9]<-4.0*psi[9]
  var.lam[10]<-4.0*psi[10]   var.lam[11]<-4.0*psi[11]     var.lam[12]<-4.0*psi[11]
  var.lam[13]<-4.0*psi[10]   var.lam[14]<-4.0*psi[11]     var.lam[15]<-4.0*psi[11]
  var.lam[16]<-4.0*psi[10]   var.lam[17]<-4.0*psi[11]     var.lam[18]<-4.0*psi[11]
  var.lam[19]<-4.0*psi[10]   var.lam[20]<-4.0*psi[11]     var.lam[21]<-4.0*psi[11]
  var.lam[22]<-4.0*psi[10]   var.lam[23]<-4.0*psi[11]     var.lam[24]<-4.0*psi[11]
  var.lam[25]<-4.0*psi[10]   var.lam[26]<-4.0*psi[11]     var.lam[27]<-4.0*psi[11]
  var.lam[28]<-4.0*psi[10]   var.lam[29]<-4.0*psi[11]     var.lam[30]<-4.0*psi[11]
  var.lam[31]<-4.0*psi[10]   var.lam[32]<-4.0*psi[11]     var.lam[33]<-4.0*psi[11]
  var.lam[34]<-4.0*psi[10]   var.lam[35]<-4.0*psi[11]     var.lam[36]<-4.0*psi[11]
  var.lam[37]<-4.0*psi[10]   var.lam[38]<-4.0*psi[11]     var.lam[39]<-4.0*psi[11]
  var.lam[40]<-4.0*psi[10]   var.lam[41]<-4.0*psi[11]     var.lam[42]<-4.0*psi[11]
  var.lam[43]<-4.0*psi[10]   var.lam[44]<-4.0*psi[11]     var.lam[45]<-4.0*psi[11]
  var.lam[46]<-4.0*psi[10]   var.lam[47]<-4.0*psi[11]     var.lam[48]<-4.0*psi[11]
  var.lam[49]<-4.0*psi[10]   var.lam[50]<-4.0*psi[11]     var.lam[51]<-4.0*psi[11]
  var.lam[52]<-4.0*psi[10]   var.lam[53]<-4.0*psi[11]     var.lam[54]<-4.0*psi[11]
  var.lam[55]<-4.0*psi[10]   var.lam[56]<-4.0*psi[11]     var.lam[57]<-4.0*psi[11]
  var.lam[58]<-4.0*psi[10]   var.lam[59]<-4.0*psi[11]     var.lam[60]<-4.0*psi[11]
  for(i in 1:60){lam[i]~dnorm(0.8,var.lam[i])}

  var.b<-4.0*psi[1]
  for(j in 1:3){gam[j]~dnorm(0.1,var.b)}

  var.c<-4.0*psi[2]
    for(j in 1:19){c[j]~dnorm(0.1,var.c)}

  #priors on precisions
  for(j in 1:P){
    psi[j]~dgamma(10,8)
    sgm[j]<-1/psi[j]
  }

  phi[1:3,1:3]~dwish(R[1:3,1:3], 30)
  phx[1:3,1:3]<-inverse(phi[1:3,1:3])
} #end of model
```

2009:

```
model{
  for(i in 1:N){
```

```
  for (j in 1:16) {
    # geo location indicator
    L[i,j] ~ dcat(pi[i,j,1:2])
    # prior for mixture probability vector
    alpha[i,j,1] <- w1[i,j]
    alpha[i,j,2] <- w2[i,j]
    pi[i,j,1:2] ~ ddirch(alpha[i,j,1:2])
  }

  #measurement equation model
  for(j in 1:16){
    r[i,j] <- L[i,j]-1
    y1[i,j]~dnorm(mu[i,j],psi[j])I(thd[1,z1[i,j]],thd[1,z1[i,j]+1])
    y2[i,j]~dnorm(mu[i,j],psi[j])I(thd[1,z2[i,j]],thd[1,z2[i,j]+1])
    y[i,j] <- r[i,j]*y1[i,j] + (1-r[i,j])*y2[i,j]
    ephat[i,j]<-y[i,j]-mu[i,j]
  }
  for(j in 17:18){
    z1[i,j+3] ~ dbin(p[i,j], z1[i,19])
    p[i,j]<-ilogit(mu[i,j])
    ephat[i,j]<-z1[i,j+3]-p[i,j]*z1[i,19]
  }
  # three factors
  mu[i,1]<- xi[i,1]                                                     # Absenteeb
  mu[i,2]<- lam[1]*xi[i,1]  + lam[16]*xi[i,2] + lam[30]*xi[i,3] +c[1]  # Allegatio
  mu[i,3]<- lam[2]*xi[i,1]  + lam[17]*xi[i,2] + lam[31]*xi[i,3] +c[2]  # Ballotrel
  mu[i,4]<- lam[3]*xi[i,1]  + lam[18]*xi[i,2] + lam[32]*xi[i,3] +c[3]  # Countingo
  mu[i,5]<- lam[4]*xi[i,1]  + lam[19]*xi[i,2] + lam[33]*xi[i,3] +c[4]  # Criminals
  mu[i,6]<- lam[5]*xi[i,1]  + lam[20]*xi[i,2] + lam[34]*xi[i,3] +c[5]  # Disabilit
  mu[i,7]<- lam[6]*xi[i,1]  + xi[i,2]                                   # Electoral
  mu[i,8]<- lam[7]*xi[i,1]  + lam[21]*xi[i,2] + lam[35]*xi[i,3] +c[6]  # IDrelated
  mu[i,9]<- lam[8]*xi[i,1]  + lam[22]*xi[i,2] + lam[36]*xi[i,3] +c[7]  # ImproperC
  mu[i,10]<-lam[9]*xi[i,1]  + lam[23]*xi[i,2] + lam[37]*xi[i,3] +c[8]  # ImproperD
  mu[i,11]<-lam[10]*xi[i,1] + lam[24]*xi[i,2] + lam[38]*xi[i,3] +c[9]  # ImproperS
  mu[i,12]<-lam[11]*xi[i,1] + lam[25]*xi[i,2] + lam[39]*xi[i,3] +c[10] # PartyList
  mu[i,13]<-lam[12]*xi[i,1] + lam[26]*xi[i,2] + xi[i,3]                # Pollingpl
  mu[i,14]<-lam[13]*xi[i,1] + lam[27]*xi[i,2] + lam[40]*xi[i,3] +c[11] # Problemwi
  mu[i,15]<-lam[14]*xi[i,1] + lam[28]*xi[i,2] + lam[41]*xi[i,3] +c[12] # Registrat
  mu[i,16]<-lam[15]*xi[i,1] + lam[29]*xi[i,2] + lam[42]*xi[i,3] +c[13] # Unspecifi

  mu[i,17]<-lam[43]*xi[i,1] + lam[44]*xi[i,2] + lam[45]*xi[i,3] +c[14] # fi
  mu[i,18]<-lam[46]*xi[i,1] + lam[47]*xi[i,2] + lam[48]*xi[i,3] +c[15] # fe

  #structural equation model
  xi[i,1:3]~dmnorm(u[1:3],phi[1:3,1:3])
}# end of i
```

```
  #thresholds
  for(j in 1:16){
    thd[j,1]<-alpbot
    thd[j,2]<-alpmid
    thd[j,3]<-alptop
  }

  for(i in 1:3){u[i]<-gam[i]}

  #priors on loadings and coefficients
  var.lam[1]<-4.0*psi[3]     var.lam[2]<-4.0*psi[3]     var.lam[3]<-4.0*psi[3]
  var.lam[4]<-4.0*psi[3]     var.lam[5]<-4.0*psi[3]     var.lam[6]<-4.0*psi[6]
  var.lam[7]<-4.0*psi[7]     var.lam[8]<-4.0*psi[8]     var.lam[9]<-4.0*psi[9]
  var.lam[10]<-4.0*psi[10]   var.lam[11]<-4.0*psi[11]     var.lam[12]<-4.0*psi[11]
  var.lam[13]<-4.0*psi[10]   var.lam[14]<-4.0*psi[11]     var.lam[15]<-4.0*psi[11]
  var.lam[16]<-4.0*psi[10]   var.lam[17]<-4.0*psi[11]     var.lam[18]<-4.0*psi[11]
  var.lam[19]<-4.0*psi[10]   var.lam[20]<-4.0*psi[11]     var.lam[21]<-4.0*psi[11]
  var.lam[22]<-4.0*psi[10]   var.lam[23]<-4.0*psi[11]     var.lam[24]<-4.0*psi[11]
  var.lam[25]<-4.0*psi[10]   var.lam[26]<-4.0*psi[11]     var.lam[27]<-4.0*psi[11]
  var.lam[28]<-4.0*psi[10]   var.lam[29]<-4.0*psi[11]     var.lam[30]<-4.0*psi[11]
  var.lam[31]<-4.0*psi[10]   var.lam[32]<-4.0*psi[11]     var.lam[33]<-4.0*psi[11]
  var.lam[34]<-4.0*psi[10]   var.lam[35]<-4.0*psi[11]     var.lam[36]<-4.0*psi[11]
  var.lam[37]<-4.0*psi[10]   var.lam[38]<-4.0*psi[11]     var.lam[39]<-4.0*psi[11]
  var.lam[40]<-4.0*psi[10]   var.lam[41]<-4.0*psi[11]     var.lam[42]<-4.0*psi[11]
  var.lam[43]<-4.0*psi[10]   var.lam[44]<-4.0*psi[11]     var.lam[45]<-4.0*psi[11]
  var.lam[46]<-4.0*psi[10]   var.lam[47]<-4.0*psi[11]     var.lam[48]<-4.0*psi[11]
  for(i in 1:48){lam[i]~dnorm(0.8,var.lam[i])}

  var.b<-4.0*psi[1]
  for(j in 1:3){gam[j]~dnorm(0.1,var.b)}

  var.c<-4.0*psi[2]
    for(j in 1:15){c[j]~dnorm(0.1,var.c)}

  #priors on precisions
  for(j in 1:P){
    psi[j]~dgamma(10,8)
    sgm[j]<-1/psi[j]
  }

  phi[1:3,1:3]~dwish(R[1:3,1:3], 30)
  phx[1:3,1:3]<-inverse(phi[1:3,1:3])
} #end of model
```

6.2.2 Credible Intervals for Additional Model Parameters

*** Tables 6, 7, 8 about here ***

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Table 1: Frequency of Postelection Complaint Types, Germany 2005 & 2009

Type	Description	2005 I[a]	2005 II[b]	2009 I[a]	2009 II[b]
AbsenteeB	Absentee-ballot Related Problem	29	29	16	17
Electoral	Electoral System	67	68	54	54
PollingPl	Polling Place Problem	24	24	15	15
Allegatio	Allegations of Official Corruption	8	8	3	3
BallotRel	Ballot Related Problem	6	6	3	3
Countingo	Counting of the Votes	6	6	6	6
CriminalS	Criminal Status Related Problem	5	5	3	3
Disabilit	Disability Access Problem	2	2	2	2
IDrelated	Identification Related Problem	6	6	13	13
ImproperC	Improper Campaigning Influence	11	11	15	15
ImproperD	Improper District Boundaries	1	1	2	2
ImproperS	Improper Statistics	4	4	8	8
PartyList	Party List Not on Ballot	19	20	22	23
Problemwi	Problems with the Creation of Party Lists	2	2	9	9
Registrat	Registration Related Problem	20	20	5	5
Unspecifi	Unspecified Other	10	10	5	5
PoliceHar	Police Harassment	1	1	0	0
VoterInti	Voter Intimidation	1	1	0	0
Dortmund	*Briefwahl in Dortmund*	12	12	0	0
Dresden	*Nachwahl in Dresden*	35	36	0	0

Note: Number of districts that have each type of complaint. [a] minimum number of districts with at least one complaint. [b] maximum number of districts with at least one complaint. Source: Compiled from archives of the Bundestag's website for the *Ausschuss für Wahlprüfung, Immunität und Geschäftsordnung* (see the Appendix). Types are described in the Appendix.

Table 2: Three-LV Model Factor Loadings, Latent Variable 1 and 2, Germany 2005

Manifest	Latent Variable 1				Latent Variable 2			
Variable	load.	lower[a]	mean[b]	upper[c]	load.	lower[a]	mean[b]	upper[c]
AbsenteeB	λ_{101}		1.0[d]					
Electoral	λ_{102}	0.398	0.7858	1.197	λ_{202}		1.0[d]	
PollingPl	λ_{103}	1.251	1.96	2.876	λ_{203}	−0.9253	−0.2523	0.3069
Allegatio	λ_{104}	1.228	2.24	3.48	λ_{204}	−0.1627	1.033	2.273
BallotRel	λ_{105}	0.4074	0.9417	1.687	λ_{205}	−0.7232	−0.03861	0.5509
Countingo	λ_{106}	−0.1417	0.2312	0.6767	λ_{206}	0.09148	0.4675	1.018
CriminalS	λ_{107}	0.798	1.572	2.562	λ_{207}	−1.505	−0.61	0.1095
Disabilit	λ_{108}	−0.07126	0.5381	1.208	λ_{208}	−0.6835	0.1612	1.004
IDrelated	λ_{109}	0.4046	0.839	1.321	λ_{209}	−0.9857	−0.3704	0.1367
ImproperC	λ_{110}	−0.3019	0.1948	0.6724	λ_{210}	0.2681	0.801	2.082
ImproperD	λ_{111}	0.735	1.531	2.522	λ_{211}	−1.198	−0.1146	0.9107
ImproperS	λ_{112}	0.3172	1.167	2.186	λ_{212}	0.1964	1.017	2.012
PartyList	λ_{113}	0.2327	0.5853	1.068	λ_{213}	0.2941	0.6268	1.114
PoliceHar	λ_{114}	−0.5507	0.287	1.242	λ_{214}	0.01936	0.893	2.21
Problemwi	λ_{115}	−1.135	−0.03143	1.152	λ_{215}	−2.304	−1.248	−0.2871
Registrat	λ_{116}	0.9576	2.037	3.466	λ_{216}	−1.329	−0.4515	0.2344
Unspecifi	λ_{117}	−1.208	−0.447	0.1728	λ_{217}	0.1099	0.7877	1.653
VoterInti	λ_{118}	−0.9039	0.0514	1.084	λ_{218}	−0.08609	0.8687	2.067
Dortmund	λ_{119}	1.295	2.469	4.116	λ_{219}	−0.008692	0.6273	1.382
Dresden	λ_{120}	1.303	2.386	3.645	λ_{220}	1.639	2.735	4.066
f_i	λ_{121}	−1.412	−0.274	0.8154	λ_{221}	−0.195	0.9611	2.152
f_e	λ_{122}	−0.7537	0.204	1.256	λ_{222}	0.2054	1.169	2.251

Note: $n = 299$. [a] 95% credible interval lower bound, [b] posterior mean, [c] 95% credible interval upper bound.
[d] fixed parameter. Parameters not shown with a value are fixed at zero.

Table 3: Three-LV Model Factor Loadings, Latent Variable 3, Germany 2005

Manifest		Latent Variable 3		
Variable	load.	lower^a	mean^b	upper^c
PollingPl	λ_{303}		1.0^d	
Allegatio	λ_{304}	-1.733	0.1121	1.869
BallotRel	λ_{305}	0.1352	1.026	2.235
Countingo	λ_{306}	-0.5995	0.016	0.7214
CriminalS	λ_{307}	-2.25	-1.052	-0.02117
Disabilit	λ_{308}	0.3623	1.451	2.853
IDrelated	λ_{309}	-2.092	-1.04	-0.2089
ImproperC	λ_{310}	-0.8385	0.09085	0.9273
ImproperD	λ_{311}	-0.9085	0.6667	2.365
ImproperS	λ_{312}	-0.8556	0.3414	1.591
PartyList	λ_{313}	-0.2578	0.2766	0.8819
PoliceHar	λ_{314}	-1.707	-0.2821	1.138
Problemwi	λ_{315}	-0.766	0.6763	2.152
Registrat	λ_{316}	0.4688	1.705	3.27
Unspecifi	λ_{317}	-3.332	-1.764	-0.1747
VoterInti	λ_{318}	0.1481	1.431	2.873
Dortmund	λ_{319}	0.3269	1.284	2.613
Dresden	λ_{320}	-0.5055	0.4277	1.461
f_i	λ_{321}	3.145	4.228	5.421
f_e	λ_{322}	0.003167	1.52	3.182

Note: $n = 299$. ^{a} 95% credible interval lower bound, ^{b} posterior mean, ^{c} 95% credible interval upper bound.
^{d} fixed parameter. Parameters not shown with a value are fixed at zero.

Table 4: Three-LV Model Factor Loadings, Latent Variable 1 and 2, Germany 2009

Manifest	Latent Variable 1				Latent Variable 2			
Variable	load.	lower[a]	mean[b]	upper[c]	load.	lower[a]	mean[b]	upper[c]
AbsenteeB	λ_{101}		1.0[d]					
Electoral	λ_{102}	−0.0455	0.6725	1.317	λ_{202}		1.0[d]	
PollingPl	λ_{103}	0.4865	1.199	2.846	λ_{203}	0.1013	0.8817	1.968
Allegatio	λ_{104}	0.5988	1.369	2.273	λ_{204}	−0.6635	0.4519	1.614
BallotRel	λ_{105}	0.5707	1.405	2.465	λ_{205}	0.07889	1.284	2.944
Countingo	λ_{106}	0.6089	1.353	2.152	λ_{206}	−0.4789	0.4752	1.884
CriminalS	λ_{107}	0.04302	0.7411	1.55	λ_{207}	−0.924	−0.02626	0.9115
Disabilit	λ_{108}	−0.155	0.7503	1.606	λ_{208}	−0.5423	0.6647	2.11
IDrelated	λ_{109}	0.1878	0.8767	1.558	λ_{209}	−1.592	−0.0373	1.694
ImproperC	λ_{110}	0.5223	1.048	1.644	λ_{210}	0.2852	0.9733	1.944
ImproperD	λ_{111}	0.1289	0.7959	1.547	λ_{211}	−0.5631	0.2945	1.188
ImproperS	λ_{112}	0.383	1.314	2.372	λ_{212}	0.1996	1.351	3.049
PartyList	λ_{113}	0.7409	2.197	5.278	λ_{213}	−0.2907	0.5515	1.439
Problemwi	λ_{114}	−1.235	−0.3591	0.5254	λ_{214}	0.3477	1.781	4.037
Registrat	λ_{115}	0.5927	1.541	3.151	λ_{215}	−0.09864	0.5939	1.419
Unspecifi	λ_{116}	0.06446	1.096	2.415	λ_{216}	0.4571	1.557	3.249
f_i	λ_{117}	0.4687	1.624	2.831	λ_{217}	−4.115	−1.965	−0.1201
f_e	λ_{118}	0.7461	1.979	3.48	λ_{218}	−2.096	−0.2933	1.204

Note: $n = 299$. [a] 95% credible interval lower bound, [b] posterior mean, [c] 95% credible interval upper bound.
[d] fixed parameter. Parameters not shown with a value are fixed at zero.

Table 5: Three-LV Model Factor Loadings, Latent Variable 3, Germany 2009

Manifest Variable	load.	Latent Variable 3 lower[a]	mean[b]	upper[c]
PollingPl	λ_{305}		1.0^d	
Allegatio	λ_{306}	-1.081	0.8721	2.792
BallotRel	λ_{307}	-0.4481	0.561	1.515
Countingo	λ_{308}	-1.564	0.3782	2.189
CriminalS	λ_{309}	-2.403	0.7121	3.278
Disabilit	λ_{310}	-0.7154	0.7446	2.636
IDrelated	λ_{311}	-3.36	1.356	4.886
ImproperC	λ_{312}	-0.1648	0.8939	2.006
ImproperD	λ_{313}	-0.4866	0.3828	1.378
ImproperS	λ_{314}	-0.328	1.255	4.116
PartyList	λ_{315}	-3.012	0.2951	2.48
Problemwi	λ_{316}	-1.14	1.332	3.019
Registrat	λ_{317}	-0.6204	0.4382	1.609
Unspecifi	λ_{318}	-0.3587	0.8552	3.189
f_i	λ_{319}	-3.362	-1.046	3.128
f_e	λ_{320}	-1.092	0.9431	2.826

Note: $n = 299$. [a] 95% credible interval lower bound, [b] posterior mean, [c] 95% credible interval upper bound.
[d] fixed parameter. Parameters not shown with a value are fixed at zero.

Table 6: Latent Variable Model Latent Variable Covariance Matrix Posterior Means

$$\mathbf{\Phi}_{2005} = \begin{bmatrix} 1.528 & 0.146 & -0.2545 \\ 0.146 & 1.633 & -0.1508 \\ -0.2545 & -0.1508 & 0.4431 \end{bmatrix}, \quad \mathbf{\Phi}_{2009} = \begin{bmatrix} 0.665 & 0.09678 & 0.01068 \\ 0.09678 & 0.9038 & -0.1178 \\ 0.01068 & -0.1178 & 0.5012 \end{bmatrix}$$

Note: $\mathbf{\Phi} = \mathbf{\Upsilon}^{-1}$. $n = 299$.

Table 7: Three-LV Model Means, Germany 2005

variable	mean	lower[a]	mean[b]	upper[c]
latent variable 1	γ_1	−2.305	−1.823	−1.46
latent variable 2	γ_2	−0.6696	−0.02793	0.5334
latent variable 3	γ_3	−0.8201	−0.2236	0.3927
Allegatio	c_4	−11.16	−7.725	−4.879
BallotRel	c_5	−2.667	−1.537	−0.642
Countingo	c_6	−3.312	−2.144	−1.3
CriminalS	c_7	−4.501	−2.697	−1.406
Disabilit	c_8	−3.738	−2.186	−0.9442
IDrelated	c_9	−4.001	−2.282	−1.196
ImproperC	c_{10}	−4.852	−2.362	−1.189
ImproperD	c_{11}	−5.902	−4.002	−2.409
ImproperS	c_{12}	−6.048	−4.062	−2.497
PartyList	c_{13}	−1.895	−1.136	−0.5309
PoliceHar	c_{14}	−7.488	−3.973	−1.925
Problemwi	c_{15}	−8.086	−5.192	−2.765
Registrat	c_{16}	−0.7122	0.12	1.15
Unspecifi	c_{17}	−7.403	−4.556	−2.02
VoterInti	c_{18}	−7.415	−4.259	−1.849
Dortmund	c_{19}	−1.557	−0.6932	0.2987
Dresden	c_{20}	−2.801	−1.538	−0.541
f_i	c_{21}	−8.447	−5.652	−2.417
f_e	c_{22}	−13.43	−10.64	−8.026

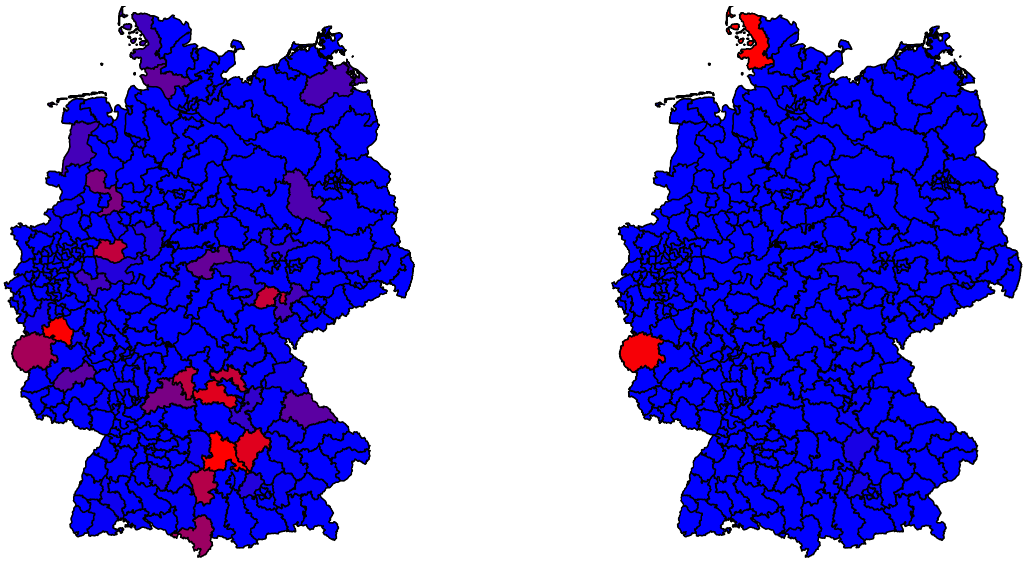
Note: $n = 299$. [a] 95% credible interval lower bound, [b] posterior mean, [c] 95% credible interval upper bound.
Parameters not shown with a value are fixed at zero.

Table 8: Three-LV Model Means, Germany 2009

variable	mean	lower[a]	mean[b]	upper[c]
latent variable 1	γ_1	−3.02	−2.265	−1.643
latent variable 2	γ_2	−1.327	0.1803	1.585
latent variable 3	γ_3	−1.597	−0.6742	0.6631
Allegatio	c_3	−9.581	−4.369	−0.3958
BallotRel	c_4	−4.209	−1.218	0.6225
Countingo	c_5	−2.717	0.2503	1.832
CriminalS	c_6	−5.353	−1.939	0.8141
Disabilit	c_7	−3.977	−1.544	0.4415
IDrelated	c_8	−5.87	−1.677	1.931
ImproperC	c_9	−1.399	0.06384	1.603
ImproperD	c_{10}	−3.653	−1.765	−0.1191
ImproperS	c_{11}	−4.503	−1.912	0.2624
PartyList	c_{12}	−1.199	1.521	3.892
Problemwi	c_{13}	−7.421	−4.323	−1.962
Registrat	c_{14}	−1.654	−0.05799	1.606
Unspecifi	c_{15}	−3.676	−1.578	0.2246
f_i	c_{16}	−3.739	−0.872	2.255
f_e	c_{17}	−8.733	−5.735	−1.998

Note: $n = 299$. [a] 95% credible interval lower bound, [b] posterior mean, [c] 95% credible interval upper bound.
Parameters not shown with a value are fixed at zero.

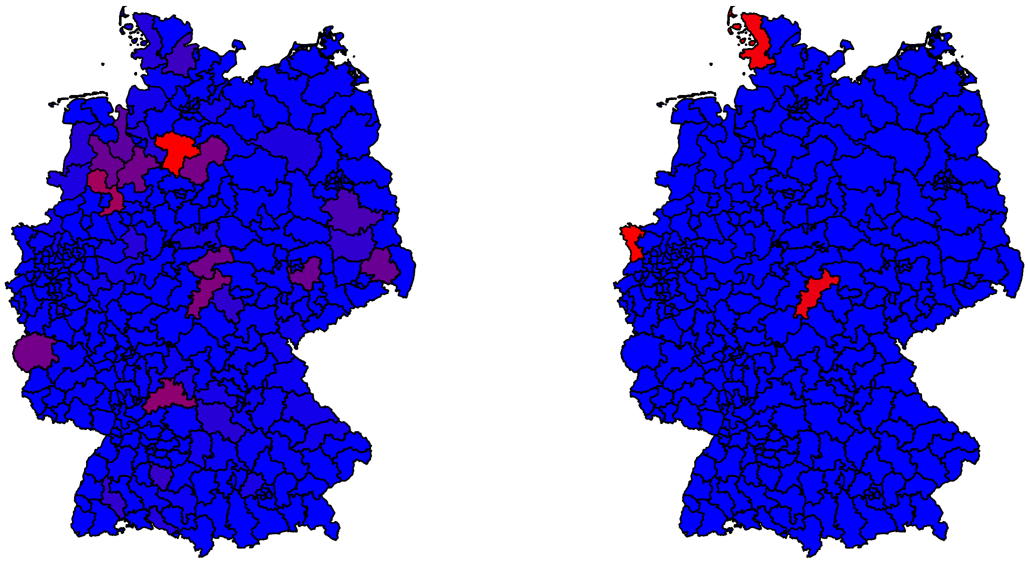
Figure 1: "Fraud" Probabilities, by District, Germany 2005 Bundestag Erststimmen



(a) Incremental (f_{i}) (b) Extreme (f_{e})

Note: $f_{\mathrm{i}i}$ and $\hat{f}_{\mathrm{e}i}$ values estimated using a finite mixture variant of the Klimek et al. (2012) model separately for each for each district, $i = 1, \ldots, 299$, using polling center data. Color red means $\hat{f}_{\mathrm{i}i}/\max(\hat{f}_{\mathrm{i}i}) = 1$ or $\hat{f}_{\mathrm{e}i}/\max(\hat{f}_{\mathrm{e}i}) = 1$, color blue means $\hat{f}_{\mathrm{i}i} = 0$ or $\hat{f}_{\mathrm{e}i} = 0$, and intermediate values have colors that are weighted mixtures of red and blue.
(a) $\max(\hat{f}_{\mathrm{i}i}) = 0.74$. (b) $\max(\hat{f}_{\mathrm{e}i}) = 0.004659684$.

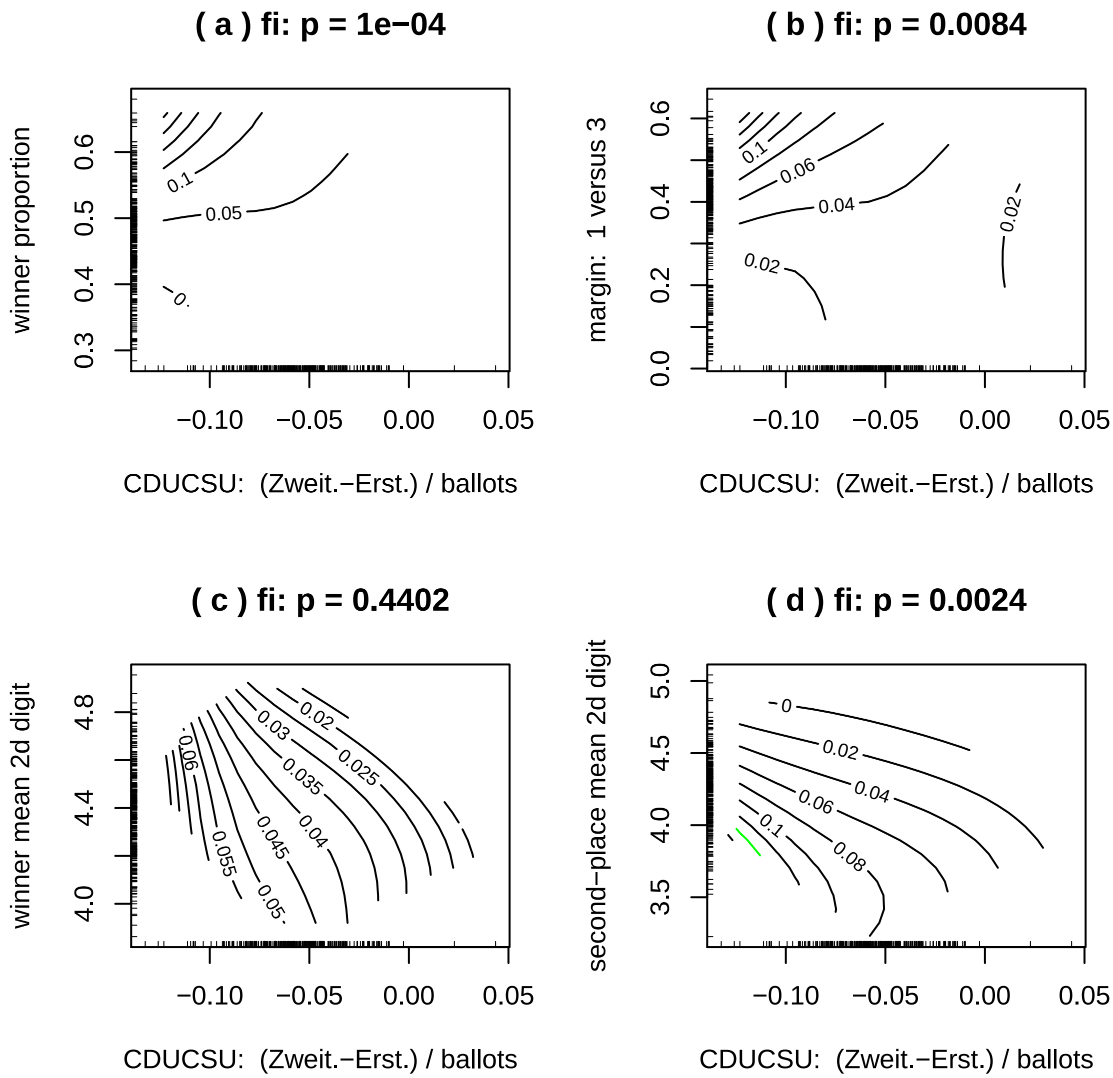
Figure 2: "Fraud" Probabilities, by District, Germany 2009 Bundestag Erststimmen



(a) Incremental (f_{i}) (b) Extreme (f_{e})

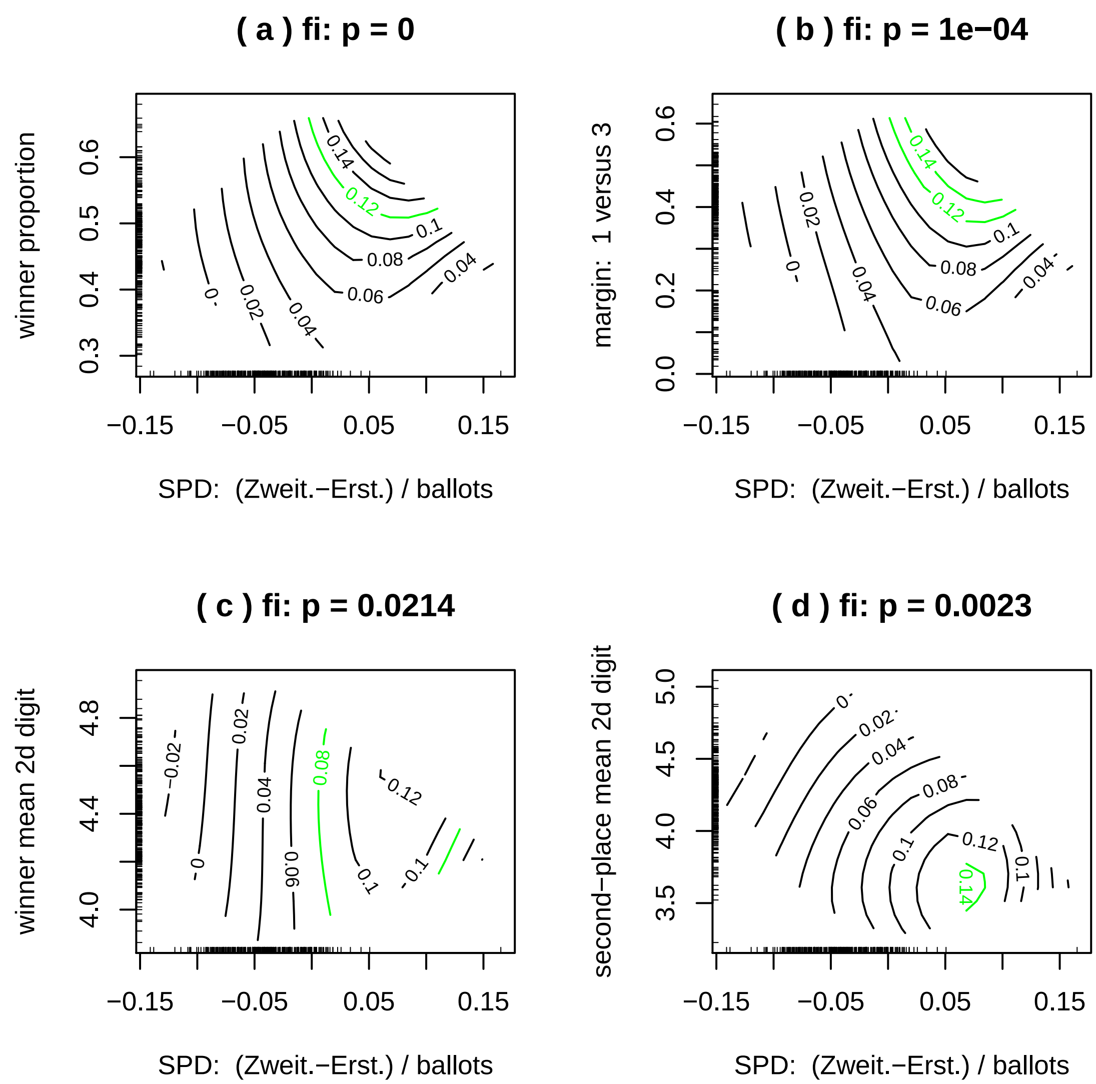
Note: $f_{\mathrm{i}i}$ and $\hat{f}_{\mathrm{e}i}$ values estimated using a finite mixture variant of the Klimek et al. (2012) model separately for each for each district, $i = 1, \ldots, 299$, using polling center data. Color red means $\hat{f}_{\mathrm{i}i}/\max(\hat{f}_{\mathrm{i}i}) = 1$ or $\hat{f}_{\mathrm{e}i}/\max(\hat{f}_{\mathrm{e}i}) = 1$, color blue means $\hat{f}_{\mathrm{i}i} = 0$ or $\hat{f}_{\mathrm{e}i} = 0$, and intermediate values have colors that are weighted mixtures of red and blue.
(a) $\max(\hat{f}_{\mathrm{i}i}) = 0.9006814$. (b) $\max(\hat{f}_{\mathrm{e}i}) = 0.003659332$.

Figure 3: Incremental Fraud Probabilities by Strategic Margins, Germany 2005 Bundestag Erststimmen, CDU-CSU



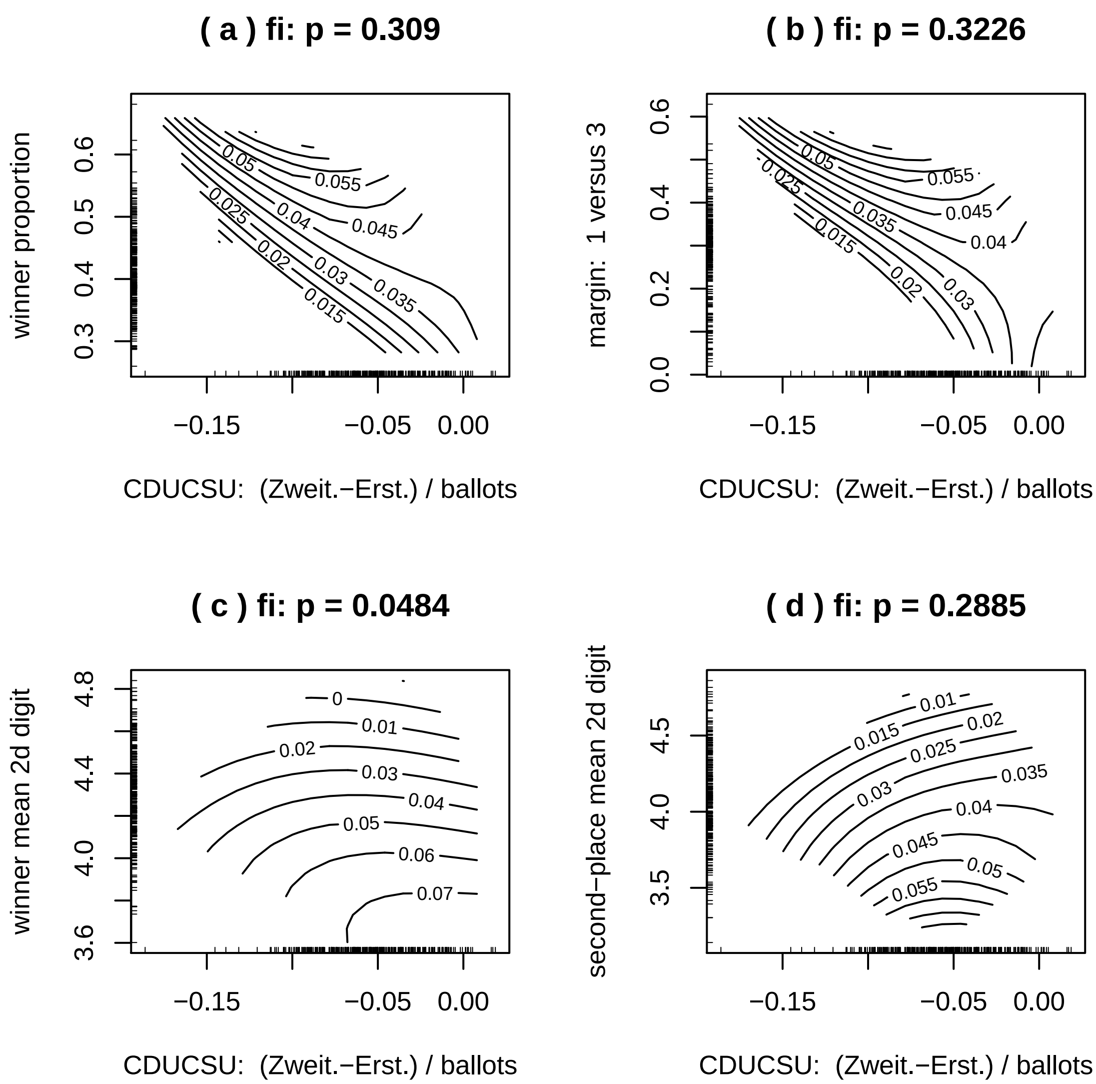
Note: nonparametric regression contours for $\hat{f}_{\text{i}i}$. "[party]: (Zweit.-Erst.)/ballots" is the total of *Zweitstimmen* cast for [party] minus the number of *Erststimmen* cast for [party] divided by the total number of ballots used in the district. "margin: 1 versus 3" is the number of *Erststimmen* for the winning party in each district minus the number of votes for the third-place party divided by the total of *Erststimmen* cast in the district. Rug plots show locations of district values. p in each subfigure heading reports the p-value for a significance test versus the model of no effects.

Figure 4: Incremental Fraud Probabilities by Strategic Margins, Germany 2005 Bundestag Erststimmen, SPD



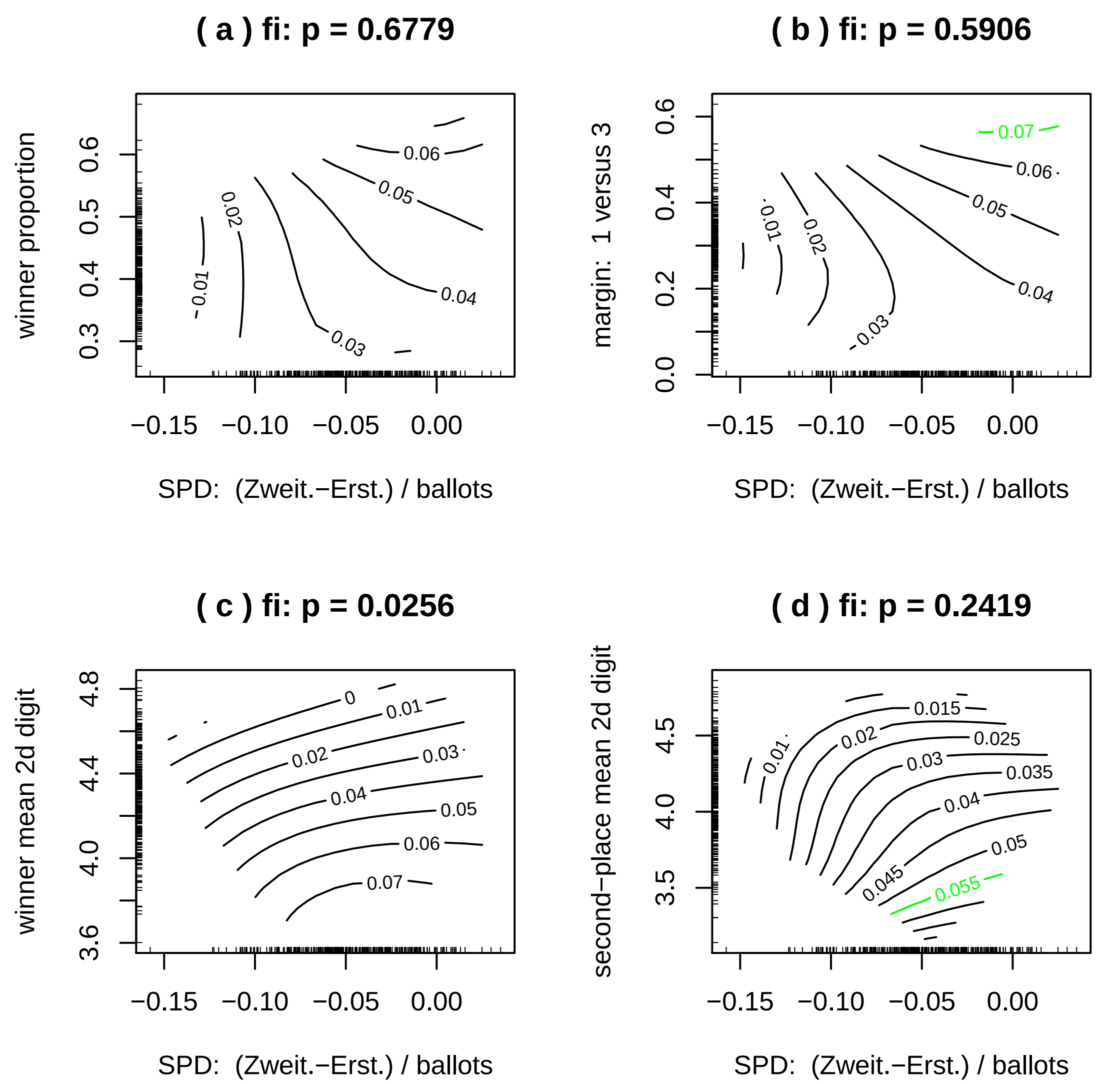
Note: nonparametric regression contours for $\hat{f}_{\mathrm{i}i}$. "[party]: (Zweit.-Erst.)/ballots" is the total of *Zweitstimmen* cast for [party] minus the number of *Erststimmen* cast for [party] divided by the total number of ballots used in the district. "margin: 1 versus 3" is the number of *Erststimmen* for the winning party in each district minus the number of votes for the third-place party divided by the total of *Erststimmen* cast in the district. Rug plots show locations of district values. p in each subfigure heading reports the p-value for a significance test versus the model of no effects.

Figure 5: Incremental Fraud Probabilities by Strategic Margins, Germany 2009 Bundestag Erststimmen, CDU



Note: nonparametric regression contours for $\hat{f}_{ei}$. "[party]: (Zweit.-Erst.)/ballots" is the total of *Zweitstimmen* cast for [party] minus the number of *Erststimmen* cast for [party] divided by the total number of ballots used in the district. "margin: 1 versus 3" is the number of *Erststimmen* for the winning party in each district minus the number of votes for the third-place party divided by the total of *Erststimmen* cast in the district. Rug plots show locations of district values. p in each subfigure heading reports the p-value for a significance test versus the model of no effects.

Figure 6: Incremental Fraud Probabilities by Strategic Margins, Germany 2009 Bundestag Erststimmen, SPD



Note: nonparametric regression contours for $\hat{f}_{ei}$. "[party]: (Zweit.-Erst.)/ballots" is the total of *Zweitstimmen* cast for [party] minus the number of *Erststimmen* cast for [party] divided by the total number of ballots used in the district. "margin: 1 versus 3" is the number of *Erststimmen* for the winning party in each district minus the number of votes for the third-place party divided by the total of *Erststimmen* cast in the district. Rug plots show locations of district values. p in each subfigure heading reports the p-value for a significance test versus the model of no effects.